Nonlinear decay of reversed shear Alfvén eigenmode and core-localized alpha-channeling

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3rd trilateral workshop on EP physics
Nov. 7 - 11, 2022

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Outline

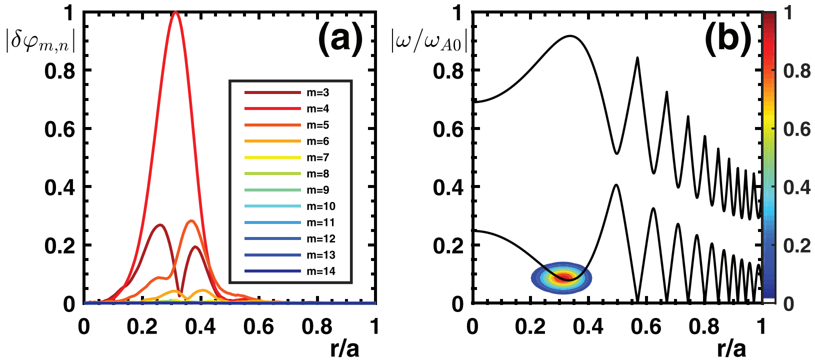
- Introduction: energetic particles (EP) and Alfvén instabilities in fusion plasmas
- General equation for SAW nonlinear coupling
- Nonlinear decay of reversed shear Alfvén eigenmode (RSAE)
- Nonlinear saturation and core-localized alpha-channeling
- Summary and Discussions

Energetic particles and Alfvén eigenmodes in fusion plasmas

- Shear Alfvén wave instabilities (SAW) excited by energetic particles (fusion-α) is an important issue in burning plasmas [Chen RMP16]: complementary- heating (alpha-channeling) + anomalous EP transport
- SAW induced EP transport rate depends on SAW amplitude & spectrum (diffusive/convective) $\Leftarrow$ SAW nonlinear saturation mechanisms
- Nonlinear mode coupling [Chen PoP13] is an important route for SAW instability nonlinear saturation in reactors with $a/\rho_h \gtrsim O(10)$: multiple-modes co-exist
- Toroidal Alfvén eigenmode (TAE) extensively studied as paradigm case for nonlinear mode coupling [Hahm&Chen 95, Chen&Zonca&Qiu 95 -]
- Reversed shear Alfvén eigenmode (RSAE) related physics expected to be crucial in reactors: reversed shear configuration + core-localized fusion-α generation

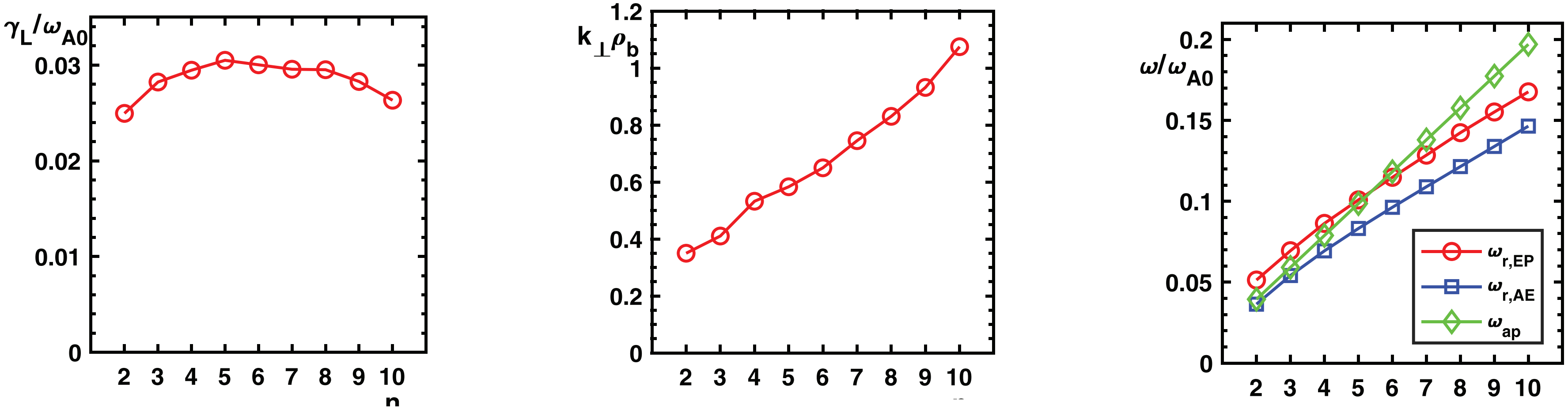
RSAE characterized by $n \gtrsim O(10)$ in reactors [Wang PoP18]

- RSAEs preferentially excited in tokamak center with (weakly) reversed shear



- DTT equilibrium $\rho_h/a \sim \mathcal{O}(10^{-2})$
- ⇐ single-n simulation with n=4
- dominated by few poloidal harmonics
- dominant drive from precession resonance

- Multiple-RSAEs co-exist with comparable linear growth rates, $k_\perp \rho_h \sim O(1)$ and frequency sensitively depend on n via $k_\parallel \propto (nq_{min} - m)$ ($q_{min} = 1.02$)



- Nonlinear mode coupling expected to play important role in RSAE saturation [Wei&Qiu 2021-].

This work: parametric decay of RSAE

- Parametric decay of RSAE into another RSAE and low frequency Alfvén mode (LFAM, e.g., BAE)
  - rich spectrum of RSAE/kRSAE (linearly stable/unstable): multiple-RSAEs excited with comparable growth rates + different frequencies
  - radially overlapped mode structures
  - matching condition easily satisfied due to dense RSAE spectrum

- Secondary LFAM with $\omega \lesssim O(v_i/(qR_0))$ excited at q_{min}:
  - $\omega \lesssim O(v_i/(qR_0))$: ion heating via Landau damping
  - narrow LFAM radial structure $\delta r \sim \sqrt{\beta}/(nq')$: core-localized heating

  $\Rightarrow$ improved fusion performance

Theoretical model

- Field variables $\delta\phi$ and $\delta\psi \equiv \omega\delta A_\parallel/(ck_\parallel)$ ($\Rightarrow \delta B_\perp$) used
- Mode equations derived from nonlinear gyrokinetic vorticity equation

$$\frac{c^2}{4\pi\omega^2} B \frac{\partial}{\partial l}\frac{k_\perp^2}{B}\frac{\partial}{\partial l}\delta\psi_k + \frac{e^2}{T_i}\langle(1-J_k^2)F_0\rangle\delta\phi_k - \sum_s\left\langle\frac{q}{\omega}J_k\omega_d\delta H_s\right\rangle$$

$$= \quad -\frac{i}{\omega}\sum_{\mathbf{k}=\mathbf{k}''+\mathbf{k}'}\underbrace{\frac{c}{B_0}\hat{\mathbf{b}}\cdot\mathbf{k}''\times\mathbf{k}'}_{\Lambda^k_{k'',k'}}\left[\langle(J_kJ_{k'}-J_{k''})\delta L_{k'}\delta H_{k''}\rangle + \frac{c^2k''^2_\perp}{4\pi}\frac{\partial_l\delta\psi_{k'}\partial_l\delta\psi_{k''}}{\omega_{k'}\omega_{k''}}\right]$$

  - derived from ∥-Ampere's law, QN condition and NL GKE
  - LHS: field line bending, inertia, ballooning-interchange
  - RHS: Reynolds stress (RS, $\delta\mathbf{v}\cdot\nabla\delta\mathbf{v}$), Maxwell stress (MX, $\delta\mathbf{j}\times\delta\mathbf{B}/c$).

- ... and quasi-neutrality condition:

$$\frac{n_0e^2}{T_i}\left(1+\frac{T_i}{T_e}\right)\delta\phi_k = \sum_s\langle q\delta H_s\rangle_k$$

- Nonadiabatic response δH derived from NL gyrokinetic equation

General equation for SAW nonlinear coupling

- Considering two SAWs, $\mathbf{\Omega}_1 \equiv \mathbf{\Omega}_1(\omega_1, \mathbf{k}_1)$ and $\mathbf{\Omega}_2 \equiv \mathbf{\Omega}_2(\omega_2, \mathbf{k}_2)$ coupling and generating a third SAW $\mathbf{\Omega}_3 \equiv \mathbf{\Omega}_3(\omega_3, \mathbf{k}_3)$: $\mathbf{\Omega}_1 + \mathbf{\Omega}_2 \to \mathbf{\Omega}_3$
- The first equation of $\mathbf{\Omega}_3$ can be derived from nonlinear vorticity equation

$$\frac{n_0 e^2}{T_i} b_{k_3}\left(-\frac{k_{\parallel,3}^2 V_A^2}{\omega_3^2}\delta\psi_{k_3} + \delta\phi_{k_3} - \frac{\omega_G^2}{\omega_3^2}\delta\phi_{k_3}\right)$$
$$\simeq \quad -\frac{i}{\omega_3}\frac{n_0 e^2}{T_i}\Lambda^{k_3}_{k_2,k_1}(b_{k_2} - b_{k_1})\left(1 - \frac{k_{\parallel,1}k_{\parallel,2}V_A^2}{\omega_1\omega_2}\right)\delta\phi_{k_1}\delta\phi_{k_2}. \tag{1}$$

- The other equation from Q.N. condition:

$$\Rightarrow \quad \delta\phi_{k_3} - \delta\psi_{k_3} = -i\Lambda^{k_3}_{k_2,k_1}\frac{1}{k_{\parallel,3}}\left(\frac{k_{\parallel,1}}{\omega_1} - \frac{k_{\parallel,2}}{\omega_2}\right)\delta\phi_{k_1}\delta\phi_{k_2}. \tag{2}$$

□ Substituting (2) into (1)

$$b_{k_3}\mathcal{E}_{k_3}\delta\phi_{k_3} = -\frac{i}{\omega_3}\Lambda^{k_3}_{k_2,k_1}\underbrace{\left[(b_{k_2}-b_{k_1})\left(1-\frac{k_{\parallel,1}k_{\parallel,2}V_A^2}{\omega_1\omega_2}\right)+b_{k_3}V_A^2\frac{k_{\parallel,3}}{\omega_3}\left(\frac{k_{\parallel,1}}{\omega_1}-\frac{k_{\parallel,2}}{\omega_2}\right)\right]}_{\alpha_{k_3}}\delta\phi_{k_1}\delta\phi_{k_2}.$$

□ General equation for SAW nonlinear interaction in torus $\mathbf{\Omega}_1+\mathbf{\Omega}_2\to\mathbf{\Omega}_3$:

- SAW DR naturally satisfied: $(\omega_1+\omega_2)\simeq(k_{\parallel,1}+k_{\parallel,2})V_A$

⇒ $\mathbf{\Omega}_3$ strongly excited if it is a normal mode ⇒ successive & significant spectral cascading

- not easy to satisfy: limited choice of $k_\parallel$ due to the periodicity of torus
- counter-part of "Hasegawa-Mima equation" of SAW in torus

- analogous to electron-DW with $\omega\simeq k_y v_*/(1+k_\perp^2\rho_s^2)$

- generalization to KAW with $\delta E_\parallel$ due to FLR effects [Chen EPL11]

⇒ $\omega/k_\perp$ cascading of KAW in solar wind? [Chen&Hasegawa 2021 ?]

Parametric decay of RSAE

- Considering the spontaneous decay of a pump RSAE $\mathbf{\Omega}_0$ into RSAE sideband $\mathbf{\Omega}_1$ and a LFAM $\mathbf{\Omega}_B$
  - $\mathbf{\Omega}_0 = \mathbf{\Omega}_1 + \mathbf{\Omega}_B$ assumed as matching condition
  - $\omega_B \lesssim O(v_i/(qR_0))$ for effective alpha-channeling $\Rightarrow \omega_B \ll \omega_0, \omega_1$, $k_{\parallel,B} \simeq 0$
  - LFAM in reversed shear region [Ma PPCF2022]

- Coupled nonlinear RSAE sideband and LFAM equations:

$$b_1\mathcal{E}_1\delta\phi_1 = -\frac{i}{\omega_1}\Lambda^{k_1}_{k_0,k_{B^*}}\alpha_1\delta\phi_0\delta\phi_{B^*}, \quad (3)$$

$$b_B\mathcal{E}_B\delta\phi_B = -\frac{i}{\omega_B}\Lambda^{k_B}_{k_0,k_{1^*}}\alpha_B\delta\phi_0\delta\phi_{1^*} \quad (4)$$

- Parametric dispersion relation for RSAE $\mathbf{\Omega}_0$ spontaneous decay:

$$\mathcal{E}_1\mathcal{E}_{B^*} = \frac{\Lambda^{k_1}_{k_0,k_{B^*}}\Lambda^{k_B}_{k_0,k_{1^*}}}{b_B b_1\omega_B\omega_1}\underbrace{\alpha_1\alpha_B}_{\alpha_N}|\delta\phi_0|^2 \quad (5)$$

- Expanding $\mathcal{E}_1 \simeq i\partial_{\omega_1}\mathcal{E}_{1,\mathcal{R}}(\partial_t + \gamma_1) \simeq (2i/\omega_1)(\gamma + \gamma_1)$ and $\mathcal{E}_{B^*} \simeq (-2i/\omega_B)(\gamma + \gamma_B)$, with $\gamma_1 \equiv -\mathcal{E}_{1,\mathcal{I}}/\partial_{\omega_1}\mathcal{E}_{1,\mathcal{R}} \Rightarrow$

$$(\gamma + \gamma_1)(\gamma + \gamma_B) = \frac{(\Lambda^{k_1}_{k_0,k_{B^*}})^2}{4b_B b_1}\alpha_N|\delta\phi_0|^2. \tag{6}$$

- Condition for spontaneous decay ($\gamma > 0$):

$$\alpha_N > 0 \quad \Rightarrow \quad \text{scattering direction} \tag{7}$$

$$\frac{(\Lambda^{k_1}_{k_0,k_{B^*}})^2}{4b_B b_1}\alpha_N|\delta\phi_0|^2 > \gamma_B\gamma_1 \quad \Rightarrow \quad \text{threshold on } \delta\phi_0 \tag{8}$$

- Expression of α_N in the simplified $k_{\parallel,B}V_A \ll \omega_B$ limit:

$$\alpha_N \simeq (b_0 - b_1)(b_0 - b_B - b_1)\left(1 - \frac{k_{\parallel,1}k_{\parallel,0}V_A^2}{\omega_0\omega_1}\right). \tag{9}$$

  - $1 - k_{\parallel,1}k_{\parallel,0}V_A^2/(\omega_0\omega_1)$: determined by q_{min} and n_0/n_1, i.e., below/above SAW CAP minimum/maximum $\Rightarrow$ $\omega/k_\parallel$ scattering
  - $(b_0 - b_1)(b_0 - b_B - b_1) = \rho_i^4(k^2_{\perp,0} - k^2_{\perp,1})(\mathbf{k}_{\perp,0}\cdot\mathbf{k}_{\perp,1} - k^2_{\perp,1})$: $k_\perp$ scattering

- Condition for maximized cross-section $(\Lambda^{k_1}_{k_0,k_{B^*}})^2 \alpha_N |\delta\phi_0|^2/(4b_B b_1)$:
  - $\Lambda^{k_1}_{k_0,k_{B^*}} \propto \mathbf{b} \cdot \mathbf{k}_{\perp,0} \times \mathbf{k}_{\perp,1}$: maximized as $\mathbf{k}_{\perp,0} \perp \mathbf{k}_{\perp,1}$
  - inertial layer with $k_{r,0} \gg k_{\theta,0} \Rightarrow$ scattering to large $k_{\theta,1}$ (high-n_1) region

  $\Rightarrow$ two potential parameter regimes for $\alpha_N > 0$

- First parameter regime: strong scattering to high-n_1 with $|k_{\perp,1} \gg k_{\perp,0}|$
  - $(b_0 - b_1)(b_0 - b_B - b_1) > 0$ as $b_B \simeq b_1 \gg b_0$
  - $1 - k_{\parallel,1}k_{\parallel,0}V_A^2/(\omega_0\omega_1) > 0$: $\mathbf{\Omega}_1$ excited above local SAW continuum maximum with $-1/2 < n_1 q_{min} - m_1 < 0$ (sufficient, not necessary)

- Second regime: scattering to moderate n_1
  - $(b_0 - b_1)(b_0 - b_B - b_1) < 0$ as $b_0 > b_1$ while $\mathbf{k}_{\perp,1} \cdot \mathbf{k}_{\perp,0} < k^2_{\perp,1}$
  - $1 - k_{\parallel,1}k_{\parallel,0}V_A^2/(\omega_0\omega_1) < 0$: $\mathbf{\Omega}_1$ excited below local SAW continuum minimum with $0 < n_1 q_{min} - m_1 < 1/2$

- Scattering to high-n_1 preferred: nonlinear drive $(\Lambda^{k_1}_{k_0,k_{B^*}})^2/(4b_B b_1))\alpha_N|\delta\phi_0|^2 \propto b_1$ for $b_1 \gg b_0$. Upper limit of n_1 by $\gamma_1(n_1)$: threshold due to $\mathbf{\Omega}_1$ damping

- Threshold on pump RSAE amplitude ($b_1 \gg b_0$)

$$|\delta\phi_0|^2 > \frac{4b_B b_1 \gamma_B \gamma_1}{\alpha_N (\Lambda^{k_1}_{k_0,k_{B^*}})^2} \Rightarrow \left|\frac{\delta B_\perp}{B_0}\right|^2 > \frac{4\gamma_1\gamma_B}{\omega_0\omega_1}\frac{k^2_{\parallel,0}}{k^2_{\perp,1}}\frac{1}{1-k_{\parallel,0}k_{\parallel,1}V_A^2/(\omega_0\omega_1)} \sim O(10^{-8})$$

rough estimation with $\gamma/\omega_0 \sim 10^{-2}$, $k_\parallel/k_\perp \sim 10^{-3}$, $1-k_{\parallel,0}k_{\parallel,1}V_A^2/(\omega_0\omega_1) \sim 10^{-2}$

- LFAM saturation level derived from coupled NL equations as

$$|\delta\phi_B|^2 = \frac{b_0 b_1 \omega_0 \omega_1 \gamma_1 \partial_{\omega_1}\mathcal{E}_{1,\mathcal{R}} \gamma_0 \partial_{\omega_0}\mathcal{E}_{0,\mathcal{R}}}{\alpha_0\alpha_1(\Lambda^{k_0}_{k_1,k_B})^2} \qquad (10)$$

- Ion heating power by LFAM Landau damping:

$$P_i = 2\gamma_B\omega_B\frac{\partial\mathcal{E}_{B,\mathcal{R}}}{\partial\omega_B}\frac{n_0e^2}{T_i}b_B|\delta\phi_B|^2 \sim 10^{-3}\gamma_0 nT \qquad (11)$$

comparable to collisional heating power estimated by $\sim nT/\tau_E$!

Summary

- A new RSAE decay and core-localized alpha-channeling mechanism proposed and analyzed
- Nonlinear decay of RSAE into another RSAE and LFAM analyzed:
  - scattering to high-n regime preferred
  - threshold on pump RSAE amplitude derived
  - LFAM saturation level estimated $\Rightarrow$ ion heating rate
- Heating power comparable to collisional heating estimated by $\sim nT/\tau_E$
- Branching ratio of different processes: LFAM generation v.s. ZFS generation v.s. ion induced scattering?
- For more details, [Wei, Wang, Chen et al, Nuclear Fusion **62**, 126038 (2022)]