Trilateral Workshop (2022)

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On Nonlinear Scatterings between Drift Waves and Toroidal Alfvén Eigenmodes †

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(A) Scattering and damping of Toroidal Alfvén Eigenmode (TAE) by Drift Waves (DWs). [CQZ, NF (2022)]

(B) Scattering of DWs by TAE [tbp]

["Reverse" of (A)]

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Summary of Highlights

(A) DWs : quasi-stationary (low-frequency) high-n (toroidal mode number) electrostatic fluctuations ⇒ TAE scattered into high-n (heavily electron Landau damped) Kinetic Alfvén Wave (KAW) quasi-mode ⇒ appreciable TAE damping and electron heating !!

(B) TAE : High-frequency long-wavelength e.m. fluctuation (pump wave) ⇒ Stimulated emission of DW and lower-sideband high-n KAW quasi-mode nearly balanced by stimulated absorption of DW and upper-sideband high-n KAW quasimode ⇒ negligible damping but appreciable nonlinear frequency shift.

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(I) Introduction

- Two significant types of low-frequency e.m. fluctuations in tokamak plasmas:

(i) Microscopic drift waves – Ion temperature gradient (ITG), trapped-electron mode (TEM)

- $|\omega_s| \sim \omega_{*s} \sim V_{ti}/L_p$, $L_p^{-1} = |\nabla \ln P|$
- $|k_{\perp s} \rho_{it}| \sim O(1)$

$\Rightarrow$ Crucial roles in thermal plasma transport

(ii) Macro/Meso-scale shear Alfvén wave (SAW)

$\Rightarrow$ Alfvén eigenmodes (AEs) in tokamaks by EPs

- $|\omega_0| \lesssim \frac{V_A}{qR} \equiv \omega_A$, q : safety factor
- $|k_{\theta 0} \rho_{EP}| \sim O(1)$

$\Rightarrow$ Crucial roles in energetic particle (EP) transport.

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⊙ Nonlinear interactions between TAE and drift waves ⇒ focus of this work

(A) ⊙ More specifically,

⇒ Given ambient electron drift waves

- $\Omega_s = (\omega_s, \underset{\sim}{k}_s)$
- $\underset{\sim}{k}_s = (n_s, m_s, k_{sr} = n_s q')$. $q' = dq/dr$ magnetic shear

⇒ How does Ω_s modify the linear stability of a test TAE

- $\Omega_0 = (\omega_0, \underset{\sim}{k}_0)$
- $\underset{\sim}{k}_0 = (n_0, m_0, n_0 q')$

via nonlinear wave-wave interactions ??

⊙ • $|n_s| \gg |n_0|$; $|m_s| \sim |n_s| \gg |n_0| \sim |m_0|$

toroidal mode number ; poloidal mode number

• $|\omega_0| > |\omega_s|$

⊙ Qualitative physics considerations :

- Scattering of Ω_0 (TAE) by Ω_s (DW)

$$\Rightarrow \Omega_+ \triangleq (\omega_s + \omega_0, \underset{\sim}{k}_s + \underset{\sim}{k}_0) \equiv (\omega_+, \underset{\sim}{k}_+)$$

$$\Omega_- \triangleq (\omega_s - \omega_0, \underset{\sim}{k}_s - \underset{\sim}{k}_0) \equiv (\omega_-, \underset{\sim}{k}_-)$$

- Ω_+ and Ω_- are short-wavelength $[|k_\perp \rho_i| \sim O(1)]$

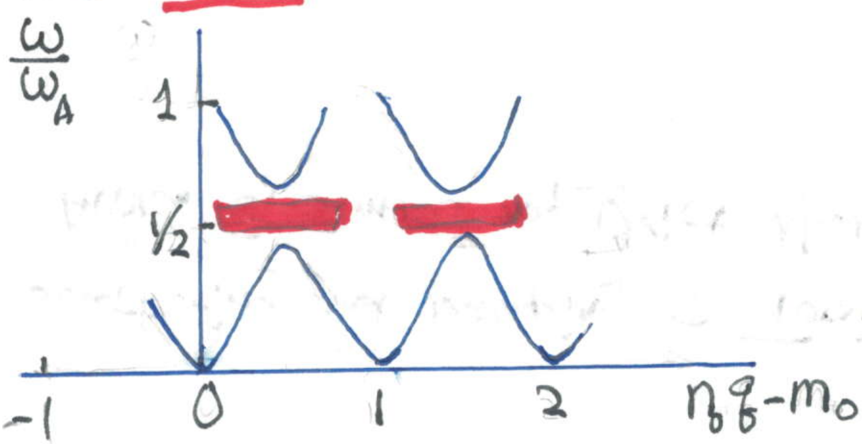
kinetic Alfvén waves (KAW)

- In tokamaks, KAW forms a continuous spectrum due to, mainly, magnetic shear.

$$\Rightarrow |b_k| < 1 \quad (\text{e.g.}, \; b_k = k_\perp^2 \rho_i^2)$$

$$\omega_{KAW}^2 \simeq k_\parallel^2 V_A^2 \left[1 + b_k \left(\frac{3}{4} + \frac{T_e}{T_i} \right) \right]$$

⊙ Ω_0 : TAE



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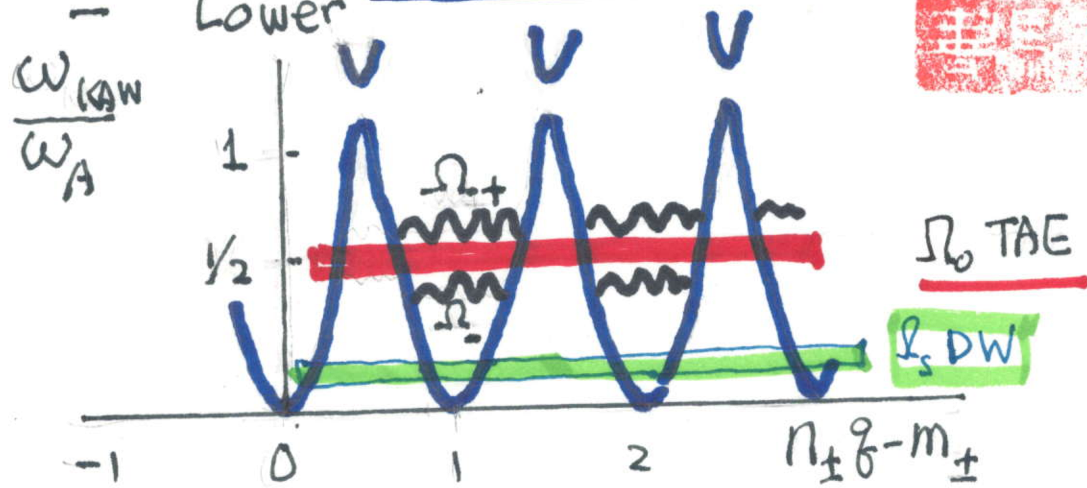
⊙ Ω_+ : upper KAW quasi-mode
$\;\;\;\,-$: Lower



⇒ $\Omega_\pm$ fall inside the KAW continuum

⇒ Mode conversion to KAW

⇒ damping and energy absorption by, mainly, electrons

⇒ energy absorption of $\Omega_\pm$

⇒ damping of Ω_0(TAE).

⇒ This work : Calculation of the damping rate.

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Theoretical Model ["General for DW-AE interactions"]

- Large-aspect-ratio circular tokamak
  - $\epsilon = r/R < 1$
- $\beta \sim \epsilon^2 \ll 1$
- Maxwellian thermal background plasma
- Field variables : $(\delta\phi, \delta A_\parallel)$
- Governing Equations :

(i) Nonlinear Gyrokinetic Eqns (NLGKE) $\Rightarrow \delta f_j^{(NL)}$

$\Downarrow$

(ii) Quasi-neutrality condition $\Rightarrow$ (NLGK Ohm's law)

$\Downarrow$

(iii) $\underset{\sim}{\nabla} \cdot \delta \underset{\sim}{J} \simeq 0$ + $\nabla^2 \delta A_\parallel = -4\pi \delta J_\parallel / c$

$\Rightarrow$ Nonlinear Gyrokinetic Vorticity Eqn. (NLGKV)

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(i) NLGKE

8 $$\boxed{\delta f_j = -(e/T)_j F_{Mj}\,\delta\phi + \exp(-\underset{\sim}{\rho}_j\cdot\underset{\sim}{\nabla})\,\delta g_j}$$

• $\underset{\sim}{\rho}_j = \underset{\sim}{v}\times\underset{\sim}{b}/\Omega_j$

a $$\boxed{\mathcal{L}_g\,\delta g_j = \left(\frac{e}{T}\right)_j F_{Mj}\,(\partial_t + i\omega_{*j})\,J_0(\lambda_j)\,\delta L - \underbrace{\langle\delta \underset{\sim}{u}_g\rangle\cdot\underset{\sim}{\nabla}\,\delta g_j}_{NL}}$$

• $\boxed{\mathcal{L}_g = (\partial_t + v_\parallel\nabla_\parallel + \underset{\sim}{v}_d\cdot\underset{\sim}{\nabla})}$

• $\underset{\sim}{v}_d = (\mu B_0 + v_\parallel^2)(\underset{\sim}{b}\times\underset{\sim}{\kappa})/\Omega_j$

• $\underset{\sim}{\kappa} = \underset{\sim}{b}\cdot\underset{\sim}{\nabla}\,\underset{\sim}{b}$

• $\boxed{\delta L = \delta\phi - v_\parallel\,\delta A_\parallel/c}$

• $\omega_{*j} = \omega_{*jn}\left[1+\eta_j\left(\frac{v^2}{v_{tj}^2}-\frac{3}{2}\right)\right]$

• $\boxed{\langle\delta\underset{\sim}{u}_g\rangle = \frac{c}{B_0}\,\underset{\sim}{b}\times\underset{\sim}{\nabla}(J_0\,\delta L)}$

• $\lambda_j = k_\perp\rho_j \Rightarrow |\lambda_e| \ll 1,\ |\lambda_j| \sim O(1)$

(ii) Quasi-Neutrality Condition [NLGKO]

$$\boxed{\sum_j (N_0 e^2/T)_j\,\delta\phi = \sum_j e_j\langle J_0\,\delta g_j\rangle_v}$$

• $e_i = e$

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(iii) NLGKV

$$\boxed{\underbrace{\underset{\sim}{b}\cdot\underset{\sim}{\nabla}\delta J_{\parallel}}_{\text{line bending}} + \underbrace{\frac{N_0 e^2}{T_i}(1-\Gamma_k)\left(\frac{\partial}{\partial t}+i\omega_{*i}\right)\delta\phi_k}_{\text{inertia}} + \underbrace{\underset{\sim}{\nabla}_\perp\cdot\sum_j\langle e_j \underset{\sim}{V}_d J_0 \delta g_j\rangle_v}_{\text{curvature-pressure}} = \sum_{\underset{\sim}{k}'+\underset{\sim}{k}''=\underset{\sim}{k}} \Lambda^{k'}_{k''}\left[\underbrace{\delta A_{\parallel k'}\delta J_{\parallel k''}}_{\text{Maxwell stress}} - \underbrace{e_i\langle (J_k J_{k'} - J_{k''})\delta L_{k'}\delta g_{ik''}\rangle_v}_{\text{GK Reynold stress}}\right]}$$

- $$\boxed{\Lambda^{k'}_{k''} = \frac{c}{B_0}(\underset{\sim}{k}''_\perp \times \underset{\sim}{k}'_\perp)\cdot\underset{\sim}{b}}$$

- $$\delta J_{\parallel k} = -\frac{c}{4\pi}\nabla_\perp^2 \delta A_{\parallel k} = \frac{c}{4\pi}k_\perp^2\delta A_{\parallel k}$$

- $\delta A_{\parallel k} \Rightarrow \left(\frac{c k_\parallel}{\omega}\right)_k \delta\psi_k$: effective induced potential

- Curvature-pressure (ballooning-interchange)

term neglected in TAE analysis.

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◎ Simplifications $\omega_{*j} \Rightarrow \omega_{*jn}$

- $\eta_i = 0$ ⇒ electron DW as a paradigm model
- Ignore trapped-electron nonlinearity

⇒ Extension to $\eta_i \neq 0$ ITG pretty straightforward and preliminary results have been obtained

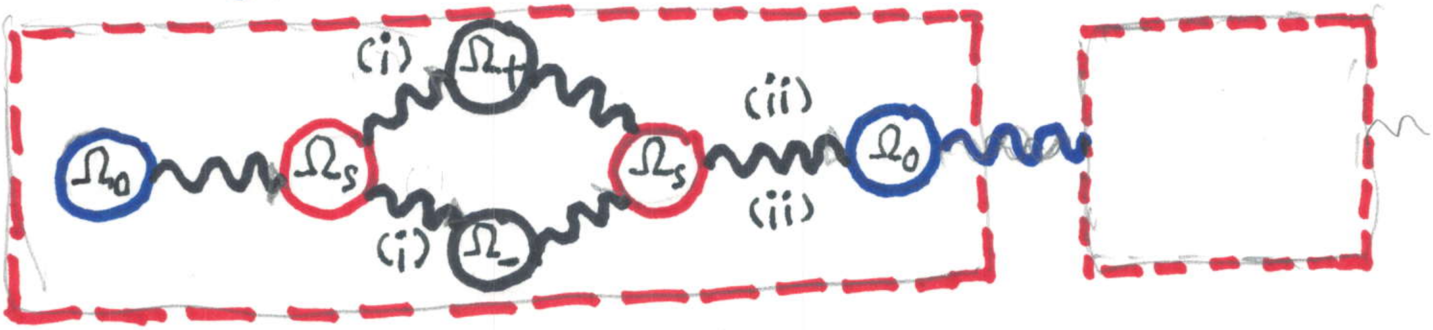
- electrons : $|\omega / k_\parallel v_{te}| \ll 1$ ⇒ adiabatic
- Ions : $|\omega / k_\parallel v_{ti}| \gg 1$ ⇒ fluid approximation adopted in nonlinear analyses
- Ω_s : DWs electrostatic fluctuations

⇒ $|\delta\psi_s| / |\delta\phi_s| \ll 1$ ⇒ Maxwell stress negligible compared with Reynold stress

(II.3) Theoretical Approach [(A) Scatterings of TAE by DW]

- Ω_s : ambient drift waves (DW)

  Ω_0 : test TAE



- Symbollically

Stage (i) • $$\boxed{\epsilon_{A\pm}\,\delta\phi_\pm = \beta_\pm\,\delta\phi_s \begin{bmatrix}\delta\phi_0\\ \delta\phi_0^*\end{bmatrix}}$$

- $\epsilon_{A\pm}$: linear KAW/SAW operator (dielectric constant)

Stage (ii) • $$\boxed{\left[\epsilon_{A0} + \alpha_0 |\delta\phi_s|^2\right]\delta\phi_0 = \left[\beta_0^+\,\delta\phi_s^*\,\delta\phi_+ + \beta_0^-\,\delta\phi_s\,\delta\phi_-^*\right]}$$

(i) & (ii) $\Rightarrow$

$$\boxed{\left[\epsilon_{A0} + \alpha_0 |\delta\phi_s|^2\right]\delta\phi_0 = \left[\beta_0^+\,\delta\phi_s^*\,\frac{\beta_+\,\delta\phi_s}{\epsilon_{A+}} + \beta_0^-\,\delta\phi_s\,\frac{\beta_-\,\delta\phi_s^*}{\epsilon_{A-}}\right]\delta\phi_0}$$

(iii) Separation of spatial scales

$$|x_s| \sim O\left(\frac{1}{n_s}\right) \ll |x_0| \sim O\left(\frac{1}{n_0}\right) \Rightarrow |n_+| \sim |n_-| \sim |n_s| \gg |n_0|$$

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$$\Rightarrow \delta\phi_0 = \Phi_0(\underset{\sim}{X}_0) + \tilde{\phi}_0(\underset{\sim}{X}_s, \underset{\sim}{X}_0) + \cdots$$

- $|\tilde{\phi}_0| / |\Phi_0| \sim O(|\delta\phi_s|^2) \ll 1$

- Averaging over the short-$\underset{\sim}{X}_s$ scales : $\langle \circ\circ\circ \rangle_s$

$$\Rightarrow \boxed{\begin{aligned} &\left[\epsilon_{A0} + \langle \alpha_0 |\delta\phi_s|^2 \rangle_s \right] \Phi_0 \\ &= \left\langle \beta_0^+ \delta\phi_s^* \frac{\beta_+}{\epsilon_{A+}} \delta\phi_s + \beta_0^- \delta\phi_s \frac{1}{\epsilon_{A-}} \beta_- \delta\phi_s^* \right\rangle_s \Phi_0 \end{aligned}}$$

(iv)
- $\dfrac{1}{\epsilon_{A\pm}} = \mathrm{Re}\left(\dfrac{1}{\epsilon_A}\right)_\pm + i\,\mathrm{Im}\left(\dfrac{1}{\epsilon_A}\right)_\pm$

- α_0 and $\mathrm{Re}\left(\frac{1}{\epsilon_A}\right)_\pm \Rightarrow$ nonlinear frequency shift : negligible

- $\mathrm{Im}\left(\frac{1}{\epsilon_A}\right)_\pm \Rightarrow$ due to mode-converted KAW damping $\Rightarrow$ damping rate $\simeq$ Alfvén wave resonance absorption rate [1974, 1976]

$$\Rightarrow \left\langle (\cdots)\,\mathrm{Im}\left(\frac{1}{\epsilon_A}\right)_\pm \right\rangle_s \propto \left\langle \delta(\epsilon_{A\pm})\,(\cdots) \right\rangle_s$$

(v) Calculate the coupling/scattering coeffients :
$\beta_\pm$, β_0^+, β_0^-, α_0, etc.

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⊙ Same with scattering with Ω_-

$$\Rightarrow \boxed{\tau b_0 [\epsilon_{A0} + i(\nu_+ + \nu_-)(k_{0r}\rho_i)^2] \Phi_0 = 0} \quad (*)$$

- Readily solved perturbatively in the ballooning space

$\Phi_0(n_0 q - m_0) \Rightarrow \hat{\Phi}(\eta)$ η : ballooning angle along $\underset{\sim}{B_0}$

$\omega_0 \Rightarrow \omega_0 + \delta\omega$

- $\boxed{\tau b_0 \hat{\epsilon}_{A0}(\eta, \omega_0) \hat{\Phi}_0(\eta) = 0}$

- $\boxed{\frac{2\delta\omega}{\omega_0} \langle \hat{\Phi}_0 b_0 \hat{\Phi}_0 \rangle_\eta = -i \langle \hat{\Phi}_0 (\nu_+ + \nu_-)(k_{\theta\theta}\rho_i)^2 \hat{s}^2 \eta^2 \hat{\Phi}_0 \rangle_\eta}$

- $|\eta| \gg 1 \Rightarrow \hat{\Phi}_0(\eta) = [A\cos(\frac{\eta}{2}) + B\sin(\frac{\eta}{2})] e^{-\lambda|\eta|} / |\eta|$

- $\lambda^2 \simeq [\frac{(1+\epsilon_0)}{4} - (\frac{\omega_0}{\omega_A})^2][(\frac{\omega_0}{\omega_A})^2 - \frac{1}{4}(1-\epsilon_0)] \sim O(\epsilon^2)$

$$\Rightarrow \boxed{\frac{\delta\omega}{\omega_0} \simeq -\frac{i}{4} \frac{(\nu_+ + \nu_-) b_{\theta\theta} \hat{s}^2}{\lambda^2}} \quad (*) \triangleq -i \frac{\gamma_{AD}}{\omega_0}$$

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(III.10) Quantative estimate

◎ $$\boxed{\frac{\delta\omega}{\omega_0} \simeq -\frac{i}{4}\cdot\frac{(\nu_+ + \nu_-) b_{0\theta}\hat{s}^2}{\lambda^2}} = -i\,\frac{\gamma_{AD}}{\omega_0}$$

◎ $$\boxed{\nu_\pm \simeq \pi\left(\frac{\Omega_{ci}}{\omega_0}\right)^2 \sum_{n_s} |A_{ns}|^2\left(\tau + \frac{\sigma_s}{2\Gamma_s}\right)^2\left(\frac{\partial\Gamma_s}{\partial b_{s\theta}}\right)^2 b_{s\theta}\hat{s}^2 \times\left(\frac{1}{\sigma_s^2 z_\pm}\right)\left|\frac{\partial\Phi_s}{\partial z_s}\right|^2_{z_\pm}}$$

◎ normalization

$$\int_{-\infty}^{\infty} dz_s |\Phi_s|^2 = 1 \Rightarrow |\Phi_s|^2 = \frac{1}{\sqrt{\pi}\Delta_s}\exp(-z_s^2/\Delta_s^2)$$

- moderately ballooning drift waves $\Rightarrow \Delta_s \gtrsim 1$
  drift wave eigenmode overlaps several m.r.s.

$$\Rightarrow \boxed{\nu_\pm \simeq \sqrt{\pi}\left(\frac{\Omega_{ci}}{\omega_0}\right)^2 \sum_{n_s} |A_{ns}|^2\left(\tau + \frac{\sigma_s}{2\Gamma_s}\right)^2\left(\frac{\partial\Gamma_s}{\partial b_{s\theta}}\right)^2 b_{s\theta}\hat{s}^2 \times\left(\frac{1}{\sigma_s^2}\right)\left(\frac{|z_\pm|}{\Delta_s^5}\right)} \quad (*)$$

◎ Typical parameters:

- $|\Omega_{ci}/\omega_0| \sim O(10^2)$ • $\sum_{n_s}|A_{ns}|^2 \sim \left|\frac{\delta n}{N_0}\right|^2 \sim O(10^{-4})$
- $b_{s\theta} \sim O(1) \sim \sigma_s \sim \Gamma_s \sim \hat{s} \sim \tau$
- $4\lambda^2 \sim O(\epsilon^2) \sim O(10^{-1} - 10^{-2})$ • $\frac{|z_\pm|}{\Delta_s^5} \sim O(10^{-1})$; $|z_\pm| < 1/2$
  $\sim (\Delta' + r/R)^2$ $\quad |\Delta_s| \gtrsim 1$

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$$\Rightarrow \boxed{\frac{\delta\omega}{\omega_0} \approx -i\, O(10^{-1})\, b_{\theta\theta} = -i\,\gamma_{AD}/\omega_0}$$

- $b_{\theta\theta} = (k_{\theta\theta}\rho_i)^2 = (k_{\theta\theta}\rho_{EP})^2 (\rho_i^2/\rho_{EP}^2)$

$$\simeq (k_{\theta\theta}\rho_{EP})^2 (T_i/T_{EP}) \approx (k_{\theta\theta}\rho_{EP})^2\, O(10^{-2})$$

- Maximizing EP drive $\Rightarrow (k_{\theta\theta}\rho_{EP})^2 \sim O(1)$

$$\Rightarrow \frac{\delta\omega}{\omega_0} \simeq -i\, O(10^{-1})\left(\frac{T_i}{T_{EP}}\right) \approx -i\, O(10^{-2} - 10^{-1}) = -i\,\frac{\gamma_{AD}}{\omega_0}$$

$\Rightarrow$ comparable to typical EP drive growth rate

$$\frac{\gamma_{EP}}{\omega_0} \approx O(10^{-2} \sim 10^{-1}) \approx \frac{\gamma_{AD}}{\omega_0}$$

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⊙ An estimate of anomalous e-heating via TAE-DW scatterings

- TAE wave energy $W_{AE} \simeq \frac{1}{8\pi}\delta B_\perp^2 + \frac{1}{2}\rho_m \delta U_\perp^2 \simeq \frac{1}{4\pi}\delta B_\perp^2$
- $dW_{AE}/dt = -2\gamma_{AD} W_{AE} \Rightarrow$ KAW e-heating
- $\boxed{d(N_e T_{e\parallel})/dt = 2\gamma_{AD} \cdot \frac{1}{4\pi}\delta B_\perp^2}$

$\Rightarrow (d\beta_{e\parallel}/dt)_{AD} \simeq 4\gamma_{AD}|\delta B_\perp/B_0|^2 \simeq O(10^{-2}-10^{-1}) \cdot 4\omega_0 |\delta B_\perp/B_0|^2$

- $|\delta B_\perp/B_0| \approx 5\times 10^{-4}$, $\omega_0 \approx 10^6$

$\Rightarrow \boxed{(d\beta_{e\parallel}/dt)_{AD} \approx O(10^{-2}-10^{-1})}$

- e-heating via EP (α-particle) slowing down

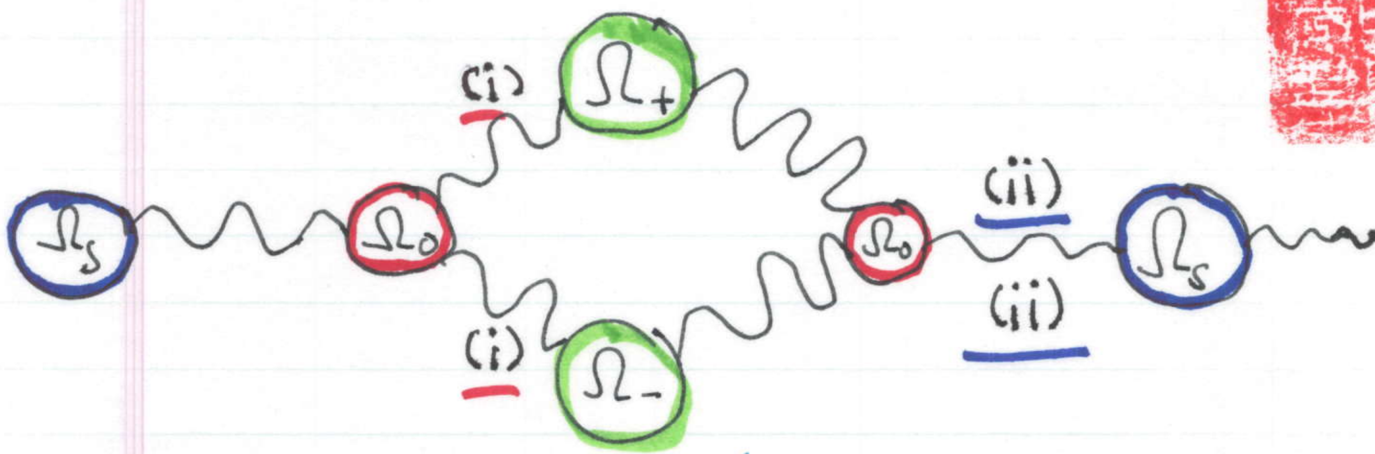
$(d\beta_e/dt)_{EP} \approx \beta_{EP}/\tau_{SD} \lesssim O(10)\beta_{EP} \lesssim O(10^{1-}) \beta_e$ burning plasma

$\Rightarrow (d\beta_e/dt)_{EP} \lesssim O(10^{-2}, 10^{-1})$ $\quad \beta_{EP}/\beta_e \sim \tau_{SD}/\tau_E \sim O(10^{-1})$

$\Rightarrow (d\beta_{e\parallel}/dt)_{AD}$ significant contribution to e heating (along $\underset{\sim}{B_0}$)

"Reverse" of (A)

(B) Scattering of DWs by TAE



◎ Ω_0 : TAE "Pump" wave

Ω_s : test DW (eDW here)

$\Omega_\pm$: high-n KAW quasi-mode sidebands

◎ Symbollically

(i) • $\boxed{\epsilon_{A\pm}\,\delta\phi_\pm = \beta_\pm\,\delta\phi_s \begin{bmatrix} \delta\phi_0 \\ \delta\phi_0^* \end{bmatrix}}$ same as (A)

(ii) • $\boxed{\left[\epsilon_s^{(1)} + \epsilon_s^{(2)}|\delta\phi_0|^2\right]\delta\phi_s = \left[\beta_s^{+}\,\delta\phi_0^{*}\,\delta\phi_+ + \beta_s^{-}\,\delta\phi_0\,\delta\phi_-\right]}$

• (i) + (ii) + Separation of spatial scales

$\Rightarrow \langle \delta\phi_s^{*}\,\epsilon_s^{(1)}\,\delta\phi_s \rangle_s = D_s$: linear DW dielectric const.

$\Rightarrow \langle \delta\phi_s^{*}\,\epsilon_s^{(2)}\,\delta\phi_s \rangle_s \Rightarrow \lambda_s^{(2)}$: NL frequency shift

$\Rightarrow$ etc.

• $\langle \cdots \rangle_s \Rightarrow$ averaging over the short $|\underset{\sim}{x}_s|$ scale.

◎ Substituting into $\delta\phi_+$ and $\delta\phi_-$ equations

$$\Rightarrow \boxed{D_s = \lambda_s^{(2)} + \alpha_{s+}^{(2)} + \alpha_{s-}^{(2)}}$$

- $$\boxed{\lambda_s^{(2)} \simeq 2\tau\left(\frac{\omega_{*i}}{\omega}\right)_s \left\langle \left(\frac{\Lambda_0^s}{\omega_0}\right)^2 \Gamma_s |\delta\phi_s|^2 \right\rangle_s |\delta\phi_0|^2}$$

- $$\boxed{\alpha_{s+}^{(2)} \simeq \left\langle \left(\frac{\Lambda_0^s}{\omega_0}\right)^2 \beta_s^+ \frac{1}{\tau b_+ \epsilon_{A+}} \beta_+ |\delta\phi_0|^2 |\delta\phi_s|^2 \right\rangle_s}$$

- $$\boxed{\alpha_{s-}^{(2)} \simeq \left\langle \left(\frac{\Lambda_0^s}{\omega_0}\right)^2 \beta_s^- \frac{1}{\tau b_- \epsilon_{A-}} \beta_- |\delta\phi_0|^2 |\delta\phi_s|^2 \right\rangle_s}$$

(P.6) • $$\beta_\pm = \tau\left\{\Gamma_s - \Gamma_0 + \left(\frac{\omega_{*i}}{\omega}\right)_s \left[F_\pm - \Gamma_s - \tau v_A^2 \left(\frac{k_\parallel b k_\parallel}{\omega^2}\right)_\pm F_\pm\right]\right\}$$

◎ Remarks

- $\lambda_s^{(2)}$: diagonal NL $\Rightarrow$ NL frequency shift
  $\Rightarrow$ implications to reactive instability

- $\alpha_{s\pm}^{(2)}$: off-diagonal term
  $\Rightarrow$ "collisionless" dissipation via $\mathrm{Im}(1/\epsilon_{A\pm})$

- $\beta_s^\pm \simeq \sigma_\pm + \tau\left(\frac{\omega_{*i}}{\omega}\right)_s F_\pm$; $F_\pm \cong \langle J_\pm J_s F_{Mi}\rangle_v / N_0$, $\sigma_k = 1 + \tau(1 - \Gamma_k)$

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(V) Stability Analyses

⊙ $D_{sr}(\omega_{sr}, n_s(r_1)) = 0$, $\omega_s = \omega_{sr} + \delta\omega_s$

$$\Rightarrow \boxed{(\delta\omega_s)\frac{\partial D_{sr}}{\partial \omega_{sr}} = \lambda_s^{(2)} + \alpha_{s+}^{(2)} + \alpha_{s-}^{(2)} - iD_{si}}$$

linear dissipation

(V.1) ⊙ NL "collisionless" dissipations via "Alfvén resonance" "causality"

$$\Rightarrow \mathrm{Im}(\alpha_{s+}^{(2)} + \alpha_{s-}^{(2)})$$

upper-sidebands

$$\boxed{\mathrm{Im}[\alpha_{s+}^{(2)}] \cong -\pi \left\langle \left(\frac{\Lambda_0^s}{\omega_0}\right)^{-2} \frac{1}{\tau b_+ \sigma_+} \hat{\beta}_+^2 \delta(\epsilon_{A+}) |\delta\phi_s|^2 \right\rangle_s |\delta\phi_0|^2}$$

$\Rightarrow$ stimulated $\boxed{\text{absorption}}$ $\quad \frac{\partial D_{sr}}{\partial \omega_{sr}} > 0$

lower-sidebands

$$\boxed{\mathrm{Im}[\alpha_{s-}^{(2)}] \simeq +\pi \left\langle \left(\frac{\Lambda_0^s}{\omega_0}\right)^{2} \frac{1}{\tau b_- \sigma_-} \hat{\beta}_-^2 \delta(\epsilon_{A-}) |\delta\phi_s|^2 \right\rangle_s |\delta\phi_0|^2}$$

$\Rightarrow$ stimulated $\boxed{\text{emission}}$

⊙ $\mathrm{Im}[\alpha_{s+}^{(2)} + \alpha_{s-}^{(2)}] \simeq O\left(\left|\frac{\omega_{sr}}{\omega_0}\right|\right) \mathrm{Im}|\alpha_{s+}^{(2)}|$

P.11 [Chinese name, illegible]

◎ Quantative estimates

- $\boxed{\delta(\epsilon_{A\pm}) \simeq \frac{1}{4\sigma_s}\delta(z_s^2 - z_\pm^2)}$, • $z_s = (n_s q - m_s)$

• $z_\pm^2 = \frac{(1-\Gamma_\pm)}{b_\pm}\frac{(\omega/\omega_A)_\pm^2}{\sigma_\pm}$

• $\omega_A = V_A / qR$

- 2-Scale-length expansion

$\Rightarrow$ • $\boxed{\hat{\beta}_\pm^2 \simeq \left[(\tau + \sigma_s/2\Gamma_s)\frac{\partial\Gamma_s}{\partial b_s}\right]^2 4\rho_i^4 (k_{0r} k_{sr})^2}$

• $\boxed{k_{\perp\pm}^2 \simeq k_{s\perp}^2 \pm (2 k_{0r} k_{sr})}$

• $i k_{sr} \Rightarrow n_s q' \frac{\partial}{\partial z_s}$

- $\boxed{\frac{c k_{0r}}{B_0}\delta\phi_0 \simeq V_A \frac{\delta B_{0\theta}}{B_0}}$ ◦ $b_+ \simeq b_s$ ◦ $\sigma_+ \simeq \sigma_s$

• $\boxed{\mathrm{Im}(\alpha_{s+}) \simeq -\pi \frac{V_A^2}{\omega_0^2}\left(\frac{\delta B_{0\theta}}{B_0}\right)^2 \frac{1}{\tau\rho_i^2}\left\langle \frac{1}{\sigma_s}\left[(\tau + \frac{\sigma_s}{2\Gamma_s})^2\frac{\partial\Gamma_s}{\partial b_s}\right]^2 \cdot 4 k_{0r}^2 \rho_i^2 \left(-\rho_i^2 \frac{\partial^2}{\partial^2 z_s^2} b_{s\theta}\hat{s}^2\right)|\delta\phi_s|^2\right\rangle_s}$

- $\delta\phi_s = \frac{1}{\pi^{1/4}\Delta_s^{1/2}}\exp(-z_s^2/2\Delta_s^2)$

$\Rightarrow$ $\boxed{\mathrm{Im}(\alpha_{s+}) = -\sqrt{\pi}\left(\frac{\Omega_i}{\omega_0}\right)^2\left(\frac{V_A^2}{c_s^2}\right)\frac{b_{s\theta}\hat{s}^2}{\sigma_s^2}\left(\tau + \frac{\sigma_s}{2\Gamma_s}\right)^2\left(\frac{\partial\Gamma_s}{\partial b_s}\right)^2\left(\frac{z_+}{\Delta_s^3}\right) \cdot (k_{0r}^2\rho_i^2)\left|\frac{\delta B_{0\theta}}{B_0}\right|^2}$

$O(10^{-7})$

$\Rightarrow |\mathrm{Im}\,\alpha_{s+}| \sim 10^4 \cdot \frac{1}{10^{-2}} b_{s\theta}\, O(1) \cdot O(10^{-1}) \cdot O(10^{-1}) \left|\frac{\delta B_{0\theta}}{B_0}\right|^2 \lesssim 10^4 \left|\frac{\delta B_{0\theta}}{B_0}\right|^2 \simeq 10^{-3}$

• $k_{0r}^2\rho_i^2 \sim k_{0\theta}^2 \rho_E^2 (T_i/T_e)/\epsilon^2 \sim O(10^{-1})$

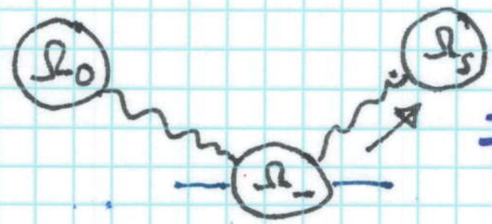
⊙ NL "Dissipation" via scatterings into high-n KAW quasimodes

$$\Rightarrow \mathrm{Im}\left[\alpha_{s+}^{(2)} + \alpha_{s-}^{(2)}\right] \simeq O(10^{4}) \left|\frac{\delta B_{0\theta}}{B_0}\right|^2 O(10^{-1})$$

$$\simeq O(10^{-4}) \text{ negligible}$$

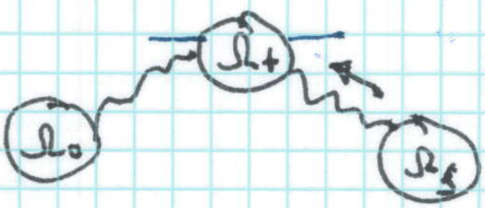
• $\left|\frac{\delta B_{0\theta}}{B_0}\right|^2 \lesssim O(10^{-7})$

⊙ PDI pictures

• 

$\Rightarrow$ $|\omega_0| > |\omega_s|, |\omega_-|$

spontaneous emission

• 

$\Rightarrow$ $|\omega_+| > |\omega_0| > |\omega_s|$

spontaneous absorption

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(V.2) Nonlinear Frequency shift

⊙ $$\boxed{\lambda_s^{(2)} \simeq 2\tau \left(\frac{\omega_{*i}}{\omega}\right)_s \left\langle \left(\frac{\Lambda_0^s}{\omega_0}\right)^2 \Gamma_s |\delta\phi_s|^2 \right\rangle_s |\delta\phi_0|^2}$$

$$\Rightarrow \; 2\left|\frac{\Omega_i}{\omega_0}\right|^2 \left|\frac{\delta B_{0\theta}}{B_0}\right|^2 \frac{b_{s\theta}}{\beta_i} \tau \left(\frac{\omega_{*i}}{\omega}\right)_s \Gamma_s$$

∘ for eDW $\quad \tau\left(\frac{\omega_{*i}}{\omega}\right)_s \Gamma_s \simeq -[1+\tau(1-\Gamma_s)] < 0$

$\Rightarrow \lambda_s^{(2)} < 0 \quad$ downward frequency shift

∘ $$|\lambda_s^{(2)}| \approx O(10^4)\, O(10^2)\, b_{s\theta}\, O(1) \left|\frac{\delta B_{0a}}{B_0}\right|^2$$

$$\approx O(10^{-1})\, b_{s\theta} \lesssim O(10^{-1})$$

$\Rightarrow$ small correction to ω_{sr}

(B) (V.3) Conclusion for eDW scattering by

TAE $\Rightarrow$ small correction (downward) in real frequency and negligible effect on "dissipation".

⊙ Indirect effect on stability via ω_r correction? Reactive instability ITG?!

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(IV) Conclusion + Discussions

(A) ⊙ Scatterings of TAE by drift waves
⇒ effective damping of TAE
⇒ damping rate comparable to typical EP-driven growth rate

⊙ Paradigm model ⇒ extensions to include $|\nabla T| \neq 0$ desirable and doable. Also, RSAE, BAE, etc.

⊙ Suggestion for simulation test : (also experiment)
employing external antenna to investigate the damping of TAE in the presence of stationary drift wave turbulence.

⊙ Implications to burning plasmas –
• DWs reduce/suppress α-particle-driven TAEs ⇒ upshifting critical ∇P_α ⇒ improved α confinement ⇒ enhance thermal plasma heating.

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- α-particle ⇒ TAE ⇒ $\begin{bmatrix} \Omega_+ \\ \Omega_- \end{bmatrix}$ ⇒ electron damping ⇒ electron heating

  high-n KAW

  → Collisionless channel of α-particle energy to electrons →

⊙ Various interesting extentions:

- ITG: reactive vs dissipative instability regimes
- Various AEs

Thanks!