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Center for Nonlinear Plasma Science CNPS Since 2020

Energetic particle nonlinear equilibria and transport processes in burning plasmas

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Acknowledgments:, A. Bottino, S. Briguglio, P. Lauber, Y. Li, G. Vlad, X. Wang

ENEA ITALIAN NATIONAL AGENCY FOR NEW TECHNOLOGIES, ENERGY AND SUSTAINABLE ECONOMIC DEVELOPMENT

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ifts 浙江大學聚变理論與模擬中心

Institute for Fusion Theory and Simulation, Zhejiang University

PSZS theory - Motivations

- Predicting the dynamics of a burning plasma on long time scales, comparable with the energy confinement time or longer, is essential in order to understand next generation fusion experiments, e.g. ITER;
- the crucial role of energetic particles as mediators of cross scale couplings, see Zonca et al. 2015b; Chen and Zonca 2016, must be properly described;
- a first-principle-based, self-consistent approach is crucial;

- Extending gyrokinetic simulations to these time scales is a challenging task from the computational resource point of view, i.e. $\sim 10^{24}$ grid points;
- simplifying assumptions based on physics understanding and first principles must be introduced leading to the definition of a reduced model for the dynamics.

Zonca F. et al. New Journal of Physics 17.1 (2015): 013052

Chen L, Zonca F. Reviews of Modern Physics 88.1 (2016): 015008

PSZS theory - Motivations

We aim at providing general expressions describing plasma transport in the phase space, particularly energetic particles, on long time scales and at introducing a framework to solve these equations within different levels of approximation.

By means of this approach it will be possible to:

- describe the physics of next generation fusion experiments where alpha particles will play a key role;
- characterize cross-scale couplings in burning plasmas.

PSZS theory - Foundations

- A multiple scales approach has been applied to extend gyrokinetic simulations to long time scales;
- macro-scales characterize radial plasma profiles, i.e. L, τ_T, while micro-scales describe "fast" variation of physical quantities, i.e. ρ_L, ω^{-1};
- transport equations for radial profiles are derived by means of a systematic separation of scales between macroscopic equilibrium and fluctuations.
- intermediate spatio-temporal scales are suppressed. Is it possible to use the same approach retaining meso-scales produced by the dynamics?

For more details see, e.g.,: Barnes et al. 2010

Barnes et al Physics of Plasmas 17.5 (2010): 056109

PSZS theory - Foundations

Zonal structures & Zonal field structures

- Mode mode coupling between fluctuating fields can generate toroidal symmetric structures in the density and temperature profiles unaffected by rapid collision-less dissipation, i.e. Landau damping, called zonal structures;
- their counterpart for vector and scalar potentials are called zonal field structures, e.g. $\delta \boldsymbol{E}_r = -\boldsymbol{\nabla}\psi \partial_\psi \delta\phi(\psi)$.

Phase space zonal structures (PSZS)

- fluctuations collision-less undamped in the phase space are called phase space zonal structures and they importantly regulate turbulence saturation level by scattering instability turbulence to shorter radial wavelength stable domain;
- they are coherent micro/meso-scale radial corrugations of the distribution function Chen and Zonca 2016. They are associated to deviations from the local thermodynamic equilibrium and thus they may enhance collisional transport.

For more details: Zonca et al 2015, Zonca et al 2021

Zonca et al. Journal of Physics: Conference Series. Vol. 1785. No. 1. IOP Publishing, 2021

PSZS theory – Orbit average

- particle motion in the reference magnetic field is characterized by three integrals of motion, i.e. $P_\phi, \mu, \mathcal{E}$;
- it follows from previous slides that PSZS equation is connected with the macro-meso- scopic component, i.e. $[\ldots]_S$, unperturbed orbit-averaged distribution function (Falessi et al. 2019):

$$\frac{\partial}{\partial t}\overline{F_{z0}} + \frac{1}{\tau_b}\left[\frac{\partial}{\partial P_\phi}\overline{\left(\tau_b \delta \dot{P}_\phi \delta F\right)_z} + \frac{\partial}{\partial \mathcal{E}}\overline{\left(\tau_b \delta \dot{\mathcal{E}} \delta F\right)_z}\right]_S = \overline{\left(\sum_b C_b^g\left[F, F_b\right] + \mathcal{S}\right)_{z\,S}}$$

where $\overline{(\ldots)} = \tau_b^{-1} \oint d\theta/\dot{\theta}\,(\ldots) = \oint d\theta/\dot{\theta}\, e^{iQ_z}(\ldots)(\bar{\psi}, \theta)$, with $\tau_b = \oint d\theta/\dot{\theta}$ and $\psi = \overline{\psi} + \delta\tilde{\psi}(\theta)$;

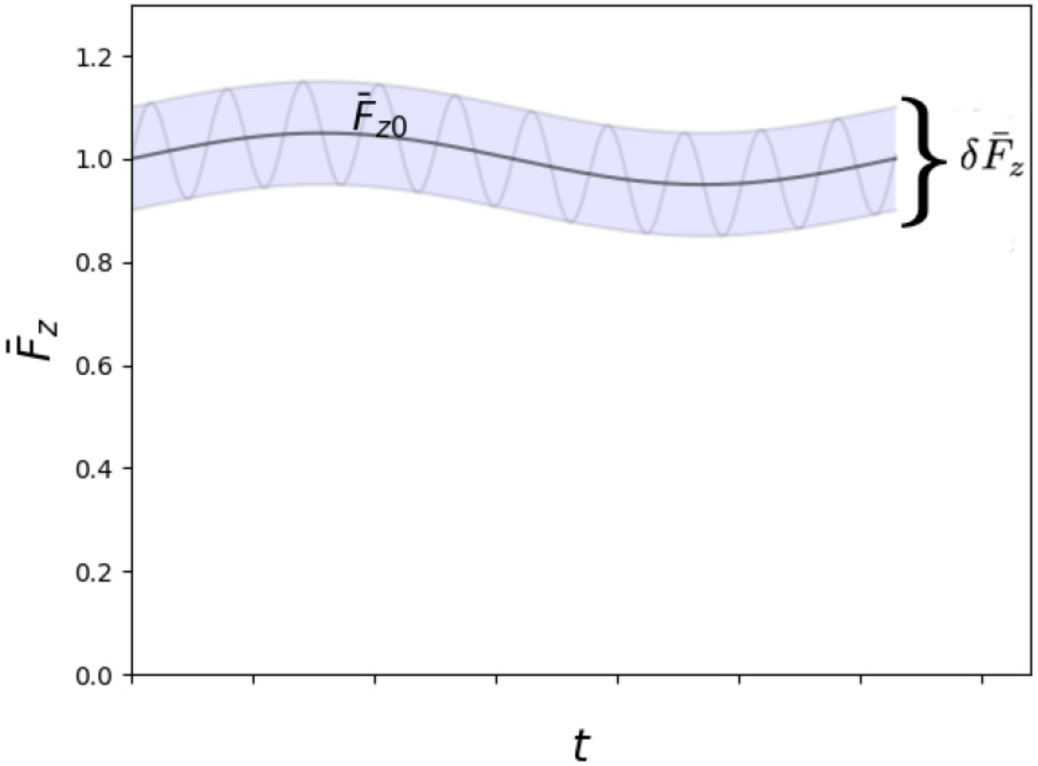
- this is equivalent to bounce/transit averaging a quantity shifted with the e^{iQ_z} operator;
- this expression describe transport processes in the phase space due to fluctuations or collisions and sources;

M. V. Falessi, F. Zonca Physics of Plasmas 26.2 (2019): 022305

Zonal state – Neighboring equilibria

- Having defined PSZS, we can decompose the toroidally symmetric distribution function;
- $\overline{F_{z0}}$ describe macro- & meso-scales;
- micro-scales are accounted by $\delta\overline{F}_z$;
- they describe system transitions between neighboring nonlinear equilibria, see Chen and Zonca 2007; Falessi and Zonca 2019;
- nonlinear equilibria, together with zonal field structures, form a zonal state, see Falessi and Zonca 2019.

$$F_z = \overline{F_{z0}} + \overline{\delta F_z} + \delta\tilde{F}_z$$



L. Chen L., F. Zonca, Nuclear Fusion 47.10 (2007): S727

Zonal State – Governing equation

$$\frac{\partial}{\partial t}\overline{F}_{z0} + \frac{1}{\tau_b}\left[\frac{\partial}{\partial P_\phi}\overline{\left(\tau_b \delta \dot{P}_\phi \delta F\right)}_z + \frac{\partial}{\partial \mathcal{E}}\overline{\left(\tau_b \delta \dot{\mathcal{E}} \delta F\right)}_z\right]_S = \overline{C^g_{z0}} + [\bar{S}]_S$$

- expanding the collision operator we obtain:

$$\bar{C}^g_{z0} = \overline{C^g_z\left[\overline{F}_{z0}, \overline{F}_{z0}\right]} + \left[\overline{C^g_z\left[\overline{F}_{z0}, \delta F_z\right]} + \overline{C^g_z\left[\delta F, \delta F\right]}\right]_S$$

- first term balance $[\bar{S}]_S$ while the second, in the presence of a Maxwellian reference state, describes neoclassical transport;
- corrections to neoclassical transport are given by the first and the third terms;

$$\frac{\partial}{\partial t}\overline{\delta F}_z + \frac{1}{\tau_b}\left[\frac{\partial}{\partial P_\phi}\overline{\left(\tau_b \delta \dot{P}_\phi \overline{F}_{z0}\right)}_z + \frac{\partial}{\partial \mathcal{E}}\overline{\left(\tau_b \delta \dot{\mathcal{E}} \overline{F}_{z0}\right)}_z\right] +$$
$$+ \frac{1}{\tau_b}\left[\frac{\partial}{\partial P_\phi}\overline{\left(\tau_b \delta \dot{P}_\phi \delta F\right)}_z + \frac{\partial}{\partial \mathcal{E}}\overline{\left(\tau_b \delta \dot{\mathcal{E}} \delta F\right)}_z\right]_F = \overline{C^g}_z - \overline{C^g}_{z_0} + [\bar{S}]_F$$

Zonal State – Multipolar expansion

- We have shown how to calculate an evolving renormalized distribution function consistent with the finite level of fluctuations;
- this is connected to the evolution of a macro-/meso- scopic corresponding CGL equilibrium . . .
- following Cary and Brizard 2009; Brizard and Hahm 2007, we write every moment of the zonal distribution function using their push-forward representation, e.g.:

$$\mathbf{J}_z(r) = e\int d\mathcal{E}d\mu d\alpha d^3\mathbf{X}D\left(T^{-1}\mathbf{v}\right)\delta(\mathbf{X}+\rho-\mathbf{r})\Big[F_0+\delta F_z-\frac{e}{m}\langle\delta L_g\rangle_z\frac{\partial}{\partial\mathcal{E}}F_0+ \\ -\frac{e}{m}\frac{\partial}{\partial\mu}F_0\langle\delta L_g\rangle_z\Big]+\frac{e^2}{m}\int d\mathcal{E}d\mu d\alpha d^3\mathbf{X}D\left[\frac{\partial}{\partial\mathcal{E}}F_0\delta\phi_z+\frac{1}{B_0}\frac{\partial}{\partial\mu}F_0\delta L_z\right]$$

- we can substitute F_z for the PSZS $\bar{F}_{z0} = [F_0 + e^{-iQ_z}\delta\bar{F}_{Bz}]_S$;
- we obtain, from a multipole expansion, an equilibrium that is consistent with an anisotropic CGL pressure tensor;

J.R. Cary, A.J. Brizard Reviews of modern physics 81.2 (2009): 693

A.J. Brizard, T.S. Hahm Reviews of modern physics 79.2 (2007): 421

Zonal State – Multipolar expansion

- the current $\mathbf{J}_z$ and the pressure tensor $\mathbf{P}$ satisfy the following force balance equation:

$$\sigma \frac{\mathbf{J}_z \times \mathbf{B}}{c} = \boldsymbol{\nabla} P_{\|} + (\sigma - 1)\boldsymbol{\nabla}(\frac{B^2}{8\pi}) + \frac{B^2}{4\pi}\boldsymbol{\nabla}_{\perp}\sigma$$

where $\sigma = 1 + \frac{4\pi}{B^2}(P_{\perp} - P_{\|})$;

- the self-consistent modification of the equilibrium magnetic field due to PSZS can be calculated solving this equation, e.g.:

$$\Delta^* \psi + \boldsymbol{\nabla} \ln \sigma \cdot \boldsymbol{\nabla} \psi = -\frac{4\pi R^2}{\sigma} - \frac{1}{\sigma^2}\frac{\partial G}{\partial \psi}$$

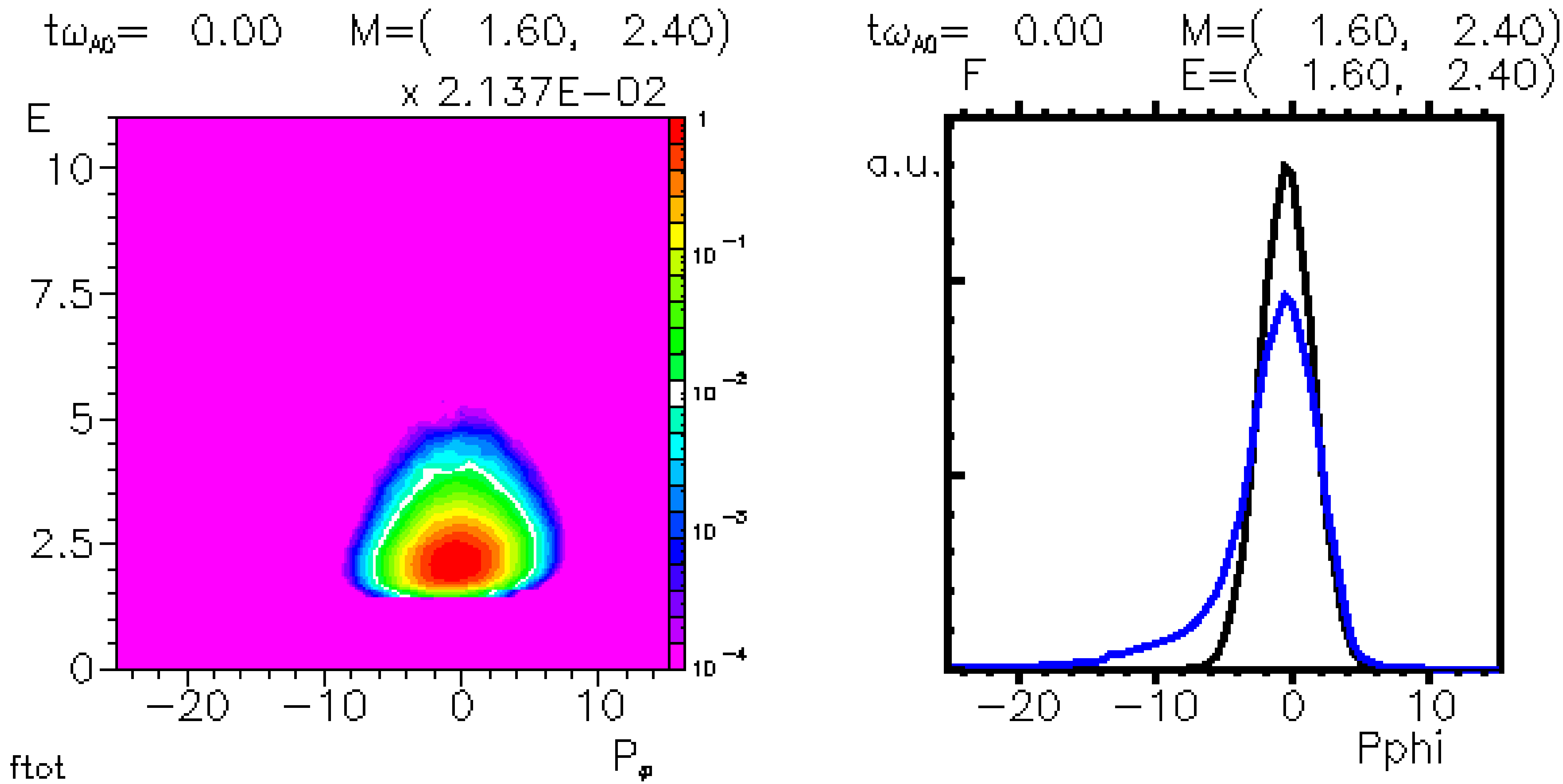
where $G \equiv \sigma F^2/2$ is a flux function;

Zonal state – recap

- PSZS are a possible definition of nonlinear equilibrium distribution function;
- together with zonal field structures and residual orbit averaged particle response they define the zonal state;
- this definition is particularly important in collisionless burning plasmas, where one cannot always describe transport via evolution of macroscopic radial profiles of a reference Maxwellian;
- present approach generally applies to near-integrable Hamiltonian systems.

Zonal state – HMGC diagnostic

- PSZS have been extracted from an EPM simulation by the HMGC hybrid code, i.e. see Briguglio et al. 1995, and more recently by HYMAGIC:



$$\frac{\partial}{\partial t}\overline{F_{z0}} + \frac{1}{\tau_b}\left[\frac{\partial}{\partial P_\phi}\overline{\left(\tau_b \delta \dot{P}_\phi \delta F\right)_z} + \frac{\partial}{\partial \mathcal{E}}\overline{\left(\tau_b \delta \dot{\mathcal{E}} \delta F\right)_z}\right]_S = 0$$

Zonal state – ORB5 diagnostic

A. Bottino et al – Varenna 2022

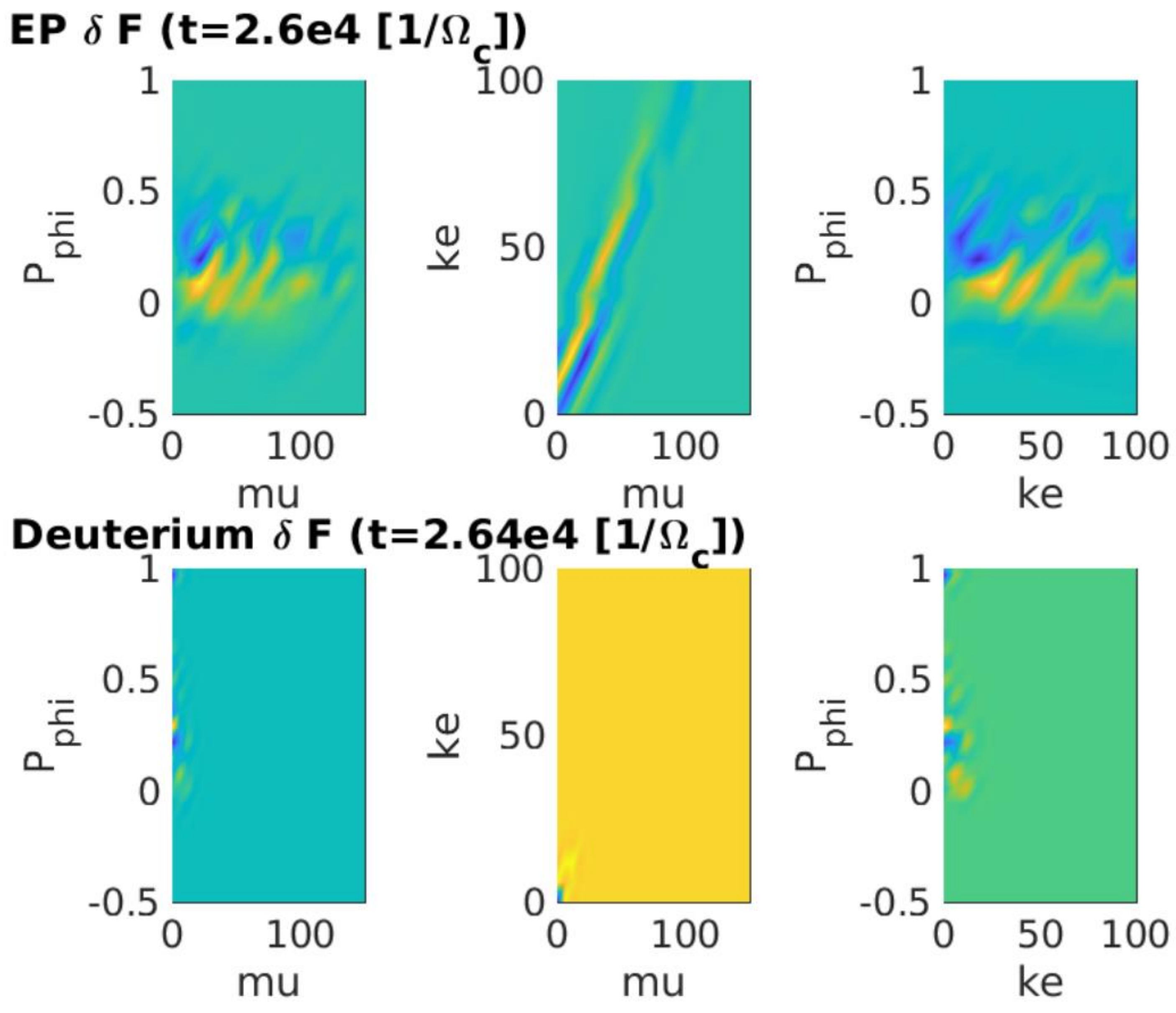


Figure 5: 2D projection of PSZS at $t = 2.6 \cdot 10^4[\Omega_c^{-1}]$, for the NLED-AUG case, for $(N_{P_\varphi} = 30, N_\mu = 30, N_\epsilon = 30)$ and $d = 1$. The axis labels are $\mathrm{P_{phi}} = \mathrm{P}_\varphi$, ke $= \epsilon$ and mu $= \mu$.

Transport models - Hierarchy

- Phase space transport has been studied by means of a hierarchy of verified and validated reduced models within several EUROfusion Enabling Research Projects with P. Lauber & F. Zonca as P.I.s;
- explicit expression of EP fluxes in PSZS equations have been calculated within the following hierarchy of simplifying assumptions:

- the zeroth level of simplification consist in the gyrokinetics description of plasma dynamics;
- the first level of simplification consist in assuming $|\omega| \gg \tau_{NL}^{-1} \sim \gamma_L$;
- the second and final level of simplification is the Quasilinear model.

Transport models - ATEP

DAEPS

- ballooning
- use mode decomposition methodology to separate radial envelope and parallel mode structures
- solve general fishbone-like dispersion relation
- calculate non-linear fluxes

LIGKA-HAGIS

- Fourier
- local and global solver
- solve linear GK equations

- use IMAS-coupled EP stability WF (HAGIS/LIGKA) to calculate orbit+zonal averaged fluxes

Transport models - DAEPS

$$\partial_t \overline{F_0} + \frac{1}{\tau_{t,b}} \partial_\psi \left[\tau_{t,b} \overline{\delta\dot{\psi}\delta F} \right] + \frac{1}{\tau_{t,b}} \partial_{\mathcal{E}} \left[\tau_{t,b} \overline{\delta\dot{\mathcal{E}}\delta F} \right] = 0$$

- Particle trajectory averaging:

$$\overline{(\cdots)} = \frac{1}{2\pi} \oint d\eta \, (\cdots) = \frac{1}{\tau_{b,t}} \int \frac{dl}{v_\parallel} (\cdots)$$

- Perturbed radial velocity, perturbed energy variation:

$$\delta\dot{\mathbf{X}}_\perp \cdot \nabla\psi \approx \frac{c}{B_0} \mathbf{b} \times \nabla \left\langle \delta\phi - \frac{v_\parallel}{c} \delta A_\parallel \right\rangle_{gc} \cdot \nabla\psi$$

$$\delta\dot{\mathcal{E}} = \frac{e_s}{m_s} \partial_t \left\langle \delta\phi - \frac{v_\parallel}{c} \delta A_\parallel \right\rangle_{gc}$$

Transport models - DAEPS

$$\delta \hat{F} = \frac{1}{2}\sum_n \left(A_n(r,t) e^{iS_n(r,t)+in\zeta} \delta \hat{f}_n + c.c. \right)$$

- Multiplication in the real space, corresponds to the convolution in the ballooning space:

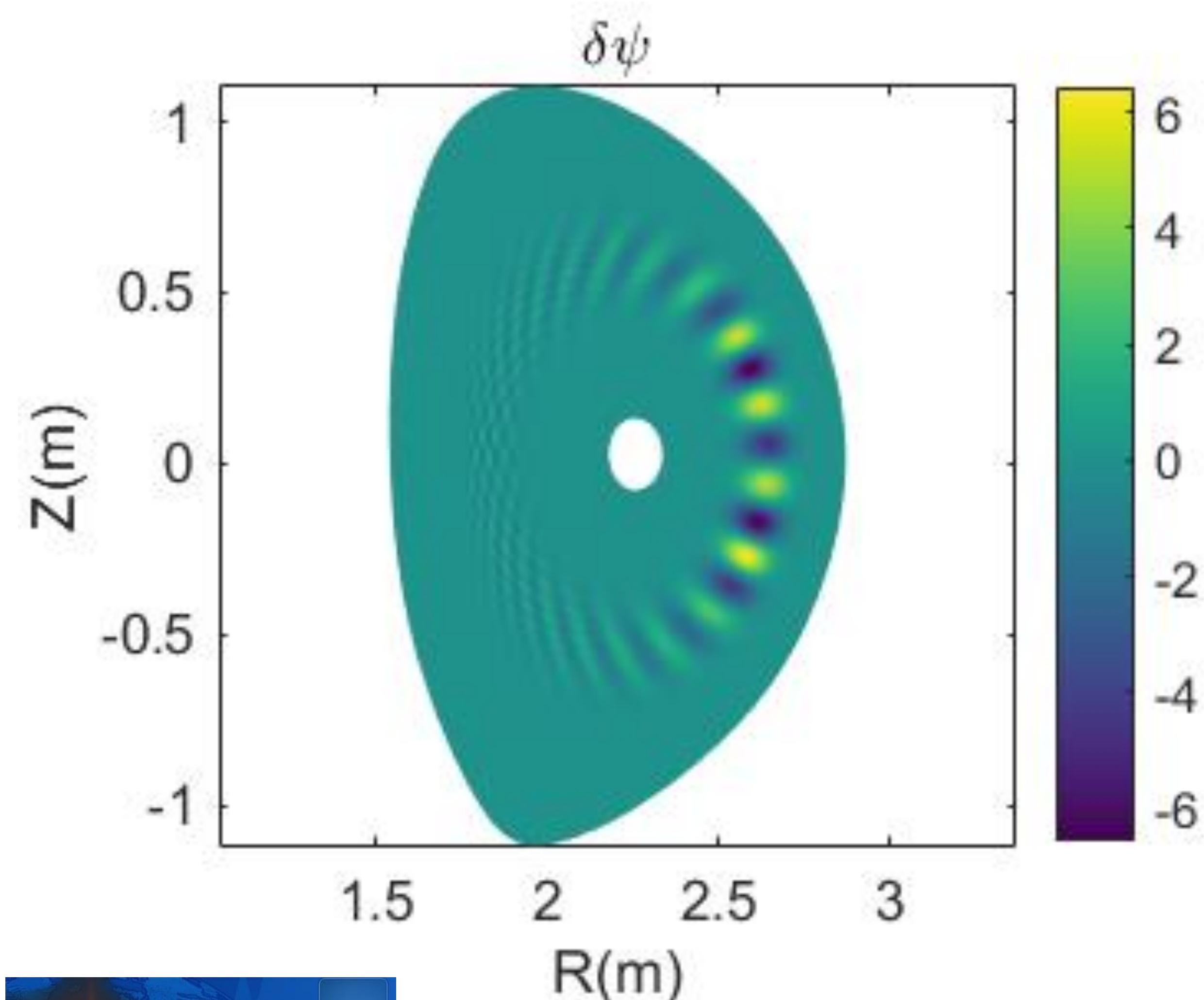
$$\overline{e^{iQ_z}\delta\dot{\psi}\delta F} = \frac{\pi}{2}\sum_{n\neq 0,\ell,j} \oint d\eta \left[e^{iQ_z} \left|A_n(r,t)\right|^2 \right.$$

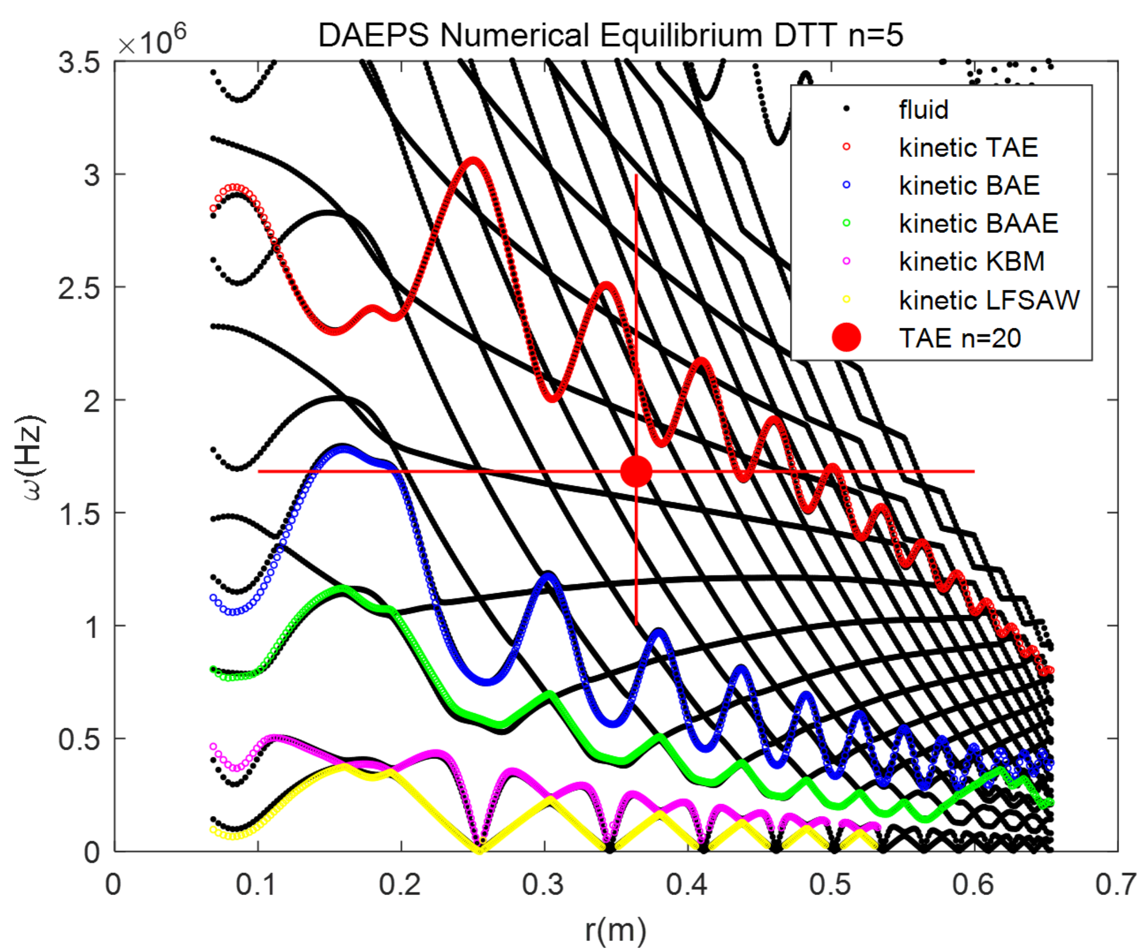
$$\left. \times e^{2\pi i n q j} \delta\dot{\hat{\psi}}_n^*(r,\theta - 2\pi\ell)\delta\hat{f}_n(r,\theta - 2\pi(\ell+j)) + c.c. \right]$$

- Extracted wave intensity dependence, leaving parallel mode structures implicit
- Orbit shift and nqj phase: microscopic scales ($j = 0$ dominant);
- Radial envelope: meso-scale and macro – scale;
- l summation, weighting on filaments (parallel mode structure).

Transport models - DAEPS

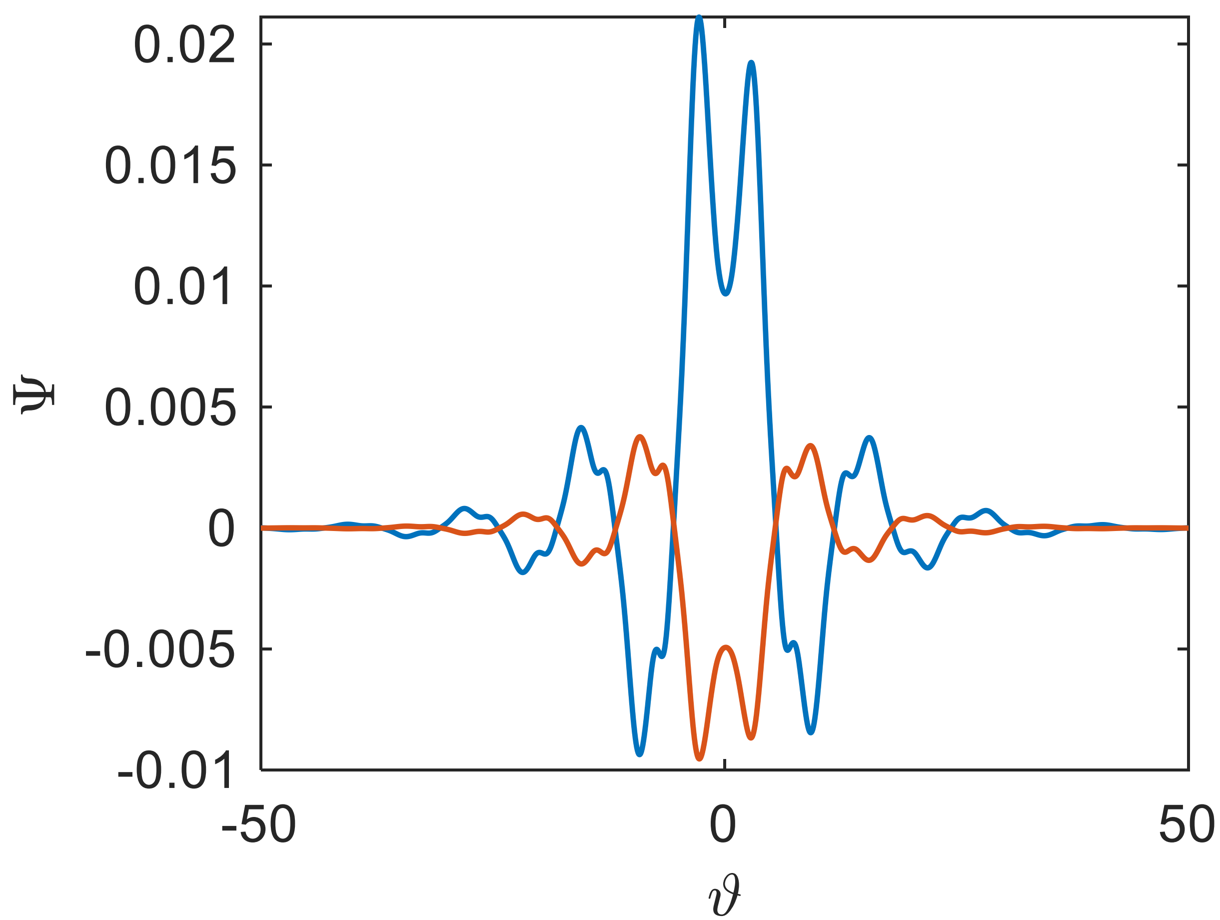
- The frequency of damped TAE mode with the fluid and kinetic continuum shows that the frequency lies inside the gap of the Alfvén continuous spectrum.
- It is also suggests that the frequency of the fluid continuous spectrum is highly consistent with the kinetic continuous spectrum for branches with strong Alfvénic polarization.

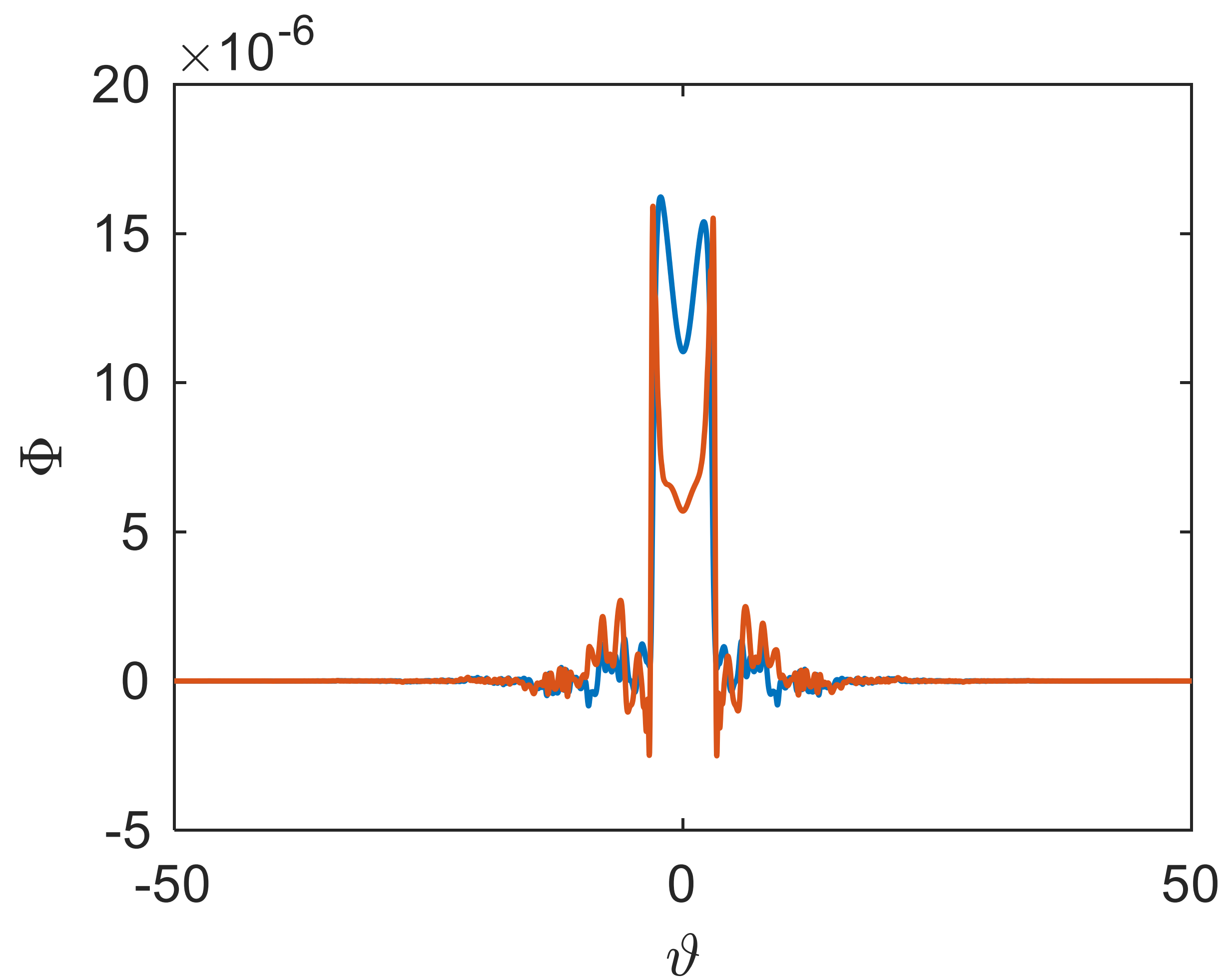




Transport models - DAEPS

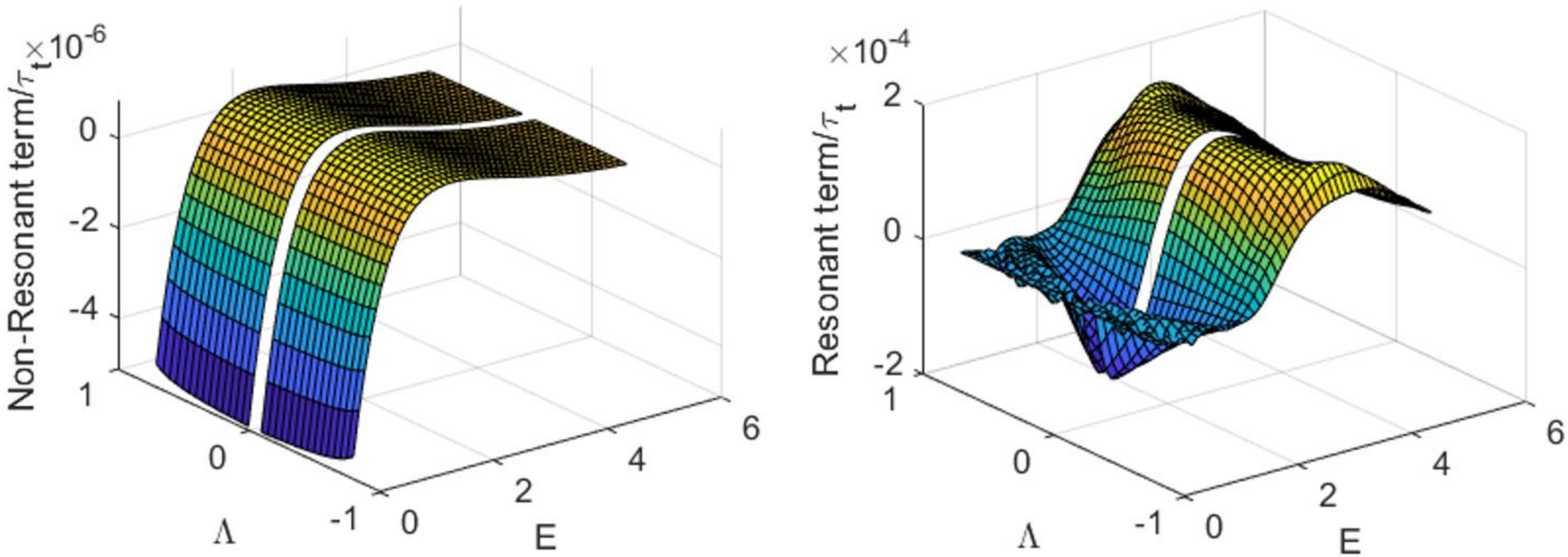
- The fast particle induced TAE is calculated with circulating particle considering general geometry, ignoring the FLR effect and considering the FOW effect of fast ion.
- $q=1.1,\ \frac{\omega}{\omega_{Ti}} = 6.86 + 0.0779i,\ \frac{\omega}{\omega_{Tf}} = 1.04 + 0.0118i$





Transport models - DAEPS

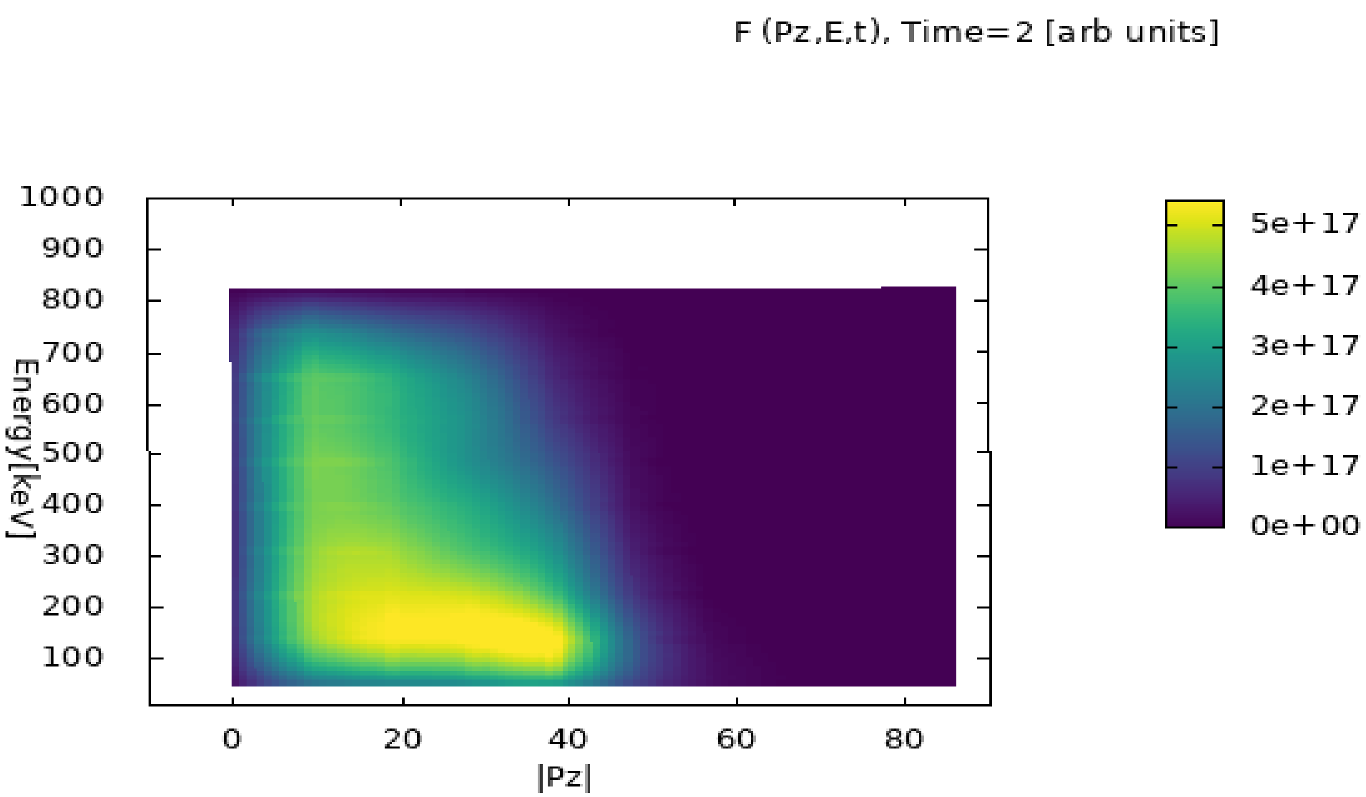
- The free energy term dominates the non-resonant particle contribution
- The kinetic compression term dominates the resonant particle contribution, which is also the main destabilization mechanism of fast particle driven TAE.

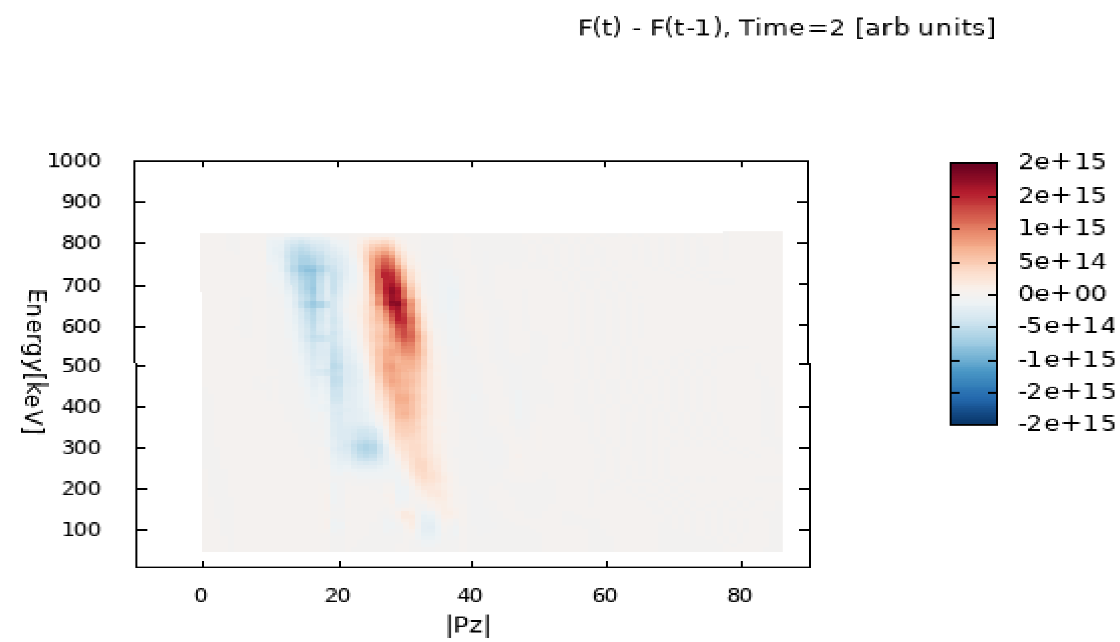


Courtesy of Y. Li

Transport models – LIGKA/Hagis

$$\frac{\partial}{\partial t}\overline{F_{z0}} + \frac{1}{\tau_b}\left[\frac{\partial}{\partial P_\phi}\overline{\left(\tau_b \delta \dot{P}_\phi \delta F\right)}_z + \frac{\partial}{\partial \mathcal{E}}\overline{\left(\tau_b\delta\dot{\mathcal{E}}\delta F\right)}_z\right]_S = \overline{\left(\sum_b C_b^g\left[F,F_b\right] + \mathcal{S}\right)}_{z\,S}$$



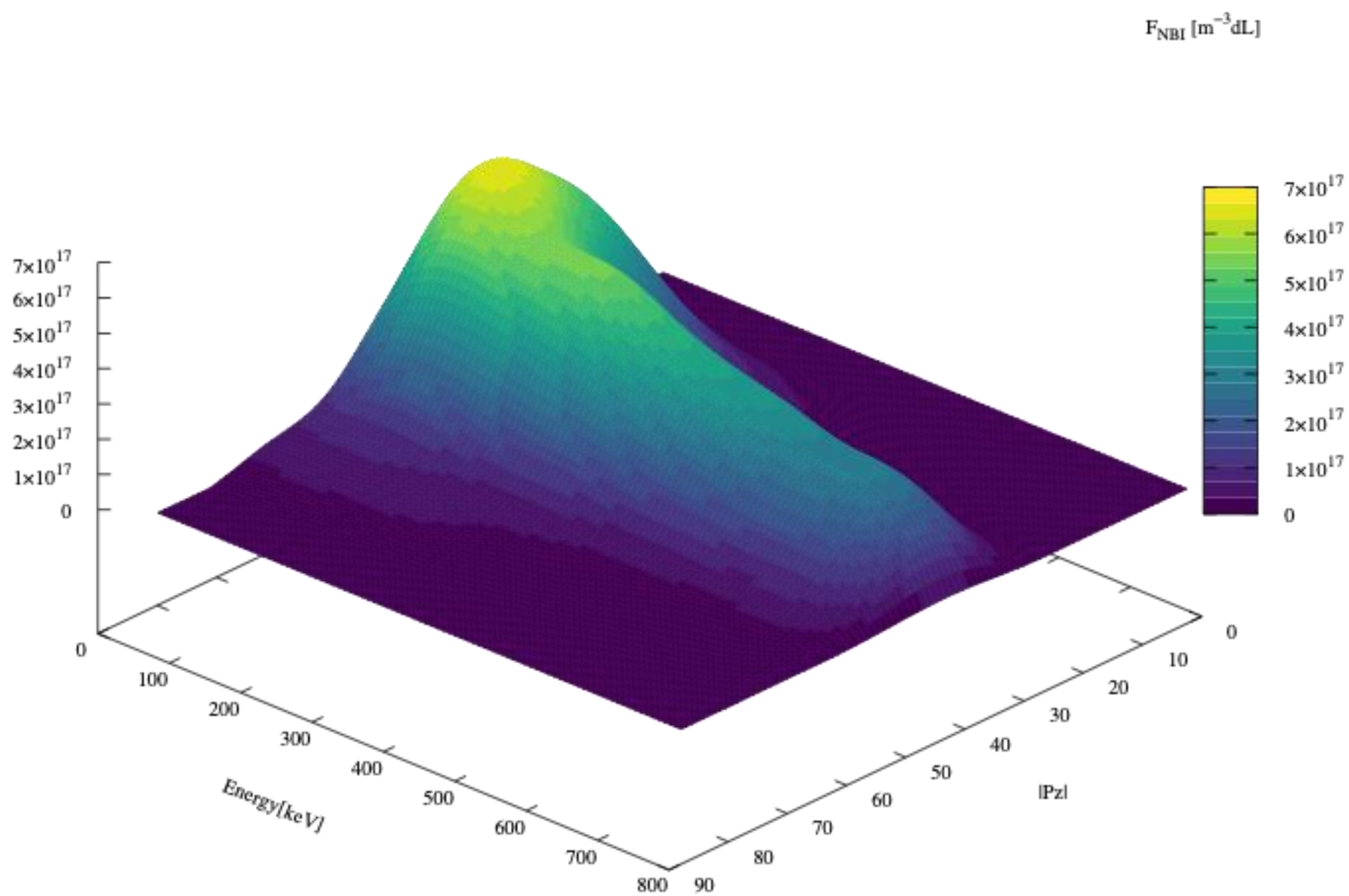


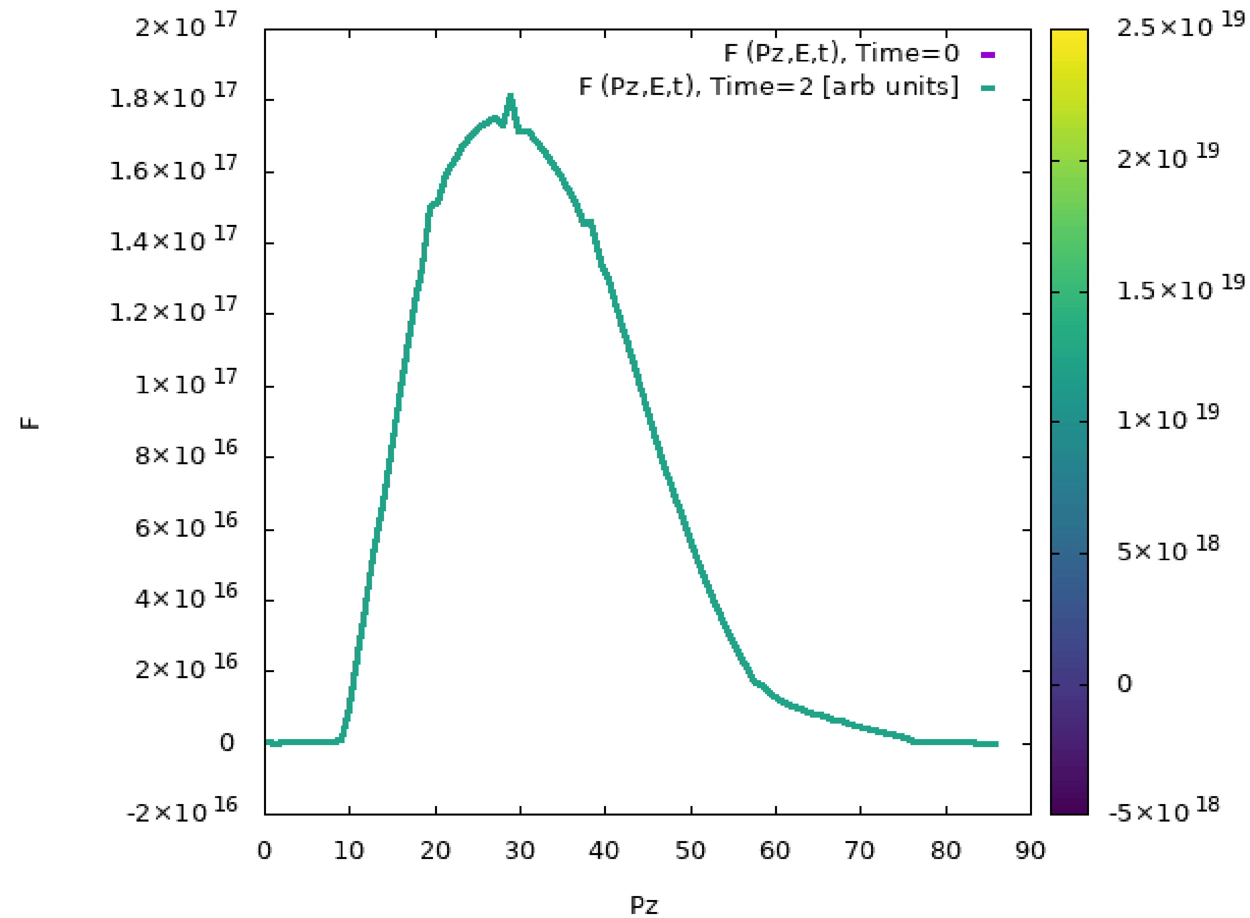
$$\frac{\partial F_{EP}}{\partial t} = \frac{\partial Pz}{\partial t}\frac{\partial F_{EP}}{\partial P_z} + \frac{\partial E}{\partial t}\frac{\partial F_{EP}}{\partial E}$$

note: $\frac{\partial^2 Pz}{\partial t \partial P_z} F_{EP}$ **term excluded so far: dPz/dt assumed constant -> kick model limit**

Courtesy of P. Lauber

Transport models – LIGKA/Hagis





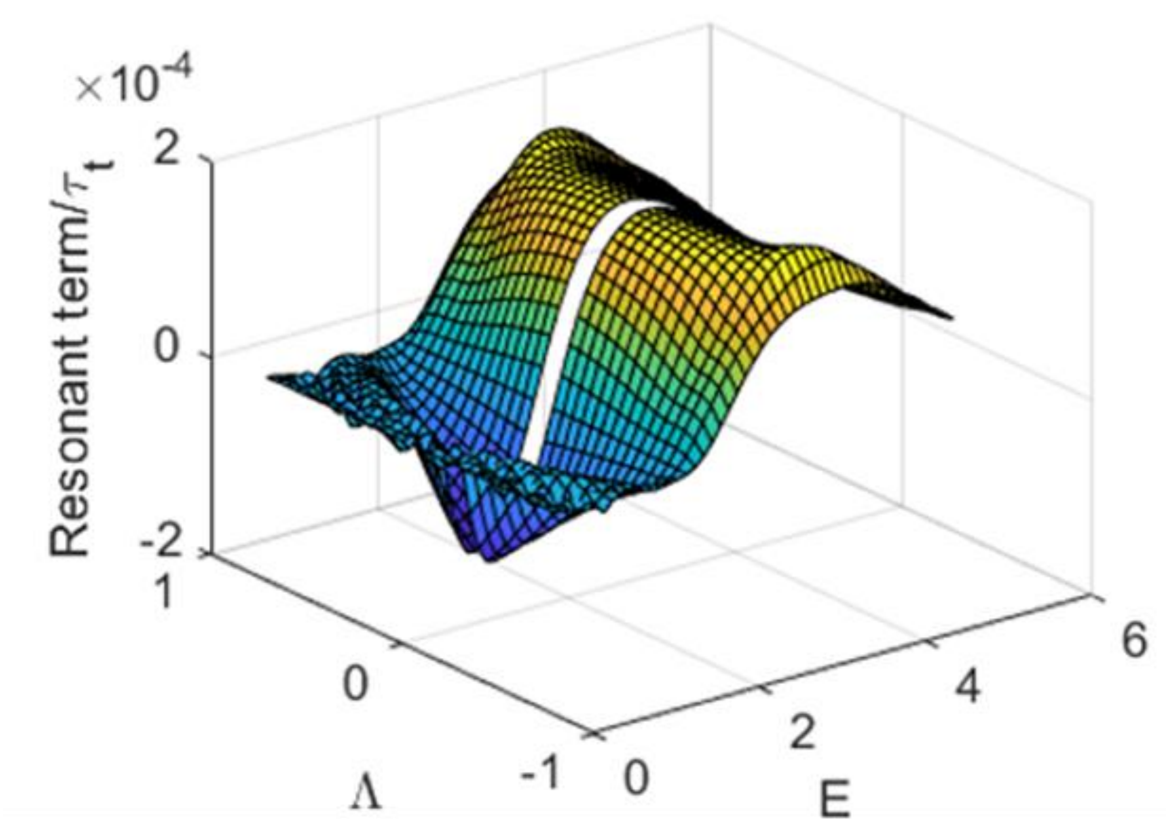
using ITER NBI off-off axis configuration

Courtesy of P. Lauber

Transport models - applications

DAEPS

$$\left[\tau_b \overline{e^{iQ_z}\delta\dot{\psi}\delta F}\right]_z$$

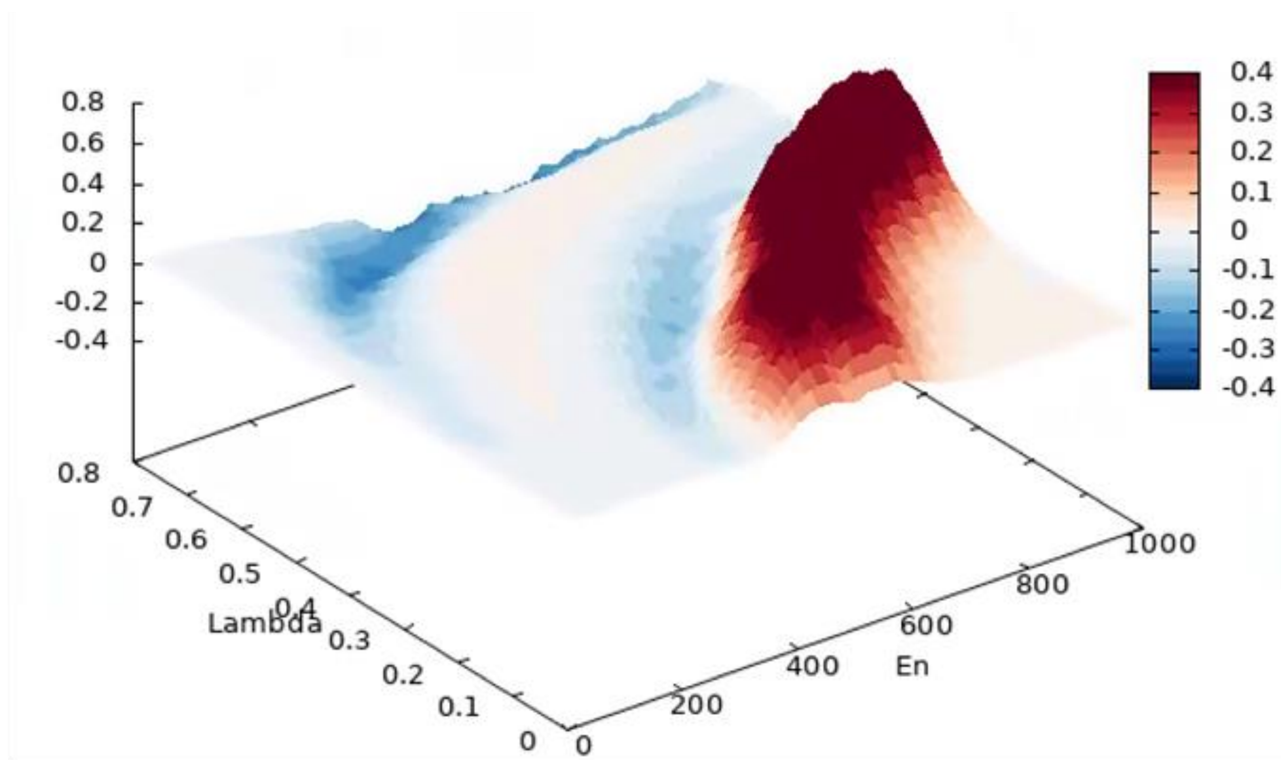


DTT, TAE

Courtesy of Y. Li

LIGKA/HAGIS

$$\left[\tau_b \overline{e^{iQ_z}\delta\dot{\psi}\delta F}\right]_z$$



ITER, TAE

Courtesy of P. Lauber

Conclusions

- PSZS are a possible definition of nonlinear equilibrium distribution function;
- together with zonal field structures and residual orbit averaged particle response they define the zonal state;
- this definition is particularly important in collisionless burning plasmas, where one cannot always describe transport via evolution of macroscopic radial profiles of a reference Maxwellian;
- PSZS have been extracted from nonlinear gyrokinetic simulations: ORB5, HYMAGYC;
- PSZS governing equation has been solved using DAEPS and LIGKA/HAGIS;

Thank you for your attention!