Entropy calculations in the nonlinear wave-particle interactions of Berk-Breizman model

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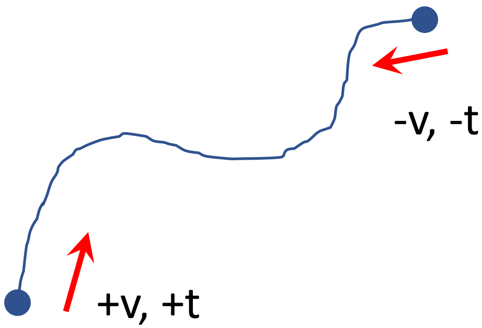
CONTENTS

Time-reversal of Vlasov-Maxwell's equation

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f + q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \nabla_v f = 0$$

$$\nabla \times \mathbf{B} = \mu_0 \left(q \int dv (vf) + \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right)$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t},$$



→ Time reversal with the opposite velocity also satisfies the equations

$$f' = f(-t, \mathbf{r}, -\mathbf{v}),\ \mathbf{E}' = \mathbf{E}(-t, \mathbf{r}) \text{ and } \mathbf{B}' = -\mathbf{B}(-t, \mathbf{r})$$

→ Entropy cannot change in the total system

Entropy can change in a set of system

- Possible reasons to break the time-reversal in a given system (not total)

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f + q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \nabla_v (f_1 + f_0) = \nu f$$

(1) Collisions:
Krook collision operator in Berk model → $f(-t,-v)$ changes only the left term, so cannot be a solution

(2) External source for distributions:
Energetic particles in Berk model → f_0 is given

Applicable to BBP model

$$\sigma = \int dv\, \dot{f}_0 f_0$$

(3) External source for fields → B_0 is given
→ $\mathbf{B}' \neq -\mathbf{B}(-t, \mathbf{r})$

Classical Entropy Production $\sigma = \sum J_a F_a$
J: Flux (e.g. energy flow, current)
F_a: Force (e.g. $\nabla(1/T)$, Efield/T)

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Generalization of Quasilinear theory and entropy

- Lagrangian for Vlasov-Maxwell equation [Low 1958]
- Oscillation Center [Kaufman 1972, Dewar 1973, Dodin 2014], Lie Transform [Ye 1992, Brizard 2007]
- Lagrangian for Free energy with dissipative term [Morrison 1989]

- Advantage of the use of Free energy or entropy

(1) Find the linear instability
[Kruskal and Oberman (1958)]

$$H = -\frac{m}{2}\int \frac{v(\delta f)^2}{df/dv}dvdx + \frac{\epsilon_0}{2}\int \delta\phi_x^2 dx$$
$$= V\frac{\epsilon_0}{4}\frac{\partial(\omega\epsilon_r)}{\partial\omega}\|\mathbf{E}(k,\omega)\|^2$$

(2) Given the external constraints, find the equilibrium (Maximum entropy production principle [MEPP])
(3) Find the positive definite relation
→ How useful to understand the Berk model?

Entropy components

- Boltzmann entropy is defined as $s = -\int dv\, f\ \log f$
- For different orders,

$$
\begin{aligned}
s_s = & -\int dv(f_{s0} + f_{s1})\left(\log f_{s0} + \log\left(1 + \frac{f_{s1}}{f_{s0}}\right)\right) \\
& \simeq -\int dv\left(f_{s0}\log f_{s0} + f_{s1}(\log f_{s0} + 1) + \frac{1}{2}\frac{f_{s1}^2}{f_{s0}}\right) \\
& \simeq s_{s0} + s_{s1} + s_{s2},
\end{aligned}
$$

- Entropy production can be defined as

$$\hat{\sigma} \propto -f_0\dot{f}_0 - f_1\dot{f}_1 - f_2\dot{f}_2$$

- Free energy of the perturbed system is $s_1 \propto \int dv \frac{f_1^2}{f_0} \propto |\phi|^2$

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1-D Berk nonlinear Landau damping

- A simplest model explaining the nonlinear saturation for an instability by the energetic particles

[Berk, Briezman, Pekker, PRL 1996]

- Diffusion and drag in collision

[Lilley et. al. PRL 2009]

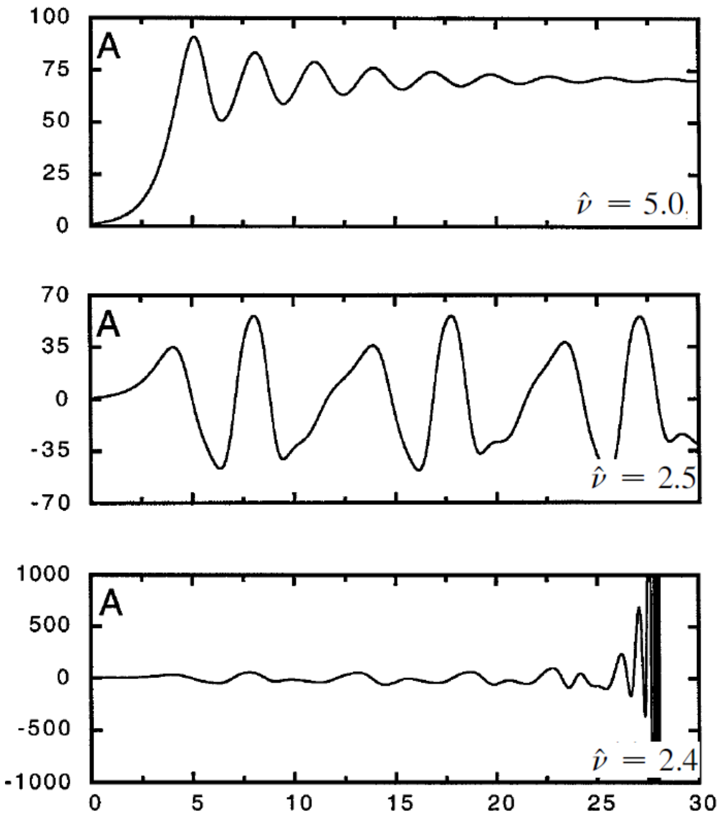
- Fokker-Planck collision

[Duarte 2017]

- Frequency Chirping

[Berk 1999, Duarte 2017,19]

$$\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial x} + \frac{q}{m}(\hat{E}\cos(kx - \omega t))\frac{\partial f}{\partial v} = -\nu f + S$$



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Assumptions for the Berk model

<Assumptions>

$$u = kv - \omega$$

- 1-D in u
- Given background distribution source (energetic particle) $f_0^{(0)} = S/\nu$
- Small perturbation but the comparable nonlinear terms $\omega_B = (ek\hat{E}/m)^{1/2}$

$\omega_B \ll \gamma_L, \gamma_D, \nu, 1/t$ and $\gamma_L - \gamma_d \ll \gamma_L$ →Possible Saturation for

$$(\gamma_L - \gamma_D)\omega_{B0}^2 - \frac{\gamma_L}{4\nu^4}\frac{\omega_{B0}^6}{2} = 0$$

- Artificial fields dissipation by γ_D
- Single wave spectrum → No stochastic by resonance overlap

$$E = \hat{E}exp(iut) + \hat{E}^* exp(-iut)$$

$$f = f_0 + f_1 exp(iut) + f_1^* exp(-iut) + f_2 exp(2iut) + f_2^* exp(-2iut)$$

Order equations of the Berk model

<Equations>

- Linear term of $O(E)$ will be balanced with the nonlinear term of $O(E^3)$
- Requiring the order equations for f_0, f_1, and f_2
- Landau damping in f_1, and f_2 → $f_1 \propto e^{-(iu+\nu)t}$ factor in solutions
- Electric fields is calculated by the energy balance (or Ampere's law)

$$\frac{\partial f_0}{\partial t} = \left(\frac{-\omega_B^2}{2}\frac{\partial f_1^*}{\partial u} + \frac{-\omega_B^{2*}}{2}\frac{\partial f_1}{\partial u}\right) - \nu f_0$$

$$\frac{\partial f_1}{\partial t} + (iu+\nu) f_1 = \frac{-\omega_B^2}{2}\frac{\partial f_0}{\partial u} - \frac{\omega_B^{2*}}{2}\frac{\partial f_2}{\partial u}$$

$$\frac{\partial f_2}{\partial t} + (2iu+\nu) f_1 = \frac{-\omega_B^2}{2}\frac{\partial f_1}{\partial u}$$

$$\frac{\partial \omega_B^2}{\partial t} = -\frac{e\omega}{\epsilon_0 mk}\int f_1 dv - \gamma_D \omega_B^2 = \gamma_L \omega_B^2 - \gamma_L \frac{2k}{\pi dF_0/du}\int (f_1^{(3)}) du - \gamma_D \omega_B^2$$

Linear growth: $\gamma_L \omega_B^2$

NonLinear Saturation: $\gamma_L \frac{2k}{\pi dF_0/du}\int (f_1^{(3)}) du$

Linear damping (External): $\gamma_D \omega_B^2$

$$\gamma_L = (\pi e^2 \omega / 2\epsilon_0 mk) df_0^{(0)}/du$$

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Distribution function solution map

<Couplings>

- The coupling between adjacent distribution orders $f_0^{(0)} \rightarrow f_{-1}^{(1)}, f_1^{(1)}$
- For each coupling, the power of E fields increases $f_1^{(1)} \rightarrow f_0^{(2)}, f_2^{(2)}$
- Collision, Landau damping factors

E fields power order

$$f_0 = f_0^{(0)} + f_0^{(2)}(t) + f_0^{(4)}(t) + ..$$

distribution order

$$\alpha = \frac{\omega_B^2}{2}\frac{\partial}{\partial u}$$

$$\alpha^* = \frac{\omega_B^{2*}}{2}\frac{\partial}{\partial u}$$

$$r_1 = \int_0^t dt' e^{(iu+\nu)(t'-t)}$$

$$r_2 = \int_0^t dt' e^{(2iu+\nu)(t'-t)}$$

k \ power of E	0	1	2	3	4
-1		$f_{-1}^{(1)}$		$f_1^{(3)}$	
0	$f_0^{(0)}$		$f_0^{(2)}$		$f_0^{(4)}$
1		$f_1^{(1)}$		$f_1^{(3)}$	
2			$f_2^{(2)}$		$f_2^{(4)}$

Entropy production components

- The entropy production is multiplication of distribution with the time derivative

a+b is power order

$$\hat{\sigma}_{c,d}^{(a,b)} = f_c^{(a)} \dot{f}_d^{(b)}$$

- For symmetry in time integral, the bounce-average of the entropy production is required [Lee PPCF 2019]

→ equivalent to the Kaufmann [1972] quasilinear diffusion form.

Because of the decorrelation time by collision or diffusion, the bounce-average is still temporally localized compared to the evolution.

$$\langle .. \rangle_b = \int \frac{dl}{v_\parallel} .. = \int dt ..$$

$$\langle Q \rangle_K = \frac{\partial}{\partial J^i}\left[\overline{\boldsymbol{D}}^{ij} \frac{\partial f_0}{\partial J^i} \right]$$

Where $\langle \ldots \rangle_K = (2\pi)^{-3} \int \int \int \ldots d\theta_1 d\theta_2\, d\theta_3$

Ch2. Entropy change in f0 system

$$\sigma_0 \sim f_0 \dot{f}_0$$

k \ *power of E*	E^2	E^4				Coll
σ_0	$\sigma_{0,0}^{(0,2)}$	$\sigma_{0,0}^{(0,4,W)}$	$\sigma_{0,0}^{(0,4,V)}$	$\sigma_{0,0}^{(2,2)}$		$\sigma_{0,coll}$
σ_1	$\sigma_{1,1}^{(1,1)}$	$\sigma_{1,1}^{(3,1,W)}$	$\sigma_{1,1}^{(3,1,V)}$	$\sigma_{1,1}^{(1,3,W)}$	$\sigma_{1,1}^{(1,3,V)}$	$\sigma_{1,coll}$
σ_2					$\sigma_{2,2}^{(2,2)}$	$\sigma_{2,coll}$

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Positive definiteness of entropy by Q and C?

$$\frac{\partial f_0}{\partial t} = \left(\frac{-\omega_B^2}{2}\frac{\partial f_1^*}{\partial u} + \frac{-\omega_B^{2*}}{2}\frac{\partial f_1}{\partial u}\right) + \nu_{scatt}^3 \frac{\partial^2 f_0}{\partial^2 u} + \nu_{drag}^2 \frac{\partial f_0}{\partial u} - \nu f_0 + S$$
$$= Q(f_0) + C(f_0) + S$$

[Question] If $\sigma^{(Q)}$ and $\sigma^{(C)}$ are positive, how can we have the steady state solution of the unperturbed f0 system?

[Answer 1] There is a hidden source in the total f0 system

$$\hat{\sigma}^{(Q)}(t) = \int_0^{t_b} dt \int dv(-\log f_0) Q(f_0)$$
$$= -r_b(t,t') \log f_0 \frac{\partial}{\partial u} E^2 \delta(u) \frac{\partial f_0}{\partial u} \geq 0.$$

$$\hat{\sigma}^{(C)}(t) = \int dv(-\log f_0) C(f_0)$$
$$= -\int dv dx (1 + \log f_0) \beta\kappa\beta f_0 \geq 0$$

$$\hat{\sigma}^{S(0)} = \int dv(-\log f_0) S < 0$$

Entropy balance between Q and C

$$f_0(t) = f_0^0 + f_0^2(t) + f_0^4(t) + \dots,$$

$$\frac{\partial f_0^2}{\partial t} = Q(f_0^0) + C(f_0^2)$$

[Question] If $\sigma^{(Q)}$ and $\sigma^{(C)}$ are positive, how can we have the steady state solution?

[Answer 2] For the higher order, There is non-symmetry, giving the balance of the positive and negative term

$$\frac{d}{dt}\left(\hat{\sigma}_{0,0}^{Q(0,2)} + \hat{\sigma}_{0,0}^{C(2,2)}\right) = 0$$

$$\hat{\sigma}^{(Q)}(t) = -r_b(t,t')\log(f_0^0 + f_0^2(t))\frac{\partial}{\partial u}E^2\delta(u)\frac{\partial(f_0^0 + f_0^2(t))}{\partial u} = \hat{\sigma}_{0,0}^{Q(0,2)} + 2\hat{\sigma}_{0,0}^{Q(2,2)}(t) + \hat{\sigma}_{0,0}^{Q(2,4)}$$

$$\hat{\sigma}_{0,0}^{Q(0,2)}(t) = -r_b(t,t')f_0^{(0)}\alpha(t')r_1^*(t',t'')\alpha^*(t'')f_0^{(0)} > 0$$

$$\hat{\sigma}_{0,0}^{Q(2,2)}(t) = -r_b(t,t')f_0^{(2)}\alpha(t')r_1^*(t',t'')\alpha^*(t'')f_0^{(0)} < 0$$

$$\hat{\sigma}_{0,0}^{Q(2,4)}(t) = -r_b(t,t')f_0^{(2)}\alpha(t')r_1^*(t',t'')\alpha^*(t'')f_0^{(2)} > 0$$

$$\hat{\sigma}^{(C)}(t) = -\int dvdx(1 + \log(f_0^0 + f_0^2(t)))\beta\kappa\beta(f_0^0 + f_0^2(t)) \geq 0$$

$$\hat{\sigma}_{0,0}^{C(0,2)}(t) = \int dvdx\frac{1}{f_0}(\beta f_0^0)\kappa(\beta f_0^0) = -\hat{\sigma}^{S(0)} \geq 0$$

$$\hat{\sigma}_{0,0}^{C(2,2)}(t) = \int dvdx\frac{1}{f_0}(\beta f_0^2)\kappa(\beta f_0^0) = \int dvdx\frac{1}{f_0}(\beta f_0^0)\kappa(\beta f_0^2) = -\hat{\sigma}^{S(2)}$$

$$\hat{\sigma}_{0,0}^{C(2,4)}(t) = \int dvdx\frac{1}{f_0}(\beta f_0^2)\kappa(\beta f_0^2) \geq 0$$

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Entropy change in f0 (unperturbed) system

- The lowest order entropy change of BBK equation is not positive-definite but larger than $\sigma^{(S)} < 0$, as expected in the open system (e.g. $\sigma = \sigma_{irrever} + \frac{\delta Q}{T}$)

order	Quasilinaer	Collision	Source
$\sigma_0^{(0)}$	$\sigma_{0,0}^{Q(0,2)}>0$	$\sigma_{0,0}^{C(0,2)}>0$	$\sigma^{S(0)}<0$
$\sigma_0^{(2)}$	$\sigma_{0,0}^{Q(2,2)} < 0$	$\sigma_{0,0}^{C(2,2)} <0$ $\sigma_{0,0}^{C(2,2)} <0$	$\sigma^{S(2)}<0$
$\sigma_0^{(4)}$	$\sigma_{0,0}^{Q(4,2)}>0$	$\sigma_{0,0}^{C(2,4)} > 0$	

$= 0$

$= 0$

$\sigma_{0,0}^{C} \geq 0$

$$\hat{\sigma}^{(Q)}(t) + \hat{\sigma}^{(C)}(t) + \hat{\sigma}^{(S)}(t)$$

$$= \left(\hat{\sigma}_{0,0}^{C(0,2)} + \hat{\sigma}^{S(0)}\right)$$

$$+ \left(\hat{\sigma}_{0,0}^{C(2,2)} + \hat{\sigma}^{S(2)}\right)$$

$$+ \left(\hat{\sigma}_{0,0}^{Q(0,2)} + \hat{\sigma}_{0,0}^{C(2,2)}\right)$$

$$\geq \hat{\sigma}^{(S)}(t) - \hat{\sigma}_{0,0}^{C(2,4)}(t)$$

$$\simeq \hat{\sigma}^{(S)}(t) = -\hat{\sigma}_{0,0}^{C(0,2)}$$

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Ch3. Symmetry in entropy internal transfer

$$f_0 \dot{f}_0^{(Q)} \, vs. \, f_1 \dot{f}_1^{(Q)}$$

k \ power of E	E^2	E^4				Coll
σ_0	$\sigma_{0,0}^{(0,2)}$	$\sigma_{0,0}^{(0,4,W)}$	$\sigma_{0,0}^{(0,4,V)}$	$\sigma_{0,0}^{(2,2)}$		$\sigma_{0,coll}$
σ_1	$\sigma_{1,1}^{(1,1)}$	$\sigma_{1,1}^{(3,1,W)}$	$\sigma_{1,1}^{(3,1,V)}$	$\sigma_{1,1}^{(1,3,W)}$	$\sigma_{1,1}^{(1,3,V)}$	$\sigma_{1,coll}$
σ_2					$\sigma_{2,2}^{(2,2)}$	$\sigma_{2,coll}$

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Entropy production (lowest order=quasilinear)

- The lowest order contribution is the second order in electric fields by

$$\hat{\sigma}^{(2)}(t) \quad = \hat{\sigma}^{(0,2)}_{0,0}(t) + \left(\hat{\sigma}^{(1,1)}_{1,1}(t)\right) + C.C)$$

$$\hat{\sigma}^{(1,1)}_{1,1}(t) \quad = -r_b(t,t')f_1^{(1)*}(t')\dot{f}_1^{(1)}(t')$$

$$= -r_b(t,t')\left(\alpha(t')f_0^{(0)}\right)\left(r_1^*(t',t'')\alpha^*(t'')f_0^{(0)}\right)$$

k \ power of E	0	1	2	
-1		$f_1^{(1)*}$		f
0	$f_0^{(0)}$		$f_0^{(2)}$	
1		$f_1^{(1)}$		f
2			$f_2^{(2)}$	

Entropy production (lowest order=quasilinear)

- The lowest order contribution is the second order in electric fields by

$$\hat{\sigma}^{(2)}(t) = \hat{\sigma}^{(0,2)}_{0,0}(t) + (\hat{\sigma}^{(1,1)}_{1,1}(t) + C.C)$$

k \ power of E	0	1	2	
-1		$f_1^{(1)*}$		f_1
0	$f_0^{(0)}$		$f_0^{(2)}$	
1		$f_1^{(1)}$		f
2			$f_2^{(2)}$	

$$\hat{\sigma}^{(1,1)}_{1,1}(t) = -r_b(t,t') f_1^{(1)*}(t') \dot{f}_1^{(1)}(t')$$

$$= -r_b(t,t')(\alpha(t') f_0^{(0)})(r_1^*(t',t'')\alpha^*(t'') f_0^{(0)})$$

- symmetry both in time integral (t and t') $r_n(t,t') = \int_0^t dt' e^{(inu+\nu)(t'-t)}$ and acceleration $\alpha(t) = \omega_B^2(t)\partial/\partial u.$

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Entropy production (lowest order=quasilinear)

- The lowest order contribution is the second order in electric fields by

$$\hat{\sigma}^{(2)}(t) = \hat{\sigma}_{0,0}^{(0,2)}(t) + (\hat{\sigma}_{1,1}^{(1,1)}(t) + C.C)$$

$$\hat{\sigma}_{0,0}^{(0,2)}(t) = \hat{\sigma}_{0,0}^{(0,2+)}(t) + \hat{\sigma}_{0,0}^{(0,2-)}(t)$$

power of E / k	0	1	2	
-1		$f_1^{(1)*}$		f_1
0	$f_0^{(0)}$		$f_0^{(2)}$	
1		$f_1^{(1)}$		f
2			$f_2^{(2)}$	

$$\hat{\sigma}_{0,0}^{(0,2+)}(t) = -r_b(t,t')f_0^{(0)*}\dot{f}_0^{(2),+}(t')$$

$$= -r_b(t,t')f_0^{(0)}\alpha(t')r_1^*(t',t'')\alpha^*(t'')f_0^{(0)}$$

Entropy production (lowest order=quasilinear)

- The lowest order contribution is the second order in electric fields by

$$\hat{\sigma}^{(2)}(t) = \hat{\sigma}^{(0,2)}_{0,0}(t) + (\hat{\sigma}^{(1,1)}_{1,1}(t) + C.C)$$

$$\hat{\sigma}^{(0,2)}_{0,0}(t) = \hat{\sigma}^{(0,2+)}_{0,0}(t) + \hat{\sigma}^{(0,2-)}_{0,0}(t)$$

$$\hat{\sigma}^{(0,2-)}_{0,0}(t) = -r_b(t,t')f_0^{(0)*}\dot{f}_0^{(2),-}(t')$$

$$= -r_b(t,t')f_0^{(0)}\alpha^*(t')r_1(t',t'')\alpha(t'')f_0^{(0)}$$

k \ power of E	0	1	2	
-1		$f_1^{(1)*}$		f_1
0	$f_0^{(0)}$		$f_0^{(2)}$	
1		$f_1^{(1)}$		f
2			$f_2^{(2)}$	

Entropy production (lowest order=quasilinear)

k \ *power of E*	E^2
σ_0	$\sigma_{0,0}^{(0,2)}$
σ_1	$\sigma_{1,1}^{(1,1)}$

- By integration by parts in u,

$$\int du\hat{\sigma}_{0,0}^{(0,2+)}(t) = -\int du\hat{\sigma}_{1,1}^{(1,1)}(t)$$

$$\int du\hat{\sigma}_{0,0}^{(0,2-)}(t) = -\int du\hat{\sigma}_{1,1}^{(1,1)*}(t)$$

- By the symmetry w/o collisions, the entropy production is positive or negative

$$\int du\hat{\sigma}_{0,0}^{(0,2)}(t)|_{\nu=0} = -\int du(\int_0^t dt' \int_0^{t'} dt'' A_0(t')^* A_0(t'') e^{\nu(t''-t')} + C.C)$$

$$= \int du \left| \int_0^t dt' A_0(t') \right|^2 > 0 \texttt{ for } \nu = 0$$

$$\int_0^t dt' \int_0^{t'} dt_1 + \int_0^t dt_1 \int_0^{t_1} dt' = \int_0^t dt' \int_0^t dt_1$$

- A small collision makes the opposite sign contribution.

For $e^{\nu t} \simeq 1 + \nu t,$

$$\int du\hat{\sigma}_{0,0}^{(0,2)}(t) - \int du\hat{\sigma}_{0,0}^{(0,2)}(t)|_{\nu=0} \simeq -\int du \left| \int_0^t dt' \nu t' A_0(t') \right|^2 < 0$$

Entropy production (Higher order=nonlinear)

- The higher order contribution is the fourth order in electric fields by

$$\hat{\sigma}^{(4)}(t) = \hat{\sigma}_{0,0}^{(0,4)}(t) + \hat{\sigma}_{0,0}^{(2,2)}(t) + \hat{\sigma}_{1,1}^{(1,3)}(t) + \hat{\sigma}_{1,1}^{(3,1)}(t) + \hat{\sigma}_{2,2}^{(2,2)}(t) + C.C$$

$$\hat{\sigma}_{0,0}^{(0,4)}(t) = -r_b(t,t')f_0^{(0)*}\dot{f}_0^{(4)}(t') = \hat{\sigma}_{0,0}^{(0,4,W+)}(t) + \hat{\sigma}_{0,0}^{(0,4,W-)}(t) + \hat{\sigma}_{0,0}^{(0,4,V+)}(t) + \hat{\sigma}_{0,0}^{(0,4,V-)}(t)$$

Entropy production (Higher order=nonlinear)

k \ power of E	0	1	2	3	4
-1		$f_1^{(1)*}$		$f_1^{(3)*}$	
0	$f_0^{(0)}$		$f_0^{(2)}$		$f_0^{(4)}$
1		$f_1^{(1)}$		$f_1^{(3)}$	
2			$f_2^{(2)}$		$f_2^{(4)}$

$$\hat{\sigma}_{0,0}^{(0,4)}(t) = -r_b(t,t')f_0^{(0)*}\dot{f}_0^{(4)}(t') = \hat{\sigma}_{0,0}^{(0,4,W+)}(t) + \hat{\sigma}_{0,0}^{(0,4,W-)}(t) + \hat{\sigma}_{0,0}^{(0,4,V)}(t)$$

- W shape + part

$$\hat{\sigma}_{0,0}^{(0,4,W+)}(t) = -r_b(t,t')f_0^{(0)*}\alpha^*(t)r_1(t',t'')\alpha(t'')r_0(t'',t_1)\alpha(t_1)r_1^*(t_1,t_2)\alpha^*(t_2)f_0^{(0)}$$

Entropy production (Higher order=nonlinear)

k \ power of E	0	1	2	3	4
-1		$f_1^{(1)*}$		$f_1^{(3)*}$	
0	$f_0^{(0)}$		$f_0^{(2)}$		$f_0^{(4)}$
1		$f_1^{(1)}$		$f_1^{(3)}$	
2			$f_2^{(2)}$		$f_2^{(4)}$

$$\hat{\sigma}_{0,0}^{(0,4)}(t) = -r_b(t,t') f_0^{(0)*} \dot{f}_0^{(4)}(t') = \hat{\sigma}_{0,0}^{(0,4,W+)}(t) + \hat{\sigma}_{0,0}^{(0,4,W-)}(t) + \hat{\sigma}_{0,0}^{(0,4,V)}(t)$$

- W shape - part

$$\hat{\sigma}_{0,0}^{(0,4,W-)}(t) = -r_b(t,t') f_0^{(0)*} \alpha^*(t) r_1(t',t'') \alpha(t'') r_0(t'',t_1) \alpha^*(t_1) r_1(t_1,t_2) \alpha(t_2) f_0^{(0)}$$

Entropy production (Higher order=nonlinear)

k \ power of E	0	1	2	3	4
-1		$f_1^{(1)*}$		$f_1^{(3)*}$	
0	$f_0^{(0)}$		$f_0^{(2)}$		$f_0^{(4)}$
1		$f_1^{(1)}$		$f_1^{(3)}$	
2			$f_2^{(2)}$		$f_2^{(4)}$

$$\hat{\sigma}_{0,0}^{(0,4)}(t) = -r_b(t,t')f_0^{(0)*}\dot{f}_0^{(4)}(t') = \hat{\sigma}_{0,0}^{(0,4,W+)}(t) + \hat{\sigma}_{0,0}^{(0,4,W-)}(t) + \hat{\sigma}_{0,0}^{(0,4,V)}(t)$$

$$\omega_B^2(t)\partial/\partial u$$

- V shape part

$$\hat{\sigma}_{0,0}^{(0,4,V)}(t) = -r_b(t,t')f_0^{(0)*}\alpha^*(t)r_1(t',t'')\alpha^*(t'')r_2(t'',t_1)\alpha(t_1)r_1(t_1,t_2)\alpha(t_2)f_0^{(0)}$$

Cancellation in the higher order

- By integration by parts in u,

$$\int du\hat{\sigma}_{0,0}^{(0,4)}(t) = -\int du\hat{\sigma}_{1,3}^{(3,1)}(t)$$

power of E / k	E^2	E^4	
σ_0	$\sigma_{0,0}^{(0,2)}$	$\sigma_{0,0}^{(0,4,W)}$	$\sigma_{0,0}^{(0,4,V)}$
σ_1	$\sigma_{1,1}^{(1,1)}$	$\sigma_{1,1}^{(3,1,W)}$	$\sigma_{1,1}^{(3,1,V)}$

- However, there is no symmetry in the time integral for $\sigma_{0,0}^{(0,4)}$

→ The sign is likely negative, but not always!, even for no collision

- The resonance condition is not available in the integral range for $\sigma_{0,0}^{(0,4,V)}$

→ No contribution from $\sigma_{0,0}^{(0,4,V)}$

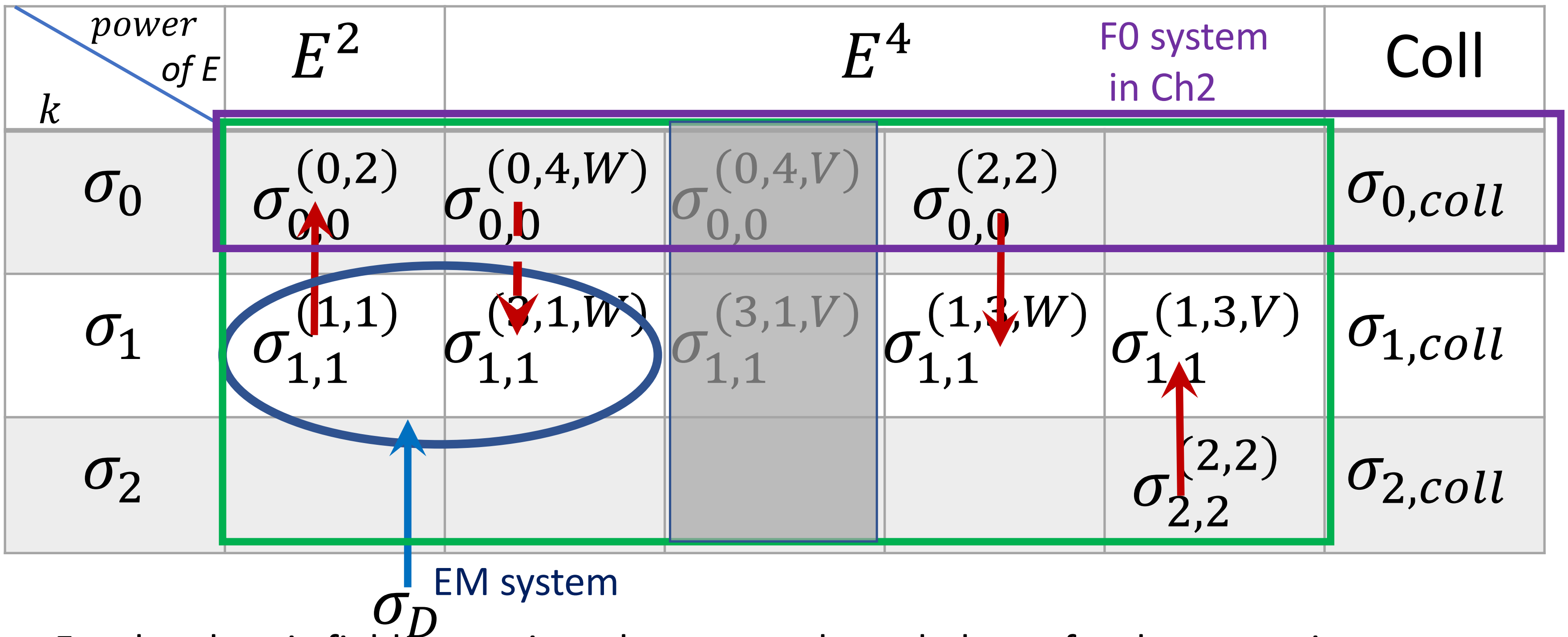
Entropy transfer in the system

k \ power of E	E^2	E^4				Coll
σ_0	$\sigma_{0,0}^{(0,2)}$	$\sigma_{0,0}^{(0,4,W)}$	$\sigma_{0,0}^{(0,4,V)}$	$\sigma_{0,0}^{(2,2)}$		$\sigma_{0,coll}$
σ_1	$\sigma_{1,1}^{(1,1)}$	$\sigma_{1,1}^{(3,1,W)}$	$\sigma_{1,1}^{(3,1,V)}$	$\sigma_{1,1}^{(1,3,W)}$	$\sigma_{1,1}^{(1,3,V)}$	$\sigma_{1,coll}$
σ_2					$\sigma_{2,2}^{(2,2)}$	$\sigma_{2,coll}$

- The entropy in the total system (green box) is conserved for any collisions
  → Only entropy transfer between sub-systems exists

- The entropy production only occurs for the collision terms

Entropy transfer in the system



- For the electric fields equation, the entropy has a balance for the saturation

$$\int du\hat{\sigma}_{1,1}^{(1,1)} + \int du\hat{\sigma}_{1,1}^{(3,1,W+)} + \int du\hat{\sigma}_D = 0$$

where $\hat{\sigma}_D = -(\gamma_D/\gamma_L)\hat{\sigma}_{1,1}^{(1,1)}$

Entropy transfer in the system

k \ power of E	E^2	E^4				Coll
σ_0	$\sigma_{0,0}^{(0,2)}$	$\sigma_{0,0}^{(0,4,W)}$	$\sigma_{0,0}^{(0,4,V)}$	$\sigma_{0,0}^{(2,2)}$		$\sigma_{0,coll}$
σ_1	$\sigma_{1,1}^{(1,1)}$	$\sigma_{1,1}^{(3,1,W)}$	$\sigma_{1,1}^{(3,1,V)}$	$\sigma_{1,1}^{(1,3,W)}$	$\sigma_{1,1}^{(1,3,V)}$	$\sigma_{1,coll}$
σ_2					$\sigma_{2,2}^{(2,2)}$	$\sigma_{2,coll}$

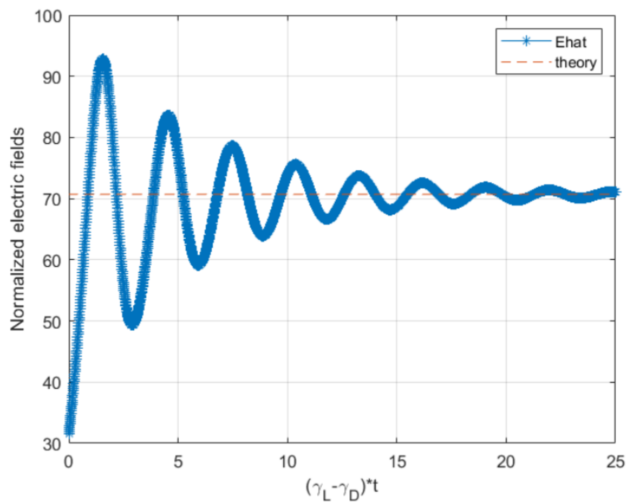
σ_D

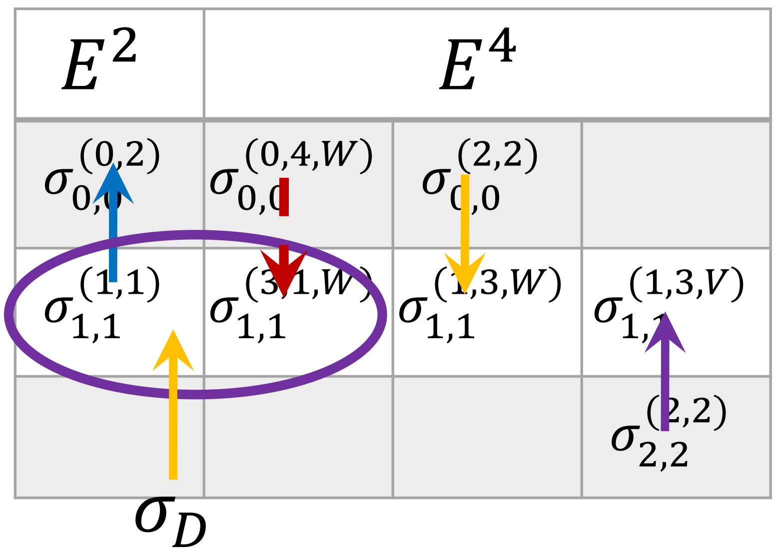
- For the electric fields equation, the entropy has a balance for the saturation

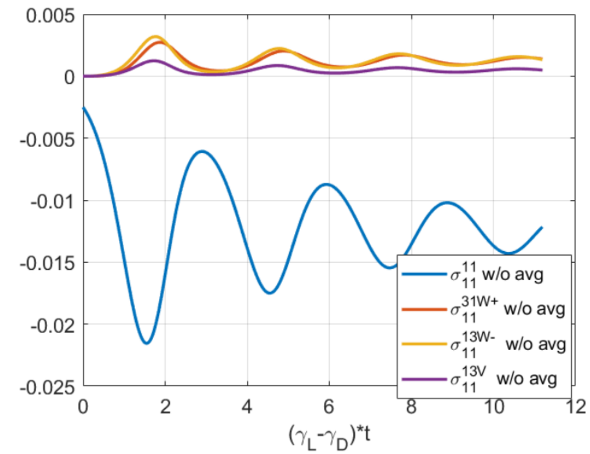
→ For saturation of the case $\gamma_L - \gamma_D > 0$, it requires $\sigma_{1,1}^{(3,1,W)} > 0$

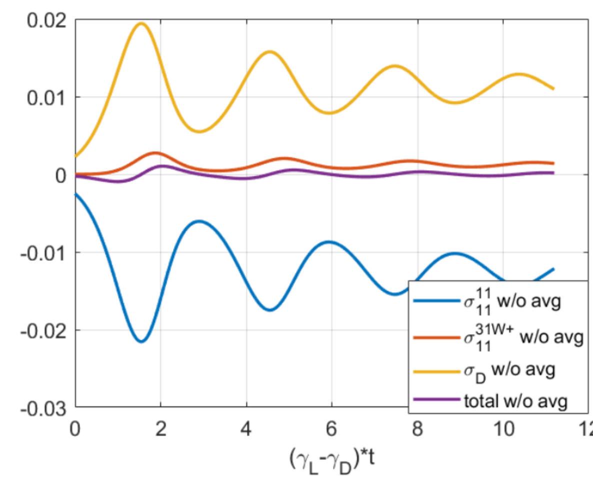
→ The non-symmetric entropy component do not guarantee the positiveness

Case 1-1. Saturation at $\hat{\nu} = \nu/(\gamma_L - \gamma_D) = 5$



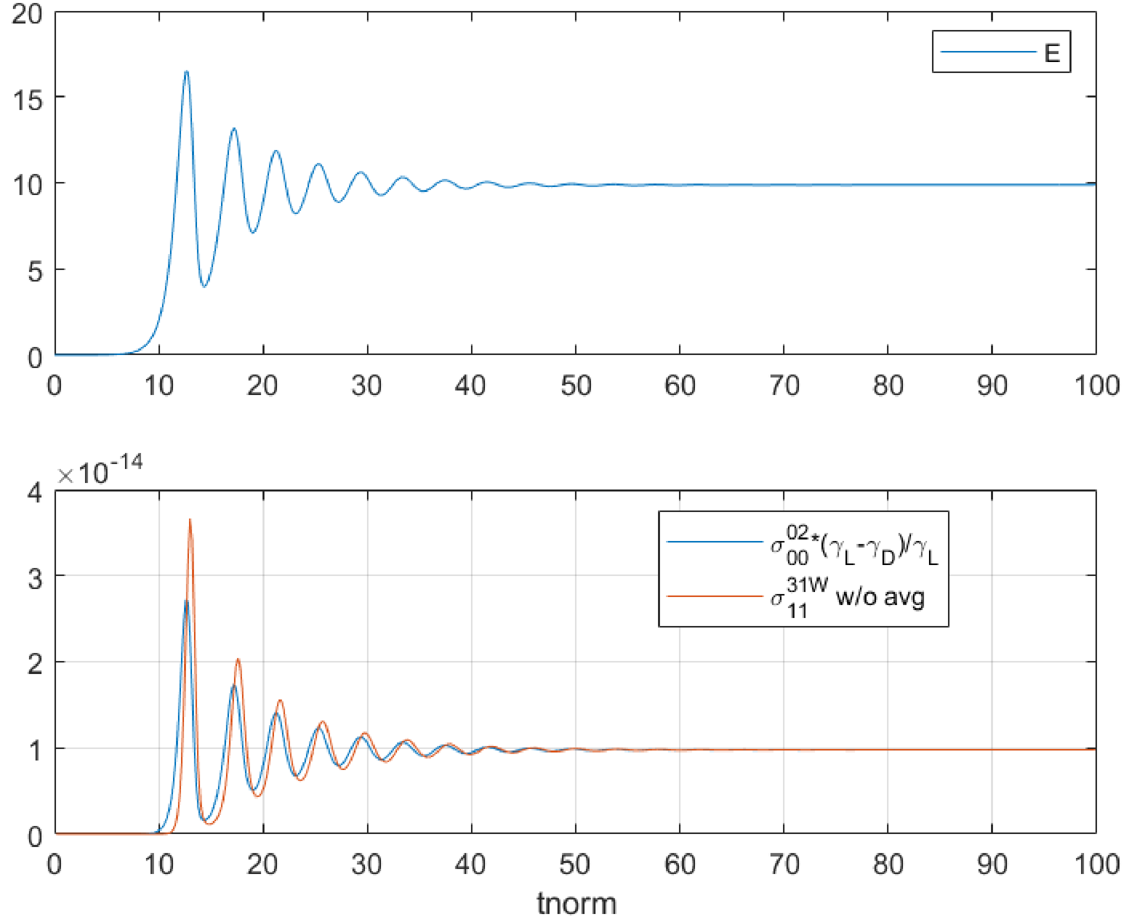




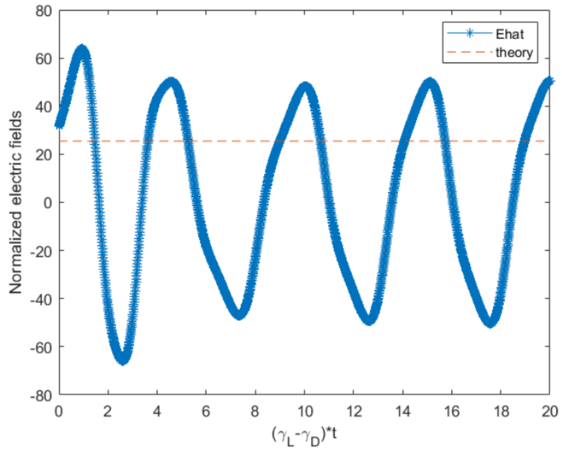


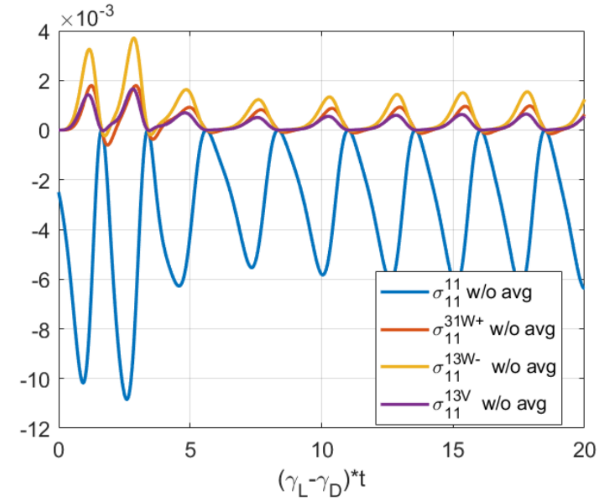
Case 1-2. Saturation at $\hat{\nu} = D^{\frac{1}{3}}/(\gamma_L - \gamma_D) = 3.0$

[Lilley et. al. PoP 2010]



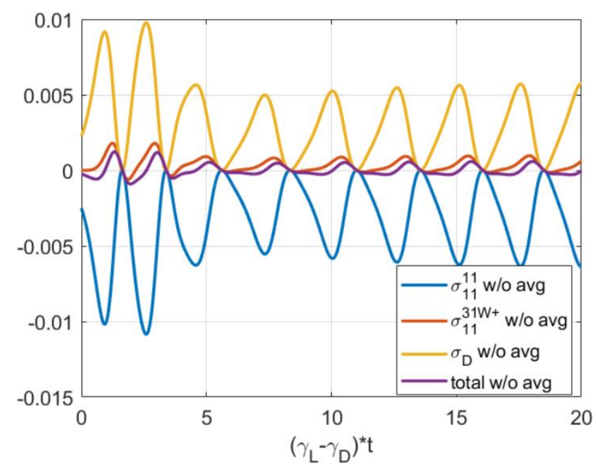
Case 2-1. limit cycle at $\hat{\nu} = \nu/(\gamma_L - \gamma_D) = 3$



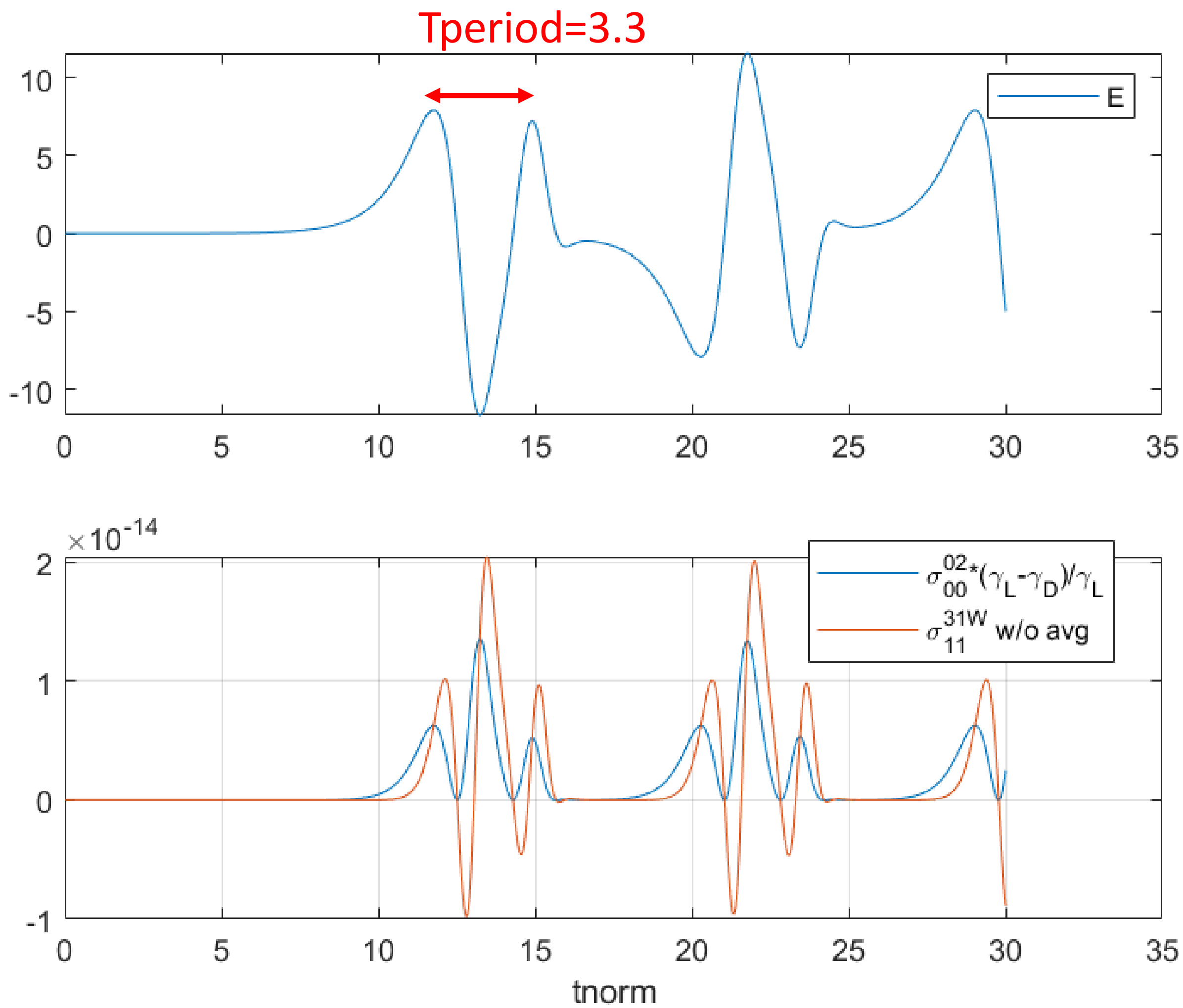


E^2	E^4		
$\sigma_{0,0}^{(0,2)}$	$\sigma_{0,0}^{(0,4,W)}$	$\sigma_{0,0}^{(2,2)}$	
$\sigma_{1,1}^{(1,1)}$	$\sigma_{1,1}^{(3,1,W)}$	$\sigma_{1,1}^{(1,3,W)}$	$\sigma_{1,1}^{(1,3,V)}$
			$\sigma_{2,2}^{(2,2)}$

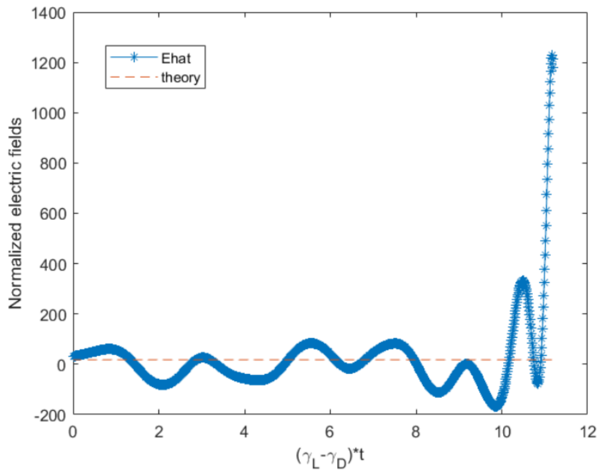
σ_D

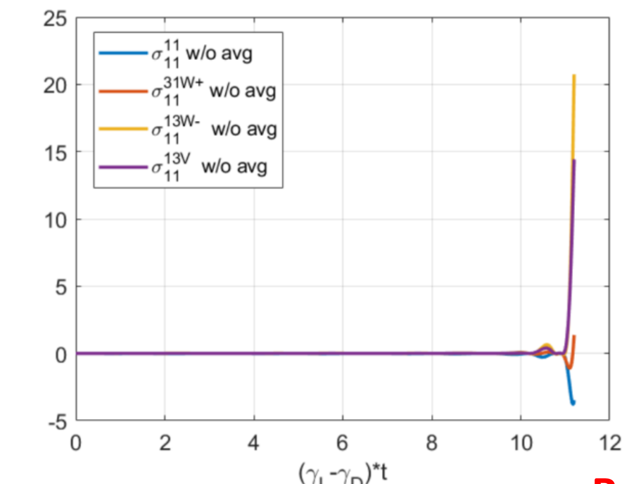


Case 2-2. limit cycle at $\hat{\nu} = D^{\frac{1}{3}}/(\gamma_L - \gamma_D) = 1.5$



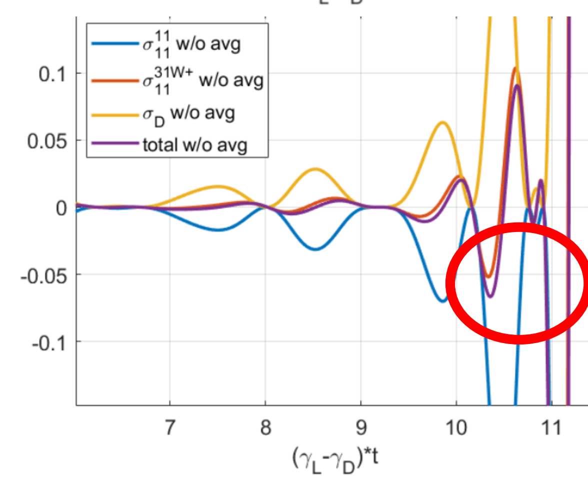
Case 3. Sudden Burst $\hat{\nu} = \nu/(\gamma_L - \gamma_D) = 2.5$





E^2	E^4		
$\sigma_{0,0}^{(0,2)}$	$\sigma_{0,0}^{(0,4,W)}$	$\sigma_{0,0}^{(2,2)}$	
$\sigma_{1,1}^{(1,1)}$	$\sigma_{1,1}^{(3,1,W)}$	$\sigma_{1,1}^{(1,3,W)}$	$\sigma_{1,1}^{(1,3,V)}$
			$\sigma_{2,2}^{(2,2)}$

σ_D



Burst when $\sigma_{1,1}^{(3,1,W)} < 0$!!

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Case 3: Precursor of the burst?

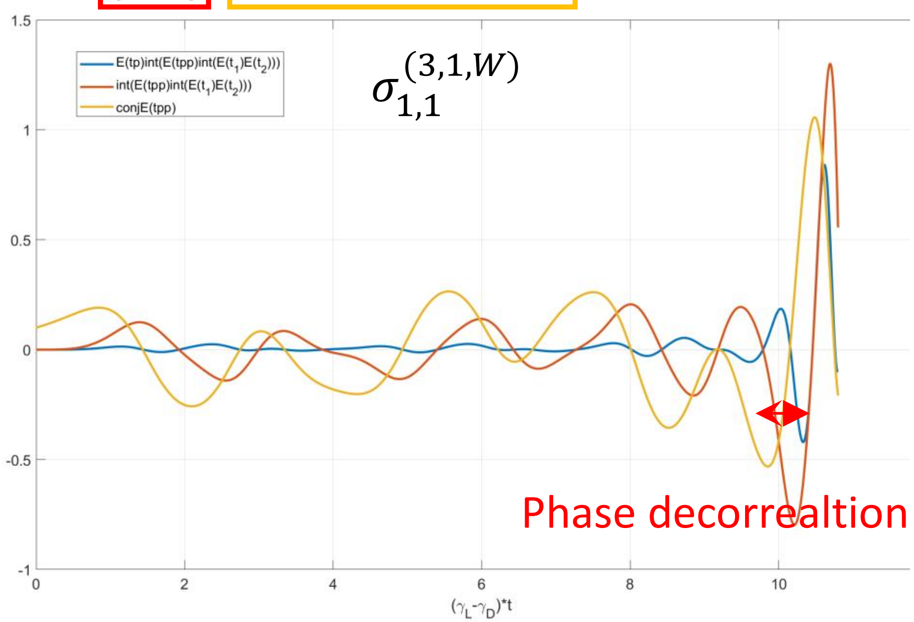
- The negative $\sigma_{1,1}^{(3,1,W)}$ coincides on the burst timing

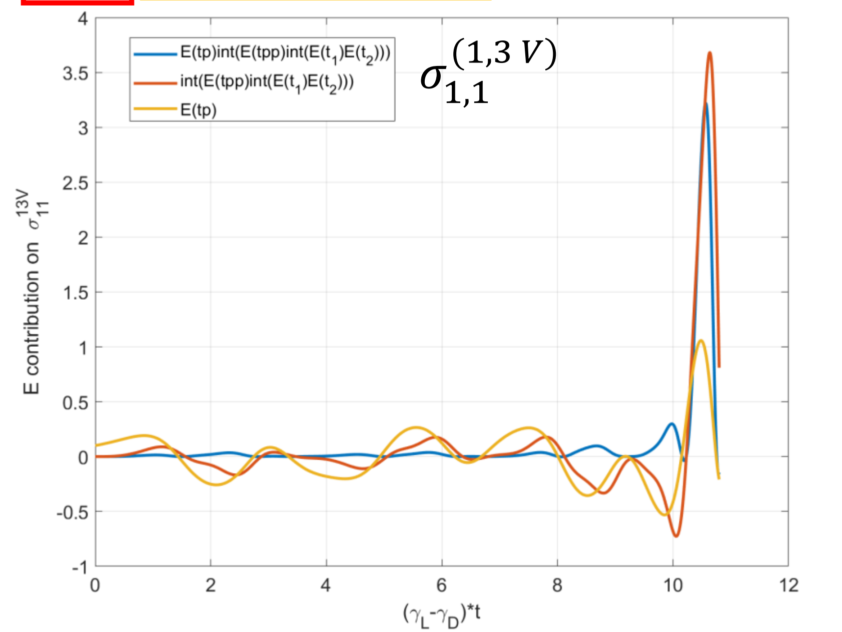
→ It cannot makes the saturation for $\int du\hat{\sigma}_{1,1}^{(1,1)} + \int du\hat{\sigma}_{1,1}^{(3,1,W+)} + \int du\hat{\sigma}_{D} = 0$

- Phase shift of the oscillating components is found → Memory mixing

$$\left(\frac{\hat{E}(t')}{2}\right)^* \frac{\hat{E}(t'')}{2}\frac{\hat{E}(t_1)}{2}\frac{\hat{E}^*(t''-t'+t_1)}{2}(t''-t')^2 e^{\nu(t_1+t''-2t')}$$

$$\left(\frac{\hat{E}^*(t')}{2}\right)\frac{\hat{E}^*(t'')}{2}\frac{\hat{E}(t_1)}{2}\frac{\hat{E}(t''+t'-t_1)}{2}(t''-t')(t''+t'-2t_1)e^{\nu(2t''-2t')}$$





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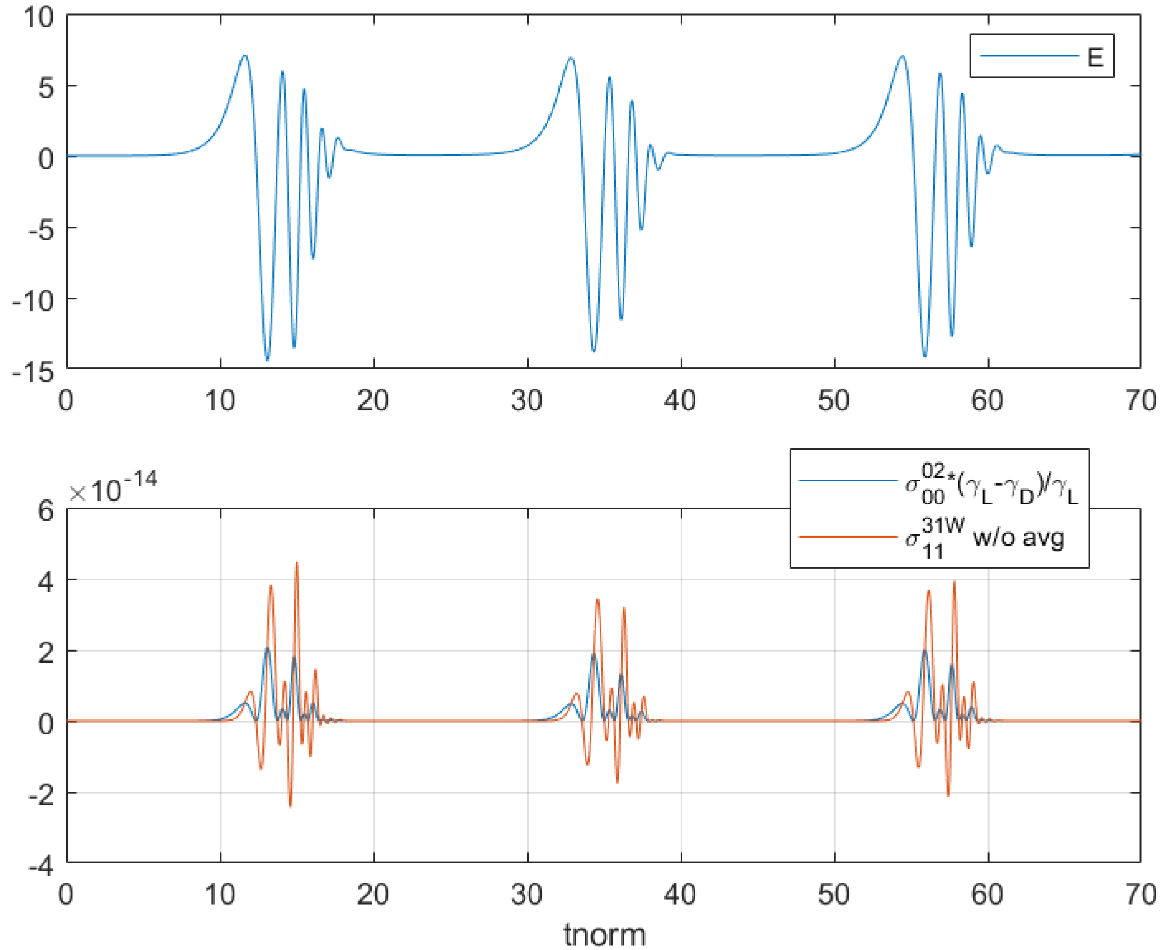
Case 4: Chaotic regime with low diffusion $\hat{\nu} = D^{\frac{1}{3}}/(\gamma_L - \gamma_D) = 1.3$

[Lilley et. al. PoP 2010]



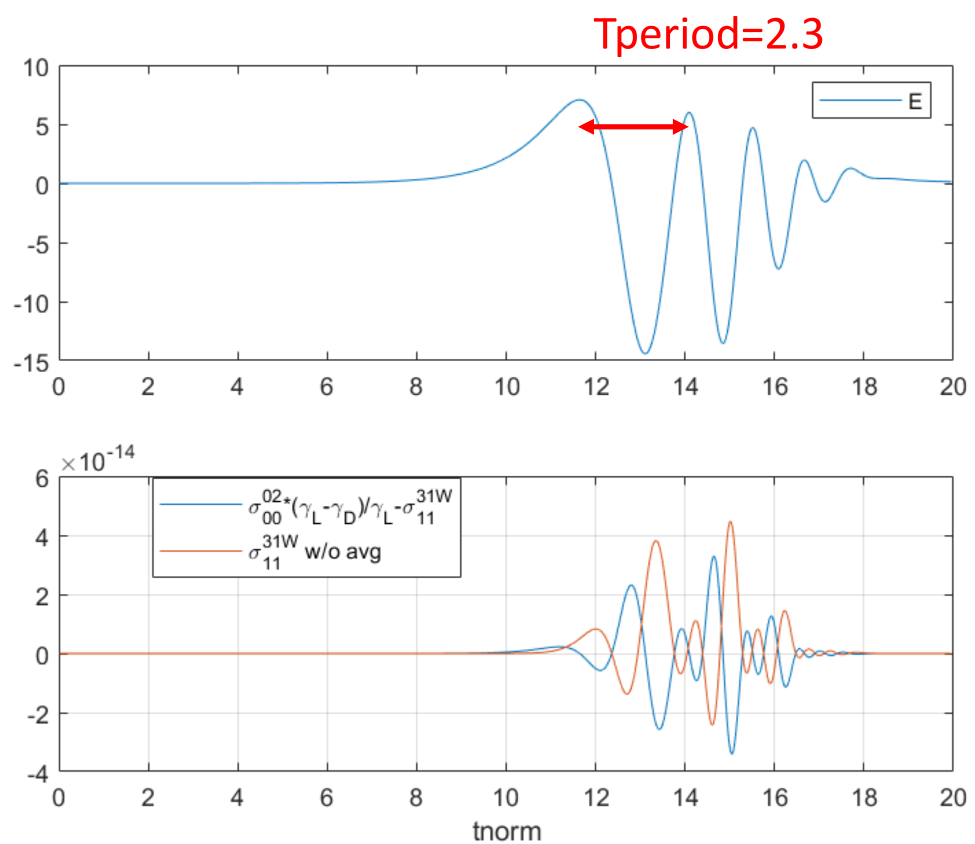
Case 4: Why showing tangent bifurcation?

- Reason of annihilation?

See the good cancellation between $\frac{1}{\gamma_L}\frac{\mathrm{d}\sigma_{0,0}^{(0,2)}}{\mathrm{dt}} = \frac{\gamma_L - \gamma_D}{\gamma_L}\sigma_{0,0}^{(0,2,W)}$-$\sigma_{1,1}^{(3,1,W)}$ and $\sigma_{1,1}^{(3,1,W)}$

→ As $\sigma_{1,1}^{(3,1,W)}$ is larger than $\frac{\gamma_L - \gamma_D}{\gamma_L}\sigma_{0,0}^{(0,2,W)}$, it is the decay equation like $\frac{dE}{dt} = -E^3$

- Then, why the nonlinear term becomes larger for this case?



Case 4: Decorrelation/asymmetry

- Temporal change of the symmetric terms is dominantly determined by

$$\frac{d\sigma^{(Q,sym)}}{dt} \propto \frac{dE}{dt}$$

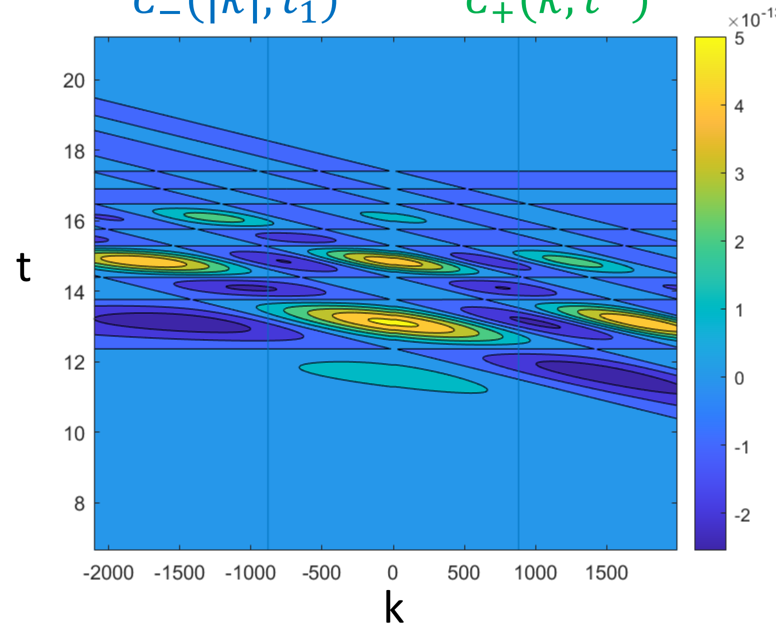
- For Non-symmetric term:

$$\frac{d\sigma^{(Q,Non-sym)}}{dt} \propto \frac{dE}{dt} + .. \frac{df}{dt}$$

→Mixed contribution will make the decorrelation

- Phase decorrelation due to the collision is well shown in the $\sigma_{1,1}^{(3,1,W+)}$

$C_-(|k|, t_1)$ $C_+(k, t'')$



$C_+(k, t'')$

$$\sigma_{1,1}^{(3,1,W+)} = \int_0^{T/2} dk k^2 e^{-2/3(Dk^3)-2\nu k+i\alpha k^2} \int_k^{T-k} dt'' \left(\hat{E}(t''+k)\right)^* \hat{E}(t'')e^{(-Dk^2+i\alpha k-\nu)t''} \times \int_k^{t''} dt_1 \hat{E}^*(t_1-k)\hat{E}(t_1)e^{(-Dk^2+i\alpha k-\nu)(-t_1)}$$

$C_-(k, t_1)$

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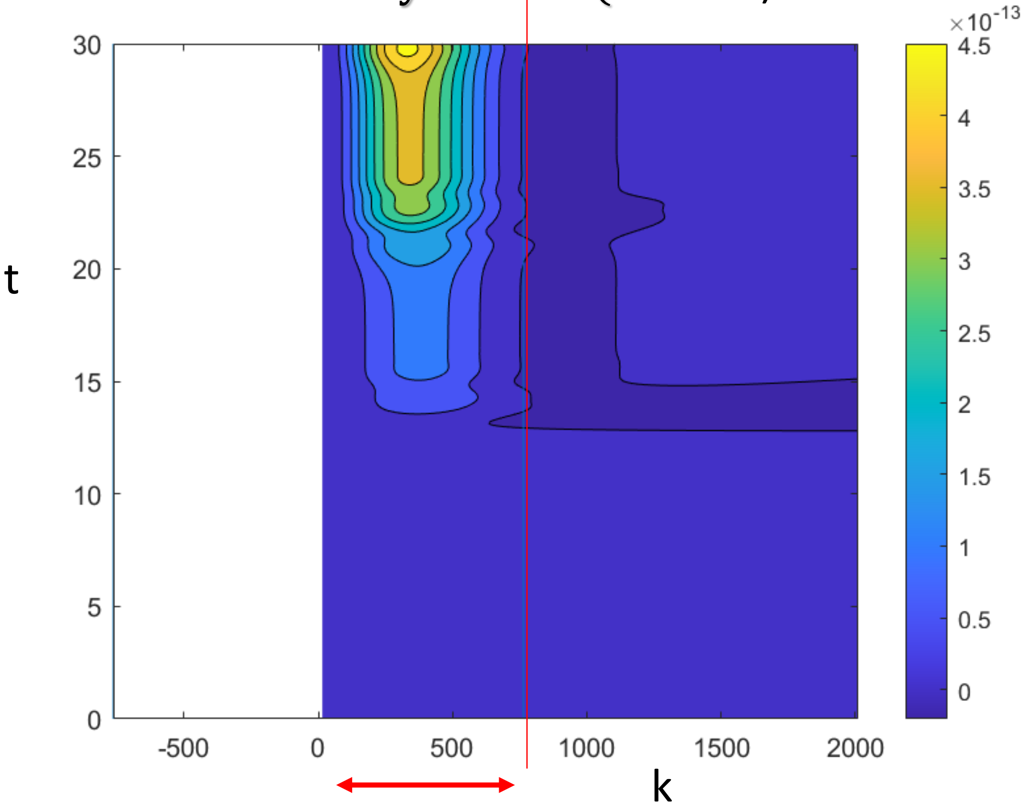
High nonlinear term when the oscillation is well alligned with the decay

Maximized when k~0.9kdecay

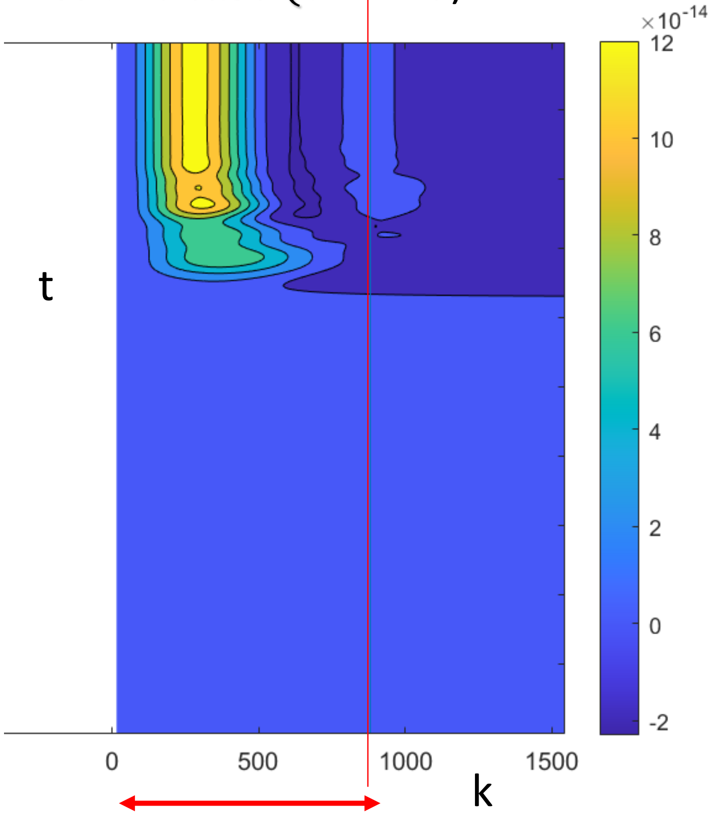
$$\sigma_{1,1}^{(3,1,W)} \sim k^2 e^{-\left(\frac{k}{kdecay}\right)^3} E(t)E(t-k)E(t)E(t+k)$$

<Limit cycle case ($\hat{\nu} = 1.5$)>



Tdecay=1.14 << Tperiod/2

<Caotic case ($\hat{\nu} = 1.3$)>



Tdecay=1.3 ~ Tperiod/2

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Conclusions

- 1-D nonlinear analysis in the given energetic particle source [Berk PRL 1997] is analyzed using the entropy production/transfer
- When the domain is f0 system, the entropy can be produced by both collisions and quasilinear interaction, while the source has the negative entropy production
- Entropy is not produced by the wave-particle interaction in total for any collisions, it is only transferred between the sub-systems
- For the entropy transfer, there are symmetric parts, which is likely positive-definite or negative-definite without collisions
- The non-symmetric part can change the sign in time, which can make the non-saturation or burst for a case of low collisions (long memory). For the annihilation of the chaotic results, the memory time is well aligned with the field oscillation, and it results in the larger nonlinear contribution than the quasilinear and dissipation

Thank you for your attention.