Energetic particle transport and 1D reduced model

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Outline

- Introduction: reduced models.
- Beam-plasma system vs. energetic particles interacting with the Alfvénic spectrum: mapping procedure.
- Application to ITER 15MA baseline scenario:
  - comparison of EP redistribution and mode dynamics;
  - transport analysis.
- Drive definition and dimensional issue.
- Comparative example with HMGC simulations.
- Conclusions.

Introduction: reduced models

Energetic particle (EP) excitation of global instabilities in burning plasmas is a key issue for the fusion sector.

$\rightarrow$ **Non-linear EP transport** processes (redistribution from the core to the edge) are crucial for the plasma heating and for the machine safety.

- These are intrinsically multi scale processes, thus **reduced models** are invoked:
  - marginal stability assumption; test-particle methods; quasi-linear models; Trapped-Gyro-Landau-Fluid model; 1.5D critical gradient model; kick model; quasi-linear gyrokinetic model (QuaLiKiz).

  For a detailed comparison between fully non-linear and reduced schemes for turbulence driven EP transport, see [N.N. Gorelenkov, et al. *Nuclear Fusion* **56**, 112015 (2016)] [C. Bourdelle et al., *PPCF* **58**, 014036 (2016)]

- Among reduced schemes, the **beam-plasma system (BPS)** can be paradigmatically addressed to describe the non-linear interactions between Alfvénic fluctuations and EP.

  [H.L. Berk and B.N. Breizman, *Phys. Fluids B* **2**, 2226 (1990)] [L. Chen, F.Zonca, *Rev. Mod. Phys.* **88**, 015008 (2016)] [B.N. Breizman and S.E. Sharapov, *PPCF* **53**, 054001 (2011)]

- The reduced **radial profile** of the burning plasma scenario can be mapped to the **velocity space** of the BPS.
- This results in a one-to-one correspondence.
- **Critical issue** - BPS: 1D uniform; 1 *d.o.f.*
  EP problem: 3D non-uniform; 2 *d.o.f.*
  Key elements: wave-particle power transfer; mode structure.
  $\rightarrow$ EP response is intrinsically different.
- **Previous applications**

  Nonlinear velocity redistribution caused by energetic-particle-driven geodesic acoustic modes:
  A. Biancalani, NC, A. Bottino, G. Montani, Z. Qiu, *J. Plasma Phys.* **84**(6), 725840602 (2018)
  Single beta-induced Alfvén Eigenmodes non-linear dynamics:
  NC, G. Montani, X. Wang and F. Zonca, *43rd EPS Conference on Plasma Physics* **40A**, P5.018 (2016)

- Hamiltonian N-body description of BPS - Fast **electron beam** injected into a **1D plasma** (**cold** linear dielectric medium) interaction with electrostatic longitudinal **Langmuir waves** ($\sim \omega_p$). Small density ratio $\eta \equiv n_B/n_p \ll 1$ (dimensionless parameter for runs):

$$\bar{x}_i' = u_i \ ,$$
$$u_i' = \sum_{j=1}^{m} \left(i\,\ell_j\ \bar{\phi}_j\ e^{i\ell_j \bar{x}_i} + c.c. \right) \ ,$$
$$\bar{\phi}_j' = -i\bar{\phi}_j + \frac{i\eta}{2\ell_j^2 N} \sum_{i=1}^{N} e^{-i\ell_j \bar{x}_i} - \bar{\gamma}_{dj}\bar{\phi}_j \ .$$

Notation: 1D plasma as a periodic slab of length L; beam particle positions and velocities labeled by x_i and v_i (N being the total particle number) and scaled as $\bar{x}_i = x_i(2\pi/L)$ and $u_i = \bar{x}_i' = v_i(2\pi/L)/\omega_p$, where $\tau = t\omega_p$ (prime indicates τ-derivative). Fourier components $\varphi_j(k_j, t)$: $\ell_j = k_j(2\pi/L)^{-1}$, $\phi_j = (2\pi/L)^2 e\varphi_j/m\omega_p^2$, $\bar{\phi}_j = \phi_j e^{-i\tau}$. Barred frequencies and growth rates are in ω_p units. External damping rate $\bar{\gamma}_{dj}$ for each mode. Positions $\bar{x}_i$ are initialized uniformly in $[0,\ 2\pi]$ ($N \simeq 10^6$), while modes at $\mathcal{O}(10^{-14})$ to ensure linear response.

- Warm beam - Initial velocities distributed with a given $F_{B0}(u)$.
- **Linearly unstable mode** - Resonance $\ell u_r = 1$ (*i.e.*, $kv_r = \omega_p$).
- **Dynamics**: mode exponential growth $\rightarrow$ non-linear mode saturation $\rightarrow$ trapped particles (phase-space rotating clumps) $\rightarrow$ f_B flattening.
- **Linear dispersion relation** ($M = \int du F_{B0}$):

$$2(\bar{\omega}_0 + i\bar{\gamma}_D - 1) - \frac{\eta}{M\ell}\int_{-\infty}^{+\infty} du \frac{\partial_u F_{B0}(u)}{u\ell - \bar{\omega}_0 - i\bar{\gamma}_D} = 0 \qquad (\bar{\omega} = \bar{\omega}_0 + i\bar{\gamma}_D) \; .$$

  - $DR(\eta) = 0 \;\Rightarrow\; \bar{\gamma}_D, \; \bar{\omega}_0$
  - $DR(\bar{\gamma}_D) = 0 \;\Rightarrow\; \eta, \; \bar{\omega}_0$

- Effective **Growth-rate**: $\bar{\gamma} = \bar{\gamma}_D - \bar{\gamma}_d$.
- **Scalings**: $|\bar{\phi}|^S \propto \bar{\gamma}^2, \;\; \bar{\omega}_B \simeq 3.31\,\bar{\gamma}, \;\; \Delta u_{NL} \simeq 8\,\bar{\gamma}/\ell$ (clump/res. width).

[NC, G. Montani, F. Zonca, *45th EPS Conference on Plasma Physics* **42A**, P5.1067 (2018)]
[NC, M. Del Prete, G. Montani, F. Squillaci, *J. Plasma Phys.* **86**, 845860503 (2020)]
[NC, G. Montani, M.V. Falessi, *J. Plasma Phys.* **86**, 845860401 (2020)]

BPS as reduced model for relevant EP simulations

Construction of a one-to-one link between the reduced **radial profile** of the EP problem and the **BPS velocity** derived from the resonance condition.

- **Single resonance** - Using two suitable normalization constants $\Omega_{1,2}$:

$$\frac{\omega^{AE}(r) - \omega^{AE}(r_r)}{\Omega_1} = \frac{k(v - v_r)}{\Omega_2} \; .$$

Notation: ω^{AE} is the mode frequency in the EP framework. The normalized Tokamak radius is $s = r/a$ (a minor radius). For frequencies (growth rates and damping) we use $\bar{\omega}^{AE} = \omega^{AE}/\Omega_1$ ($\bar{\gamma}^{AE} = \gamma^{AE}/\Omega_1$) with $\Omega_1 = \omega_{A0}$ ($\omega_{A0} = v_{A0}/R_0$). Furthermore n^{AE} will denote the toroidal mode number.

- **Local map** trough the expansion $\bar{\omega}^{AE}(s) = \bar{\omega}^{AE}(s_r) + (s - s_r)\partial_s\bar{\omega}^{AE}$:

$$v = v_r - \left|\Omega_2 \partial_s \bar{\omega}^{AE}\right| (s - s_r)/k \; .$$

Imposing boundaries ($s = 0 \;\mapsto\; v = v_{max}$ and $s = 1 \;\mapsto\; v = 0$) and resonance conditions, we finally get the dimensionless map:

$$\boldsymbol{u} = (1 - \boldsymbol{s})/\ell_1 \; ,$$

ℓ_1 parameter is arbitrarily fixed (periodicity length L of 1D BPS slab).

- **Drive definition** - BPS is closed by fixing η. Critical issue: EP problem: 3D $\Rightarrow$ BPS: 1D (higher dimensionality)
  $\rightarrow$ EP response intrinsically different: in BPS no modulation of power exchange (one class of resonant parts.), **transport more efficient.**
  Drives (normalized to the mode frequencies) are imposed as

$$\frac{\bar{\gamma}_D}{\bar{\omega}_0} = \alpha \, \frac{\bar{\gamma}_D^{AE}}{\bar{\omega}^{AE}} \,, \quad \text{with } \alpha \leqslant 1 \,.$$

  η is now given by **the dispersion relation** with $F_{H0}(s) \rightarrow F_{B0}(u)$.

- **Multiple modes** - Set reference resonance for fixing η w/o ambiguity
  $\rightarrow$ whole spectrum addressed using the proper resonance conditions:

$$\ell_j = \ell_1/(1 - s_{rj}) \,.$$

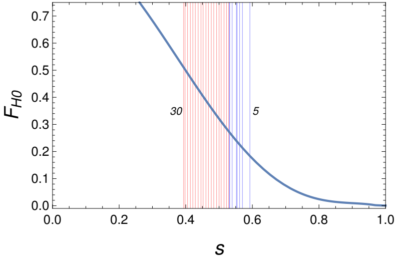
- **Damping** - To preserve asymptotic mode decay, we assume

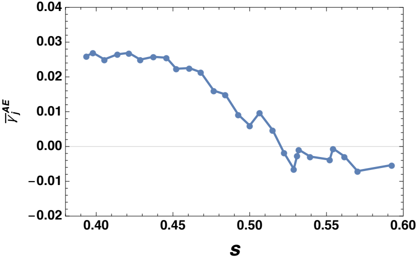
$$\bar{\gamma}_{dj}/\bar{\omega}_{0j} = \bar{\gamma}_{dj}^{AE}/\bar{\omega}_j^{AE} \,.$$

Application to ITER scenario

Reduced ITER 15MA beseline scenario:
M. Schneller, Ph. Lauber, S. Briguglio, *PPCF* **58**, 014019 (2016) **[SLB16]**
(EP interacting with Toroidal Alfvén Eigenmodes (TAEs)).

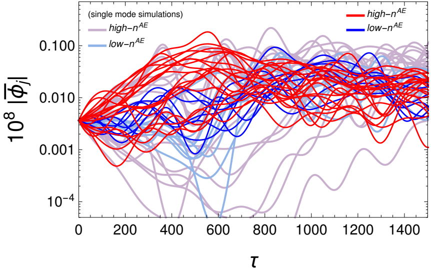
- **Setup and linear analysis** - Initial **EP slowing down** with least damped **27 TAEs**: $n^{AE} \in [12, 30]$ for the main branch (red) and $n^{AE} \in [5, 12]$ for the low branch (blue) almost **linearly stable**.

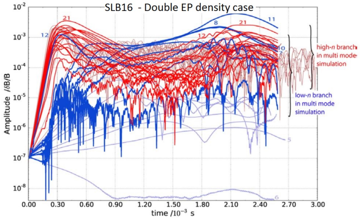




Reference resonance: $n^{AE} = 21$. **Optimization for** $\alpha = 0.4$.

- **Mode evolution** - Comparison of the multi mode simulations (bright colors) with respect to the 27 runs of single mode (opaque colors):

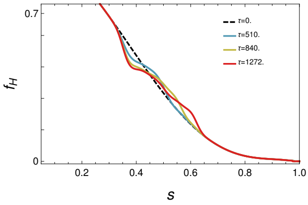


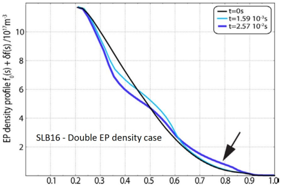


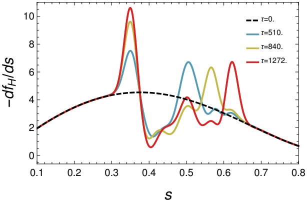
- Agreement with **SLB16**: low-n^{AE} branch (blue) is more efficiently and rapidly excited in multi-mode sims (despite negative drive).

Self-consistent multi mode simulations:
avalanche transport of particles in regions that can excite (resonate w/) stable modes.

- **EP redistribution** - Spectral evolution reflects on the EP profile:







- **Convective-like transport** toward the plasma edge: second peak (low branch) is shifting in time toward $s \simeq 0.65$.

- BPS fixed set of modes: the transport process is **bounded** due to the absence of further unstable modes.

- **SLB16**: outer redistribution triggered toward $s \simeq 0.85$. Importance of the **poloidal harmonics** spectrum (low branch has been simulated with 12 poloidal harmonics, the high branch with 2).

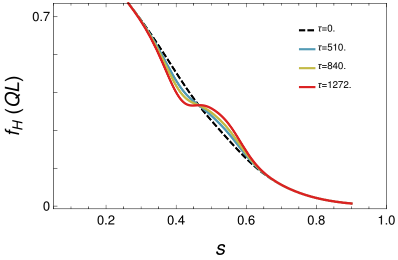
- **Quasi-linear (QL) model comparison** - QL equations:

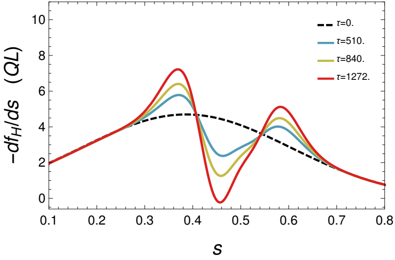
[NC, A.V. Milovanov, M.V. Falessi, G. Montani, D. Terzani and F. Zonca, *Entropy* **18**, 143 (2016)]
[G. Montani, F. Cianfrani, NC, *Plasma Phys. Contr. Fus.* **61**, 075018 (2019)]

$$f_H = F_{H0} - \pi \bar{\mathcal{N}} R \partial_s [(1-s)^{-5} \bar{\mathcal{I}}] \,,$$

$$\partial_t \bar{\mathcal{I}} = -\frac{\eta}{R}(1-s)^2 \, \bar{\mathcal{I}} \, \partial_s F_{H0} + \pi \eta (1-s)^2 \bar{\mathcal{N}} \, \bar{\mathcal{I}} \, \partial_s^2 [(1-s)^{-5} \bar{\mathcal{I}}] \,.$$

Notation: $R = \int ds F_{H0}$, while the spectrum is $\bar{\mathcal{I}}(\tau, s) = \ell_1^5 |\bar{\phi}|^2/\eta$. $\bar{\phi}(\tau, s)$ is the continuous mode spectrum derived from the discrete one, specified by means of the resonance conditions. $\bar{\mathcal{N}} = m/\Delta\ell$ is spectral density and $\Delta\ell = \text{Max}[\ell] - \text{Min}[\ell]$ the spectral width.





- Absence of avalanche: two well **localized peaks** avoiding the outer redistribution. QL model is not predictive, in agreement with SLB16.

Drive definition

- Dimensional reduction issue and role of α parameter.
- BPS: 1D uniform, 1 dof $\rightarrow$ AE problem: 3D non-uniform, 2 dof.
  - **Single toroidal mode number:** definition of motion constants and dimensional reduction

$$\boldsymbol{K} = \boldsymbol{E} - \omega \boldsymbol{P}_\phi / \boldsymbol{n} \, , \qquad \boldsymbol{\mu}$$

[S. Briguglio et al. *Phys. Plasmas* **21**, 112301 (2014)]

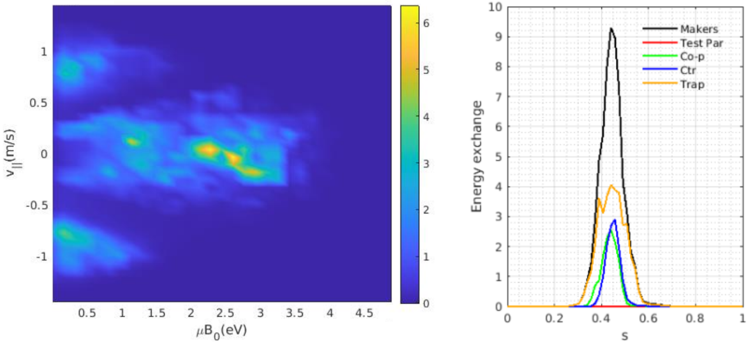
- Fix two constants of motions: cutting the phase space into infinitesimal **slices** that **do not mix together** (also nonlinearly)

  $\Rightarrow$ slice dynamics analyzed by **1D self consistent model**.

- Single **slice power-exchange** (proper maximizing sample)

  $\Rightarrow$ **quantitative** calibration of $\boldsymbol{\alpha}$.

- **n=25 TAE single resonance** [details in G. Meng talk].
  - $s_r = 0.43$; $\bar{\gamma}_D^{AE}/\bar{\omega}^{AE} = 0.034$; $\bar{\gamma}_d^{AE}/\bar{\omega}^{AE} = 0.009$; $\bar{\gamma}^{AE}/\bar{\omega}^{AE} = 0.025$

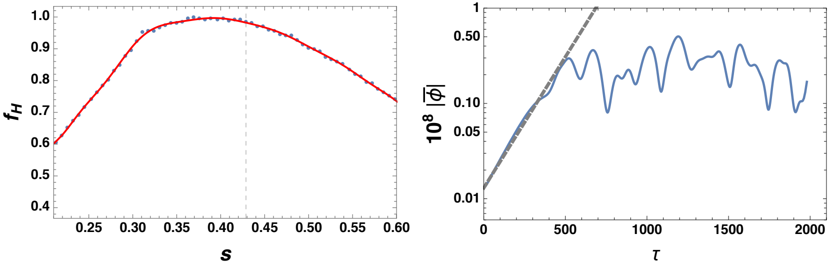


  - **Energy exchange fraction**: Trap=0.59; **Co-p=0.21**; Ctr=0.2.
  - **Slice: most resonant Co-p tracers** ($K = 1.25E03keV$; $\mu = 164keV$).

- Co-p pow. exchange fract. 0.21 = **slice contribution to linear drive**

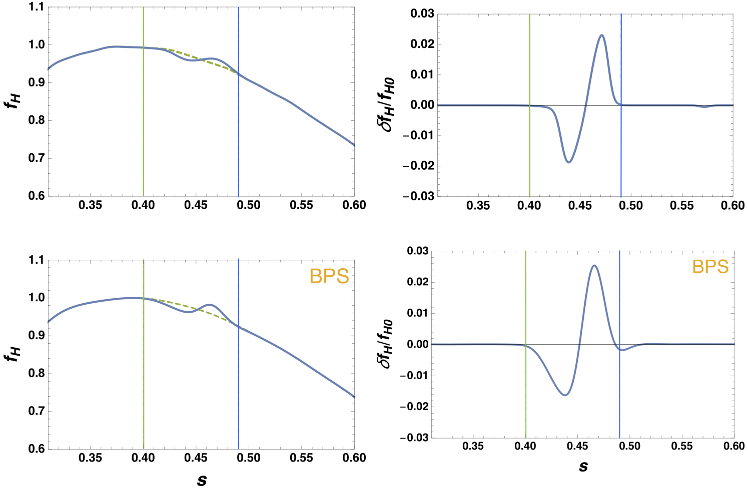
  $\Rightarrow$ $\alpha = 0.21$ **for linear drives** (integrates $DR(\bar{\gamma}_D) = 0$ provides η)

- Initial distribution of most resonant slice markers and mode evolution:



  - Difficult simulations due to flat profile around resonance and to drive reduction to avoid radial decoupling.

- **Flattening at saturation** $s_1 = 0.4$ (green) $s_2 = 0.49$ (blue):



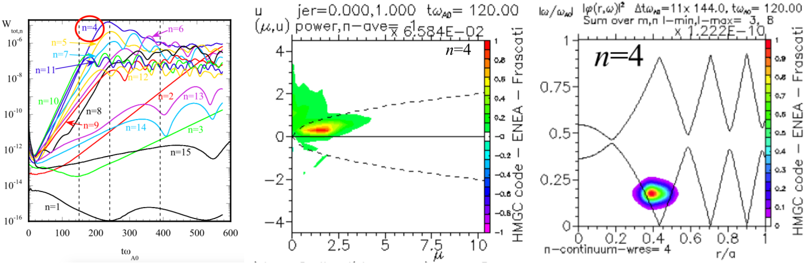
→ Very good predictive character of the self consistent BPS.

→ Power exchange and slice description provide a **quantitative definition of the mapping parameter α.**

Application to HMGC simulations

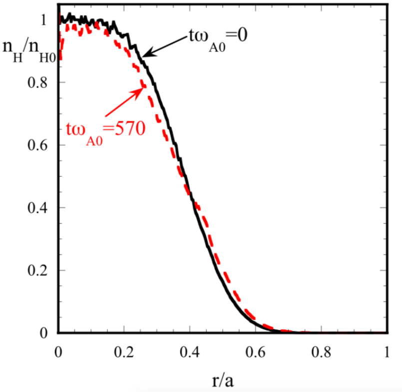
- G. Vlad *et al.*, *Nucl. Fusion* **58**, 082020 (2018) **V18**:
  multi- Vs single-mode for a generic symmetric static equilibrium.

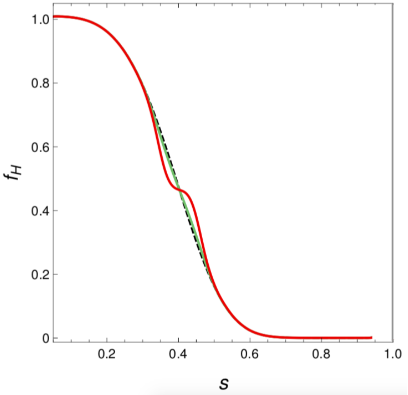
- **Single resonance** - Consider only the large saturated mode $n = 4$:



- A **single population** dominates the power exchange (**Trap**).

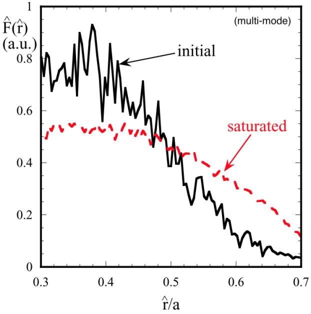
- Axisymmetric radial profile global redistribution $\Rightarrow \alpha \simeq 0.13$

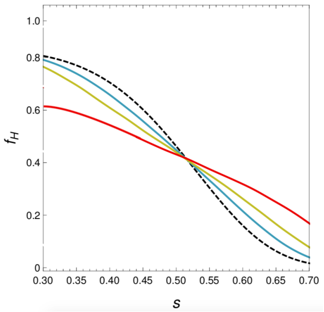




EP global response is due to average procedure including all particle (also) with negligible power-exchange.

- **Most resonant particles** (maximization of power exchange) **only**: morphology represented using **equal drives**: $\alpha = 1$ (**only one population** dominates wave-particle power exchange).





- Future developments: global response described by **coherent summation over many slice dynamics** mapped to reduced model.

Conclusions and Outlooks

- **Mapping procedure** between the velocity space of BPS and the radial profile of fast ions interacting with Alfvénic fluctuations.

  Advantages:

  - computational demand;
  - proper prediction of peculiar transport features: avalanche processes;
  - deviation from QL model and characterization of non-diffusive dynamics (frequently used in literature).

  Critical issues:

  - linear expansion of mode frequency (smooth radial frequency modification);
  - radial mode structure (non-uniformity) not accounted;

- **Outlooks** Summation of slice dynamics and description of the global transport features by means of the 1D BPS.