CNPS Workshop on Multi-scale Energetic particle Transport in fusion devices & MET Final Workshop – March 3-5, 2021

Flux surface averaged expressions of energetic particle fluxes for global transport studies of phase space zonal structures in tokamaks

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Background I

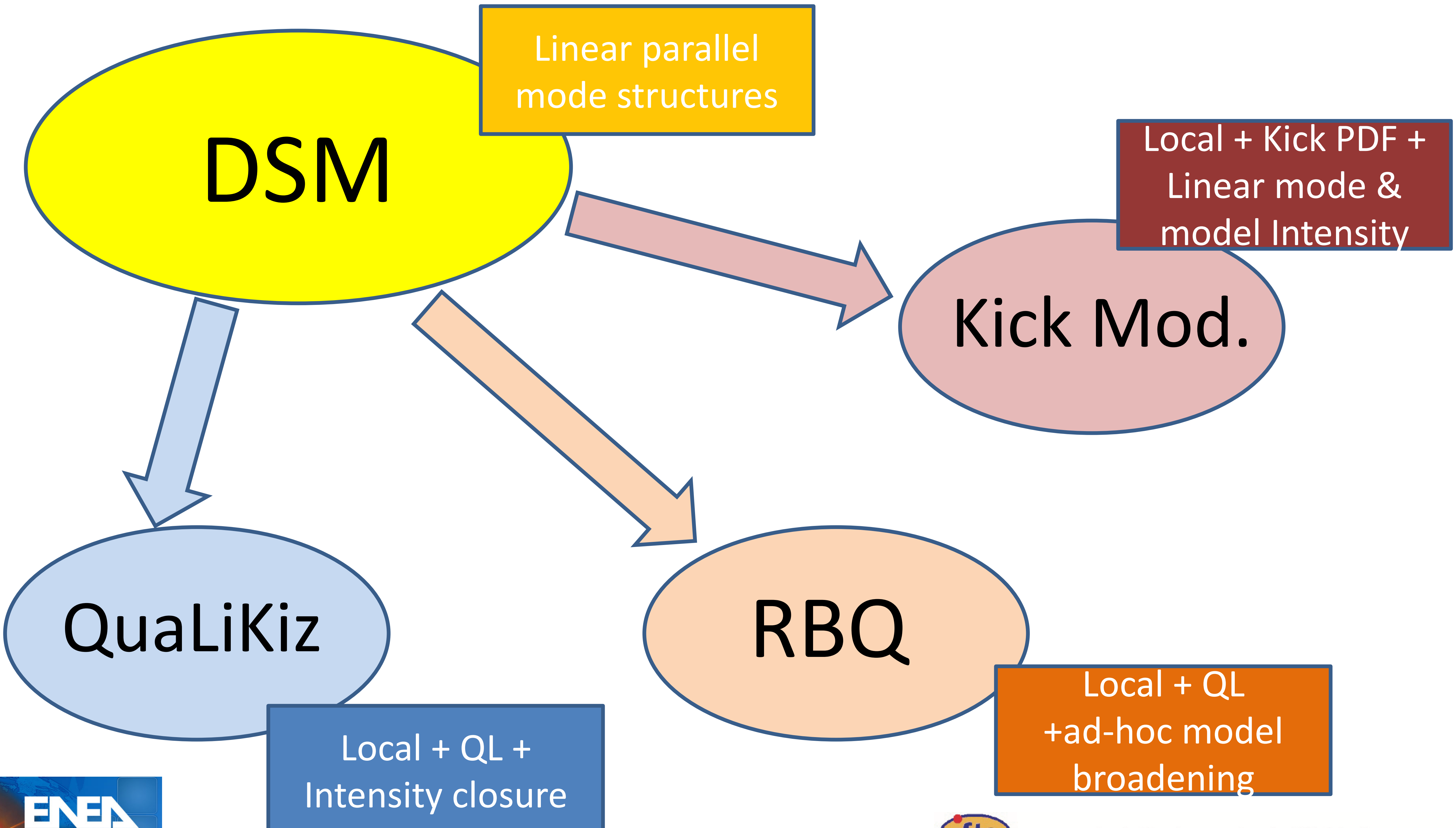
- **Energetic particles (EPs) in fusion plasmas are the major source of free energy in self-sustained burning plasma conditions**
  - Main source of instabilities in the Alfvénic fluctuation spectrum
  - Nonlinearly excite zonal field structures (ZFS)
  - Thus, they act as mediators of cross-scale couplings and regulate thermal plasma transport
  - Central physics for MET Project [C&Z RMP16, NJP15, PPCF15]
- **Importance of resonant processes**
  - Need for analyses of phase space features of fluctuation induced fluxes on long time scales
  - Naturally leads to the notion of neighboring nonlinear equilibria [C&Z NF07] and of Phase Space Zonal Structures (PSZS) [C&Z NJP15; Falessi et al. POP19, NJP21, PPCF21]
  - Long time scale transport described by PSZS dynamic evolution

Background II

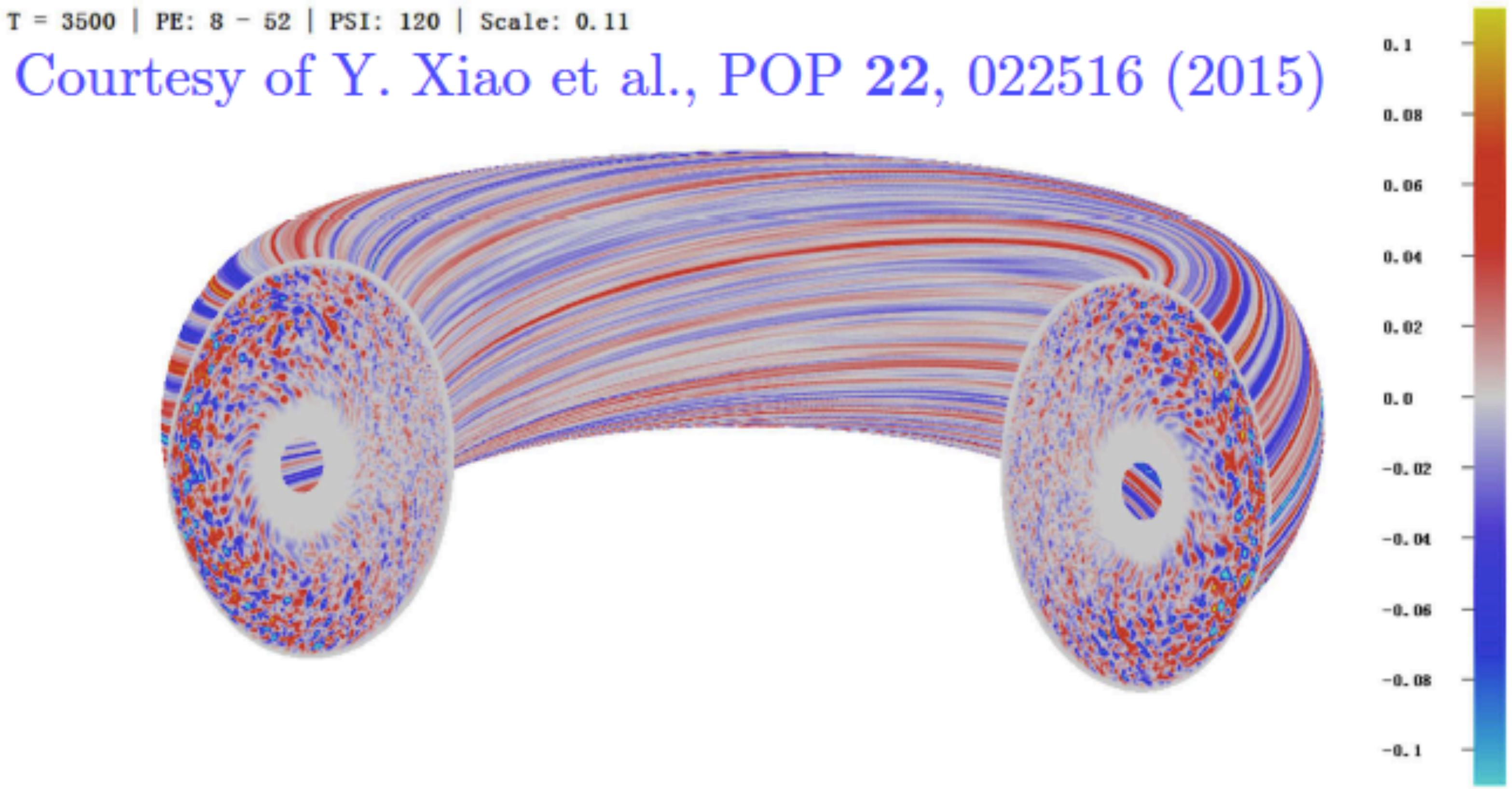
- **Multi-scale transport studies of PSZS**
  - Need for global e.m. non-perturbative approach, but still feasible for routine studies
    - Full NL GK approach still extremely costly
  - Multi-level reduced description proposed by MET rigorously formalized by the Dyson Schrödinger Model (DSM) [ZCFQ JPCS21]
    - M. Falessi: NL radial envelope equations
    - Y. Li: Kinetic effects and parallel mode structures
    - A. Zocco: Extension to stellarators
  - **Overarching and unified theoretical framework** for different levels of approximation
    - complete NL GK
    - Dyson Schrödinger Model (DSM)
    - Quasilinear transport

Background III

From DSM to: …



Structure formation in tokamaks I



Courtesy of Y. Xiao et al., POP **22**, 022516 (2015)

- ψ = poloidal magnetic flux

$$\boldsymbol{B}_0 = F(\psi)\boldsymbol{\nabla}\phi + \boldsymbol{\nabla}\phi \times \boldsymbol{\nabla}\psi$$

$$\equiv \boldsymbol{\nabla}\zeta \times \boldsymbol{\nabla}\psi$$

$$q \equiv \frac{\boldsymbol{B}_0 \cdot \boldsymbol{\nabla}\zeta}{\boldsymbol{B}_0 \cdot \boldsymbol{\nabla}\theta} = q(\psi)$$

- Filaments $\Rightarrow$ Parallel mode structures $\Rightarrow$ weighing of linear and nonlinear interactions [Zonca et al, PPCF15], [C&Z RMP16]
- Correspondingly, nonlinear interactions can take the forms of mode coupling between two toroidal mode numbers, and modulation of the radial envelope.
- Proper description of fluctuation spectrum in toroidal geometry requires extending standard wave-kinetic equation to a non-linear Schrödinger-like equation (NLSE) [C&Z RMP16].

Structure formation in tokamaks II

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- **Back-scattering and focusing/defocusing of wave packets**
  - Linear and nonlinear processes enter on the same footing
    - Need to properly modify WKE into NL Schrödinger-like envelope equation
  - Fundamental qualitative and quantitative difference
    - Envelope gives the superposition rule for filaments, which exists as local solutions characterized by prescribed (calculated) dispersiveness, polarization and group velocity. All consistent with the toroidal geometry and radial non-uniformity
    - These quasi-particles are continuously emitted and reabsorbed [Z&C PPCF15] and are fundamentally different from the plane waves usually involved in WKE

Structure of turbulent fluxes I

- **Adopt mode structure decomposition approach** [Z. Lu 12]

$$\begin{bmatrix} \delta\phi(r,\theta,\zeta) \\ \delta\psi(r,\theta,\zeta) \end{bmatrix} = 2\pi \sum_{n,\ell\in\mathbb{Z}} e^{in\zeta - inq(\psi)(\theta-2\pi\ell)} \begin{bmatrix} \delta\hat{\phi}_n(r,\theta-2\pi\ell) \\ \delta\hat{\psi}_n(r,\theta-2\pi\ell) \end{bmatrix}$$

- Introduce $\delta\hat{\phi}_{\parallel n} \equiv \delta\hat{\phi}_n - \delta\hat{\psi}_n$, radial envelope, A_n and polarization vector $\hat{\boldsymbol{e}}_n$

$$\begin{pmatrix} e\delta\hat{\psi}_n(r,\vartheta;t)/T_{0i} \\ e\delta\hat{\phi}_{\parallel n}(r,\vartheta;t)/T_{0i} \end{pmatrix} \equiv A_n(r,t)e^{iS_n(r,t)} \begin{pmatrix} e_1(r,t)y_1(r,\vartheta) \\ e_2(r,t)y_2(r,\vartheta) \end{pmatrix} .$$

with eikonal representation and normalizations

$$\int_{-\infty}^{\infty} |y_{1,2}(r,\vartheta)|^2 \, d\vartheta = 1 \, . \qquad \hat{\boldsymbol{e}}_n^{+} \cdot \hat{\boldsymbol{e}}_n = 1 \, .$$

- **Allows accounting for all spatiotemporal structures of turbulent fluxes** [FCQZ JPCS21,NJP21,PPCF21]

Structure of turbulent fluxes II

⊙ Since PSZS are undamped by (fast) collisionless dissipation mechanisms, they are naturally expressed as functions of invariants of motion (nearly integrable Hamiltonian system). Separating fast ($[...]_F$) from slow variations,

$$F_z \equiv \bar{F}_0 + e^{-iQ_z}\left(\overline{e^{iQ_z}\delta F_z}\Big|_F + \delta\tilde{F}_{Bz} \right) ,$$

macro- ⊕ meso-scale (CGL) micro-scale collisionless damped

where $\overline{[...]} = \oint d\ell/v_\parallel [...]/\oint d\ell/v_\parallel$, $\overline{\widetilde{[...]}} = 0$, F_z is the $n = 0$ gyrocenter particle distribution function. $e^{-iQ_z} \Rightarrow$ nonlocal (integral) particle response

$$\partial_t \overline{e^{iQ_z}\bar{F}_0} = - \overline{e^{iQ_z}\frac{F(\psi)}{B_0}\partial_t \left\langle \delta A_{\parallel g}\right\rangle_z \frac{\partial}{\partial\bar{\psi}}\bar{F}_0}\Big|_S - \frac{1}{\tau_b}\frac{\partial}{\partial\psi}\left[\tau_b \overline{e^{iQ_z}\delta\dot{\psi}_z\delta F_z}\right]_S$$

$$-\frac{1}{\tau_b}\frac{\partial}{\partial\mathcal{E}}\left[\tau_b \overline{e^{iQ_z}\delta\dot{\mathcal{E}}_z\delta F_z}\right]_S - \frac{1}{\tau_b}\frac{\partial}{\partial\psi}\left[\tau_b \overline{e^{iQ_z}\delta\dot{\psi}\delta F}\right]_{zS} - \frac{1}{\tau_b}\frac{\partial}{\partial\mathcal{E}}\left[\tau_b \overline{e^{iQ_z}\delta\dot{\mathcal{E}}\delta F}\right]_{zS}$$

$$+ \overline{e^{iQ_z}\left[C_g + \mathcal{S}\right]}\Big|_{zS} .$$

[M.V. Falessi I-14 Varenna 2020]

Structure of turbulent fluxes III

- **Spatiotemporal structures of turbulent fluxes** [FCQZ JPCS21,NJP21,PPCF21]
  - Describe only the collisionless undamped component (not the whole flux surface averaged phase space fluxes)
  - Describe both "slow" (CGL equilibrium evolution) and "fast" (NL spatiotemporal scale; e.g. avalanches) evolution
  - All accounted for by adopted mode structure decomposition

→ convolution integral & action angle variables [NJP15]

$$\int \frac{d\ell}{v_\parallel} = \int \frac{d\theta}{\dot{\theta}} \Rightarrow \frac{\tau_b}{2\pi} \int d\eta$$

Second invariant

$$J = \oint v_\parallel d\ell \Longleftrightarrow \eta = \frac{2\pi}{\tau_b} \int_0^{\vartheta} \frac{d\theta}{\dot{\theta}}$$

ballooning space map

Structure of turbulent fluxes IV

- **Adopt normalizations as in** [Chen JGR99] (e.g.; particle flux)

$$\delta \hat{F} = \frac{1}{2} \sum_n \left(A_n(r,t) e^{iS_n(r,t)+in\zeta} \delta \hat{f}_n + c.c. \right)$$

$$\overline{e^{iQ_z} \delta\dot{\psi} \delta F} = \frac{\pi}{2} \sum_{n\neq 0,\ell,j} \oint d\eta \left[e^{iQ_z} |A_n(r,t)|^2 \right.$$
$$\left. \times e^{2\pi i n q j} \delta\hat{\dot{\psi}}^*_n(r,\theta - 2\pi\ell) \delta \hat{f}_n(r,\theta - 2\pi(\ell+j)) + c.c. \right]$$

- Extracted wave intensity dependence, leaving parallel mode structures implicit
  - Orbit shift and ~ nqj phase: microscale (j=0 dominant)
  - Radial envelope: mesoscale and macro scale
  - ℓ summation: weighing on filaments (parallel mode structure)

$$2\pi \sum_\ell \simeq \int_{-\infty}^{\infty} d\vartheta$$

Structure of turbulent fluxes V

- **Description is fully consistent with global e.m. gyrokinetic/ hybrid approach** [Wang&Briguglio, this Workshop]
  - Verified in EPM simulations by HMGC [Briguglio&Wang] and HYMAGYC [Vlad&Fusco]
  - Assuming parallel filament structures from linear theory is justified as long as the nonlinear time scale is longer than the inverse mode frequency **Dyson Schrödinger Model (DSM) [ZCFQ JPCS21]**
  - More details on DSM [FCQZ, this Workshop]
  - Works not only for **SAW excited by EP but also for DW turbulent transport**
    - Chen et al. POP2000, 2001, etc. (see [PPCF2015])
    - Recent numerical studies by C. Gillot (PhD Thesis; 2020), comparing QuaLiKiz vs GISELA

Dyson Schrödinger Model I

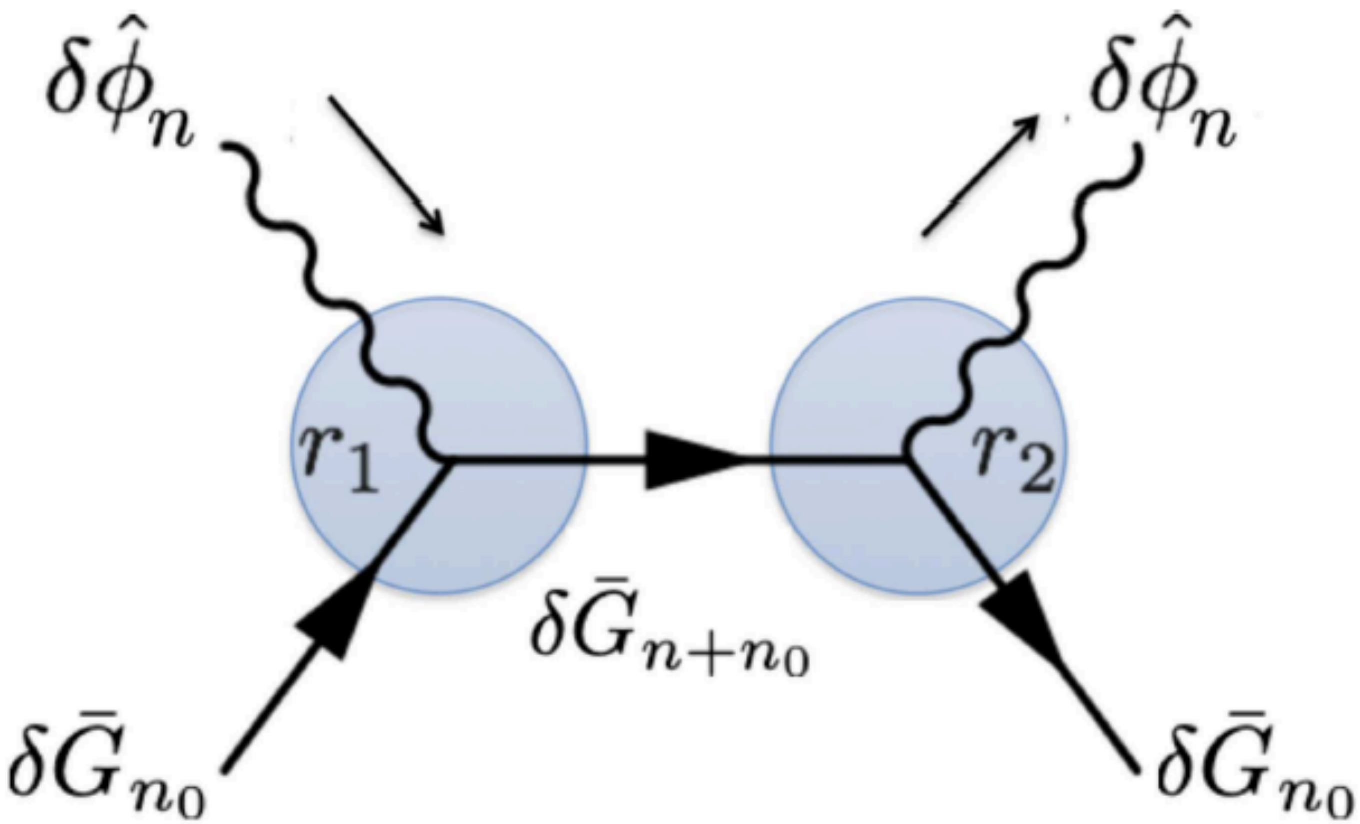
- Having adopted action angle variables, the solution of **Dyson Schrödinger Model (DSM)** is technically challenging but straightforward
  - Present results refer to the fishbone paradigm (precession resonance) for simplicity. General result discussed in [FCQZ JPCS21,NJP21,PPCF21]
  - Perturbation expansion of particle response in the weak field intensity limit, yields secular terms dominated by emission and absorption of the same k-fluctuation (called diagonal interactions).
  - Extension to stellarators [Zocco et al, this Workshop]

Dyson Schrödinger Model II

- Particle response to test k_0 fluctuation $\Rightarrow$ renormalization [Dupree 66], [Mima 73], [Laval & Pesme 84,99].

$$\left[i\left(\bar{\omega}_{d0}(r) - \omega_{k_0} - i\partial_t\right) - i\frac{cn_0}{d\psi/dr}\overline{e^{iQ_{k_0}}\partial_r \left\langle \delta L_g \right\rangle_z e^{-iQ_{k_0}}} - \Delta - F\frac{\partial}{\partial r} - \frac{\partial}{\partial r}D\frac{\partial}{\partial r}\right]\delta\bar{G}_{k_0}$$

$$= i\overline{\left[e^{iQ_{k_0}}\left(\frac{e}{m}Q\bar{F}_0\left\langle \delta L_g\right\rangle_{k_0}\right)\right]} - i\frac{cn_0}{d\psi/dr}\overline{e^{iQ_{k_0}}\left\langle \delta L_g\right\rangle_{k_0} e^{-iQ_z}}\frac{\partial \delta\bar{G}_z}{\partial r} + [\text{NL OFF DIAG.}]$$



- $Q\bar{F}_0 = i\partial_\mathcal{E}\bar{F}_0\partial_t - i\Omega^{-1}\mathbf{b}\times\nabla\bar{F}_0\cdot\nabla$
- ∂_t accounts for the temporal meso-scale variation
- toroidal geometry effects through the radial profile of $\bar{\omega}_{d0}(r)$, precession frequency dependence on the constants of motion, and bounce averaging

Bounce averaged expressions left implicit: to be made explicit as shown above

Dyson Schrödinger Model III

- Physical interpretation [Dupree 66]:
  - Δ: nonlinear complex frequency shift
  - F: nonlinear anti-symmetric resonance distortion (in radius)
  - D: nonlinear resonance broadening (in radius)

$$\Delta = -\sum_{k\neq -k_0}\left(1+\frac{n}{n_0}\right)\left(\frac{n_0 c}{d\psi/dr}\right)^2 \overline{e^{iQ_{k_0}}\partial_r \left\langle \delta L_g\right\rangle_{-k} e^{-iQ_{k+k_0}}}\frac{i}{(\bar{\omega}_d-\omega)_{k+k_0}}\overline{e^{iQ_{k+k_0}}\partial_r \left\langle \delta L_g\right\rangle_{k} e^{-iQ_{k_0}}}$$

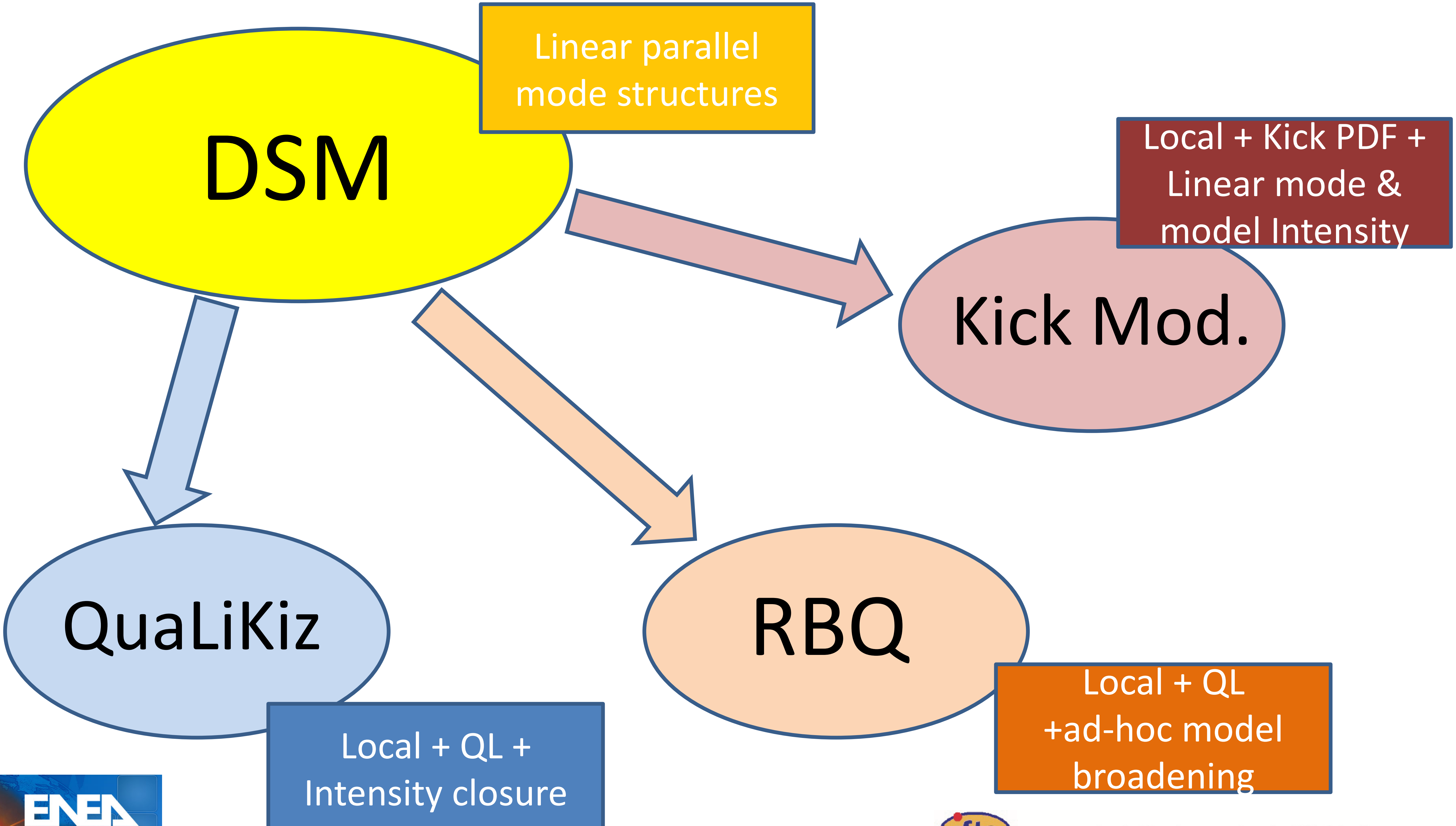
$$F = \sum_{k\neq -k_0} n n_0 \left(\frac{c}{d\psi/dr}\right)^2 \overline{e^{iQ_{k_0}}\partial_r \left\langle \delta L_g\right\rangle_{-k} e^{-iQ_{k+k_0}}}\frac{i}{(\bar{\omega}_d-\omega)_{k+k_0}}\overline{e^{iQ_{k+k_0}} \left\langle \delta L_g\right\rangle_{k} e^{-iQ_{k_0}}}$$

$$-\sum_{k\neq -k_0} n n_0 \left(\frac{c}{d\psi/dr}\right)^2 \overline{e^{iQ_{k_0}} \left\langle \delta L_g\right\rangle_{-k} e^{-iQ_{k+k_0}}}\frac{i}{(\bar{\omega}_d-\omega)_{k+k_0}}\overline{e^{iQ_{k+k_0}}\partial_r \left\langle \delta L_g\right\rangle_{k} e^{-iQ_{k_0}}}$$

$$D = \sum_{k\neq -k_0} \left(\frac{nc}{d\psi/dr}\right)^2 \overline{e^{iQ_{k_0}} \left\langle \delta L_g\right\rangle_{-k} e^{-iQ_{k+k_0}}}\frac{i}{(\bar{\omega}_d-\omega)_{k+k_0}}\overline{e^{iQ_{k+k_0}} \left\langle \delta L_g\right\rangle_{k} e^{-iQ_{k_0}}}$$

Dyson Schrödinger Model IV

Recovering QL limit: … for a broad spectrum

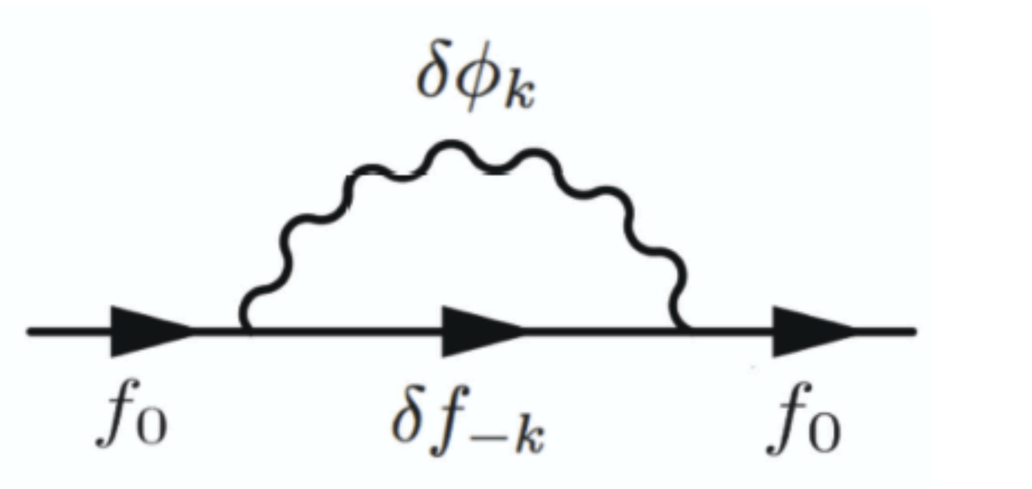


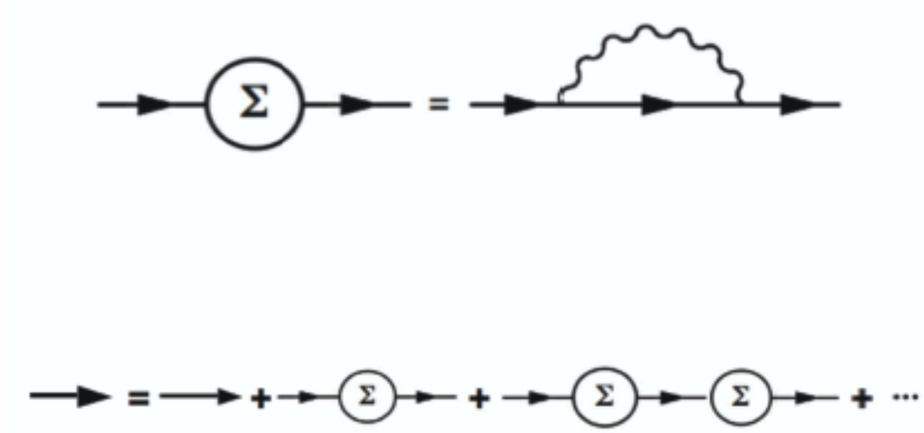
Evidence of avalanches

- Dropping for simplicity radial modulations by ZS, and writing formal solution for the particle response ($\delta\bar{G}_k$) keeping relevant NL terms

$$\partial_t \delta \bar{G}_z \sim -i \sum_{k} \frac{nc}{d\psi/dr}\frac{\partial}{\partial r}\left[\overline{e^{iQ_z}\langle\delta L_g\rangle_{-k}\, e^{-iQ_k}}\, \frac{(nc)/(d\psi/dr)}{(\bar{\omega}_{dk} - \omega_k - i\partial_t + i\Delta \ldots)}\, \overline{e^{iQ_k}\langle\delta L_g\rangle_{k}\, e^{-iQ_z}}\, \frac{\partial}{\partial r}\delta\bar{G}_z\right]$$

- Importance of fluctuation spectrum in determining NL PSZS evolution:
  - broad spectrum ⇒ QL diffusion [Al'tshul' & Karpman 65]
  - narrow (quasi-coherent) spectrum: ballistic transport of phase-locked particles and convective amplification of radially propagating wave-packets (NLSE) [C&Z RMP16]





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Conclusions

- Overarching and unified theoretical framework suggested for MET with different levels of approximation for self-consistent description of EP transport in tokamaks has been successfully derived and demonstrated
  - complete NL GK/Hybrid
  - Dyson Schrödinger Model (DSM)
  - Quasilinear transport
- Numerical applications have been made for selected reference cases and verification of the approach is complete
- Applications to realistic cases of practical interest (CFETR, DTT) are in progress along with the extension to 3D stellarator equilibria [A. Zocco et al.]