Stability analysis of plasma waves driven by runaway electrons in the hybrid frequency range via Cerenkov resonance

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Observations of RE driven instabilities

- RE driven instabilities have been documented since the early ’70 [Vlasenkov et al. NF 1973]
- Often accompanied by jumps in $\perp$ electron energy [Causa et al NF 2019]
- Recently radio-emissions have been detected in 100-200 MHz frequency range in DIII-D (whistler waves) [Spong et al PRL 2018]
- Radio-emissions have been detected in FTU [Buratti et al. to be submitted]

Why we study RE-driven instabilities

- Diagnostic tools

Radio-emission during the current rise suggest the occurrence of RE before any other signal

- Method of mitigation by injection of RF waves

Warning: LH injection might increase RE

Theoretical analysis (I)

- Two main candidate cold plasma waves:

  Whistler (fast waves) [1],…

  Magnetized EPW = LH waves (slow waves) [2],…

- Two RE-waves interactions

  Anomalous Doppler resonance: $\omega - k_{\|} V_{\|} + \frac{|\omega_c|}{\gamma} = 0,$

  Cherenkov resonance: $\omega - k_{\|} V_{\|} = 0$

- Higher frequency waves (X electron waves) have been also considered [3]

[1] Fulop et al PoP (2006)
[2] Parail and Pogutse NF (1978)
[3] Pokol et al PoP (2014)

Theoretical analysis (II)

- The time evolution of the distribution function under RE-wave interaction can bring new instabilities

  [Parail and Pogutse NF 1978]

  [Liu et al. PRL 2018]

- WW are dominant compared to LHW
- Only AD for WW and LHW (standard fdd)

  [Aleynikov and Breizman NF 2015]

Outline

- Kinetic evaluation of the RE susceptibility near Cherenkov resonance
- Growth rates of slow and fast wave instability in the hybrid frequency range
- Collisional damping and instability threshold

Electrostatic susceptibility

$$\chi_b = \frac{\boldsymbol{k}\cdot\bar{\bar{\chi}}\cdot\boldsymbol{k}}{k^2} = \frac{\omega_{po,b}^2}{\omega\Omega_e}\frac{k_\parallel^2}{k^2}\int_0^{+\infty} dp_\perp\, 2\pi p_\perp \int_{-\infty,\mathcal{L}}^{+\infty} dp_\parallel \frac{\Omega}{\omega - k_\parallel \mathrm{v}_\parallel} J_o^2\left(\frac{k_\perp \mathrm{v}_\perp}{\Omega}\right) p_\parallel \frac{\partial f_b}{\partial p_\parallel}$$

$$f_b \cong \frac{1}{\pi}\frac{e^{-p/p_o}}{p_o p^2}\frac{e^{-\theta^2/\theta_o^2}}{\theta_o^2}$$

$\sigma >> 1$

$$f_b = \frac{1}{2\pi}\frac{e^{-p/p_o}}{p_o p^2} c_\sigma \sigma e^{-\sigma[1-\cos(\theta)]}$$

$$\lim_{\sigma\to+\infty} f_b = \frac{1}{2\pi}\frac{e^{-p/p_o}}{p_o p^2}\frac{\delta(\theta)}{\sin(\theta)}$$

$$\chi_b = \frac{\omega_{po,b}^2}{k^2 c^2}\frac{k_\parallel c}{\omega}\int_0^{\pi} d\theta \int_{0,\mathcal{L}}^{+\infty} du\; I_\theta(u)$$

$p = m_e\, c\, u$

Electrostatic susceptibility

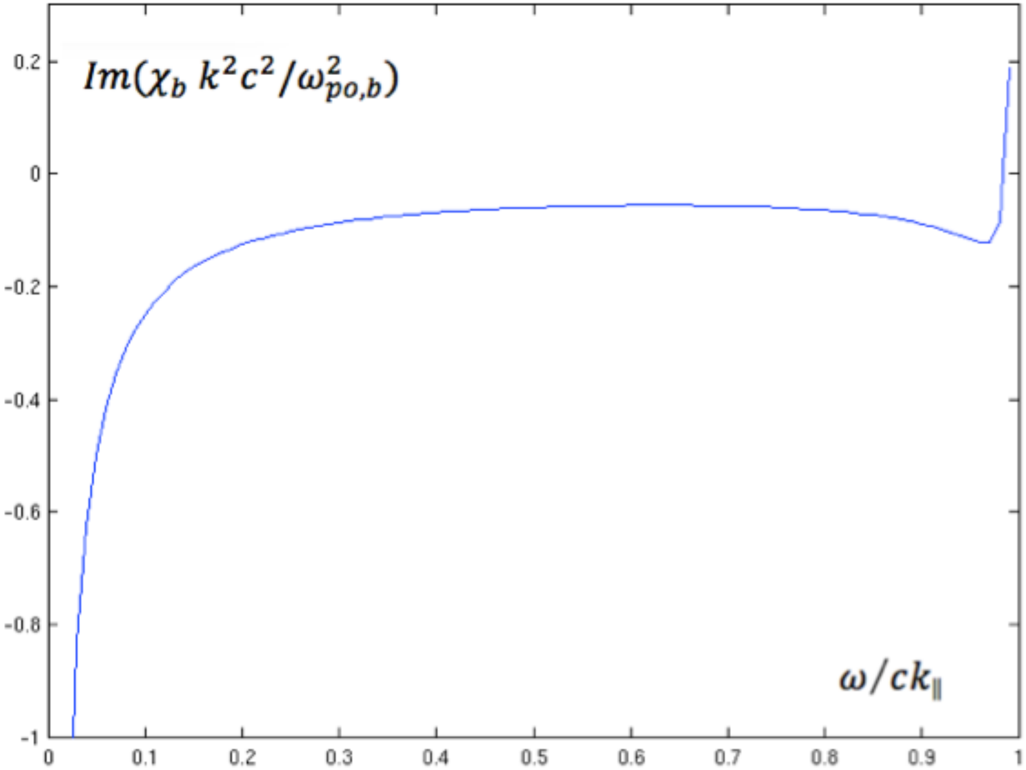
Analytic evaluation for real ω, $N_{//}>1$, $\sigma>>1$

$$Im(\chi_b) \cong \pi s' \frac{\omega_{po,b}^2}{k^2c^2}\frac{u_{po}^2}{u_o} N_\parallel^3 e^{-u_{po}/u_o}\left[\frac{u_{po}}{u_o}\left(1-\frac{3}{\sigma}\right)-\frac{4}{\sigma}\right]$$

For $k_{//}>0$, $\omega>0$

Negative values

Unstable modes



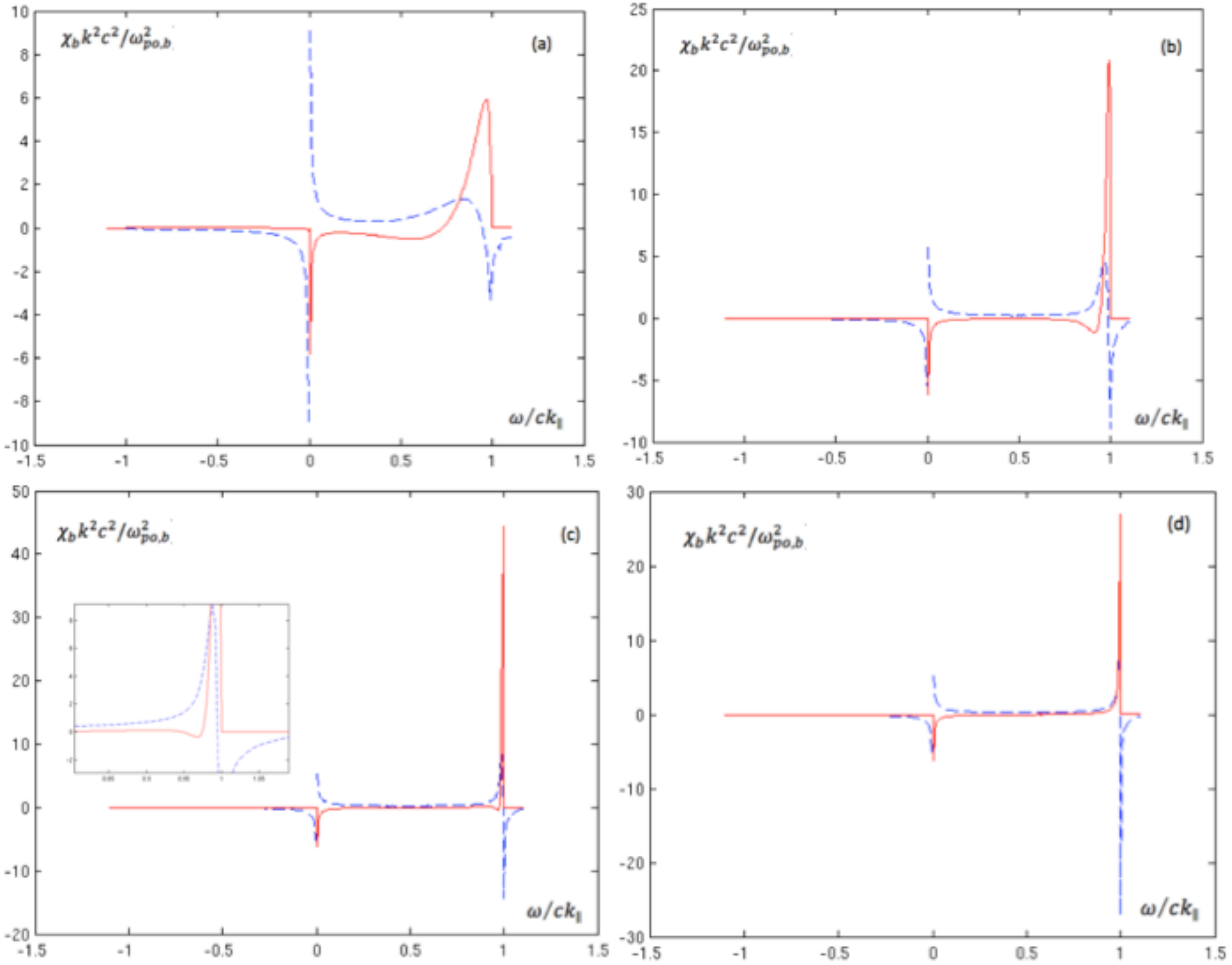
$$u_{po} = \left(N_\parallel^2-1\right)^{-1/2}$$

$$N_\parallel = k_\parallel c/\omega$$

$$s' = sgn(k_\parallel)$$

Figure 1. Analytic approximation of the imaginary part of the electrostatic susceptibility of RE. Here we assume $\sigma = 20$, corresponding to the angular spread $\theta_o \cong 0.32$, $\gamma_o = 25$ and $k_\parallel > 0$.

Electrostatic susceptivity Numerical evaluation (real ω)



σ=8,50,200,1250

The instability disappear
for very large anisotropy
(0.04 rad pitch angle)

Red = imaginary part
Blue = real part

Electrostatic susceptibility
Effect of RE energy

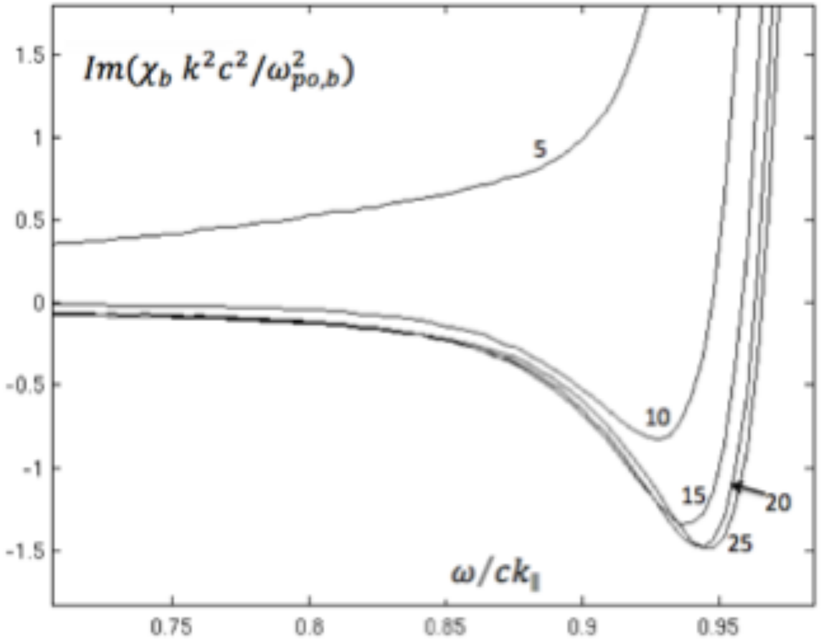
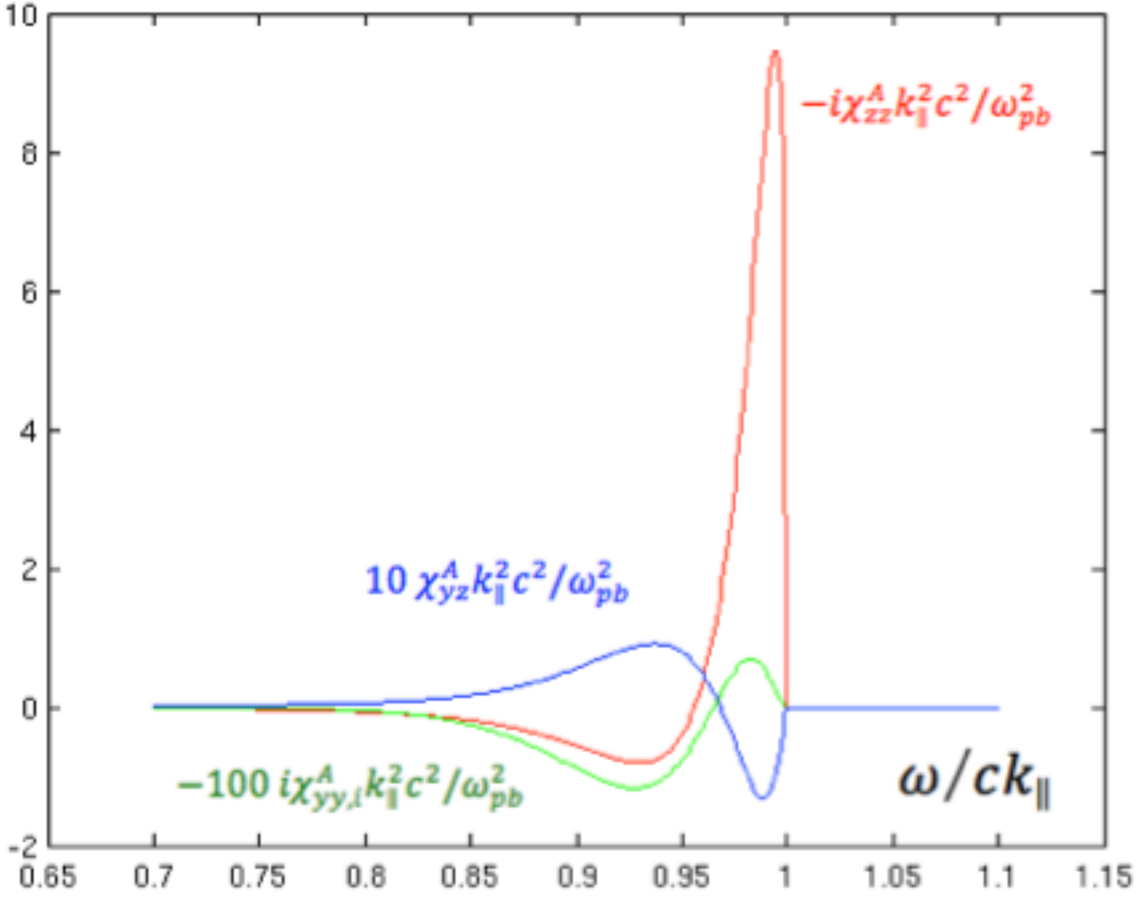


Figure 4. Imaginary part of the RE electrostatic susceptibility vs. the angular frequency (real). Here $\sigma = 72$, corresponding to the angular spread $\theta_o = 1/6$. Different values of γ_o, as indicated by the numbers near the curves, are considered, corresponding to average kinetic energies of RE from about 2.62 MeV up to about 12.8 MeV. Other parameters are as in Fig. 5.

Anti-Hermitian parts of the suscetibility tensor (for real ω)



$\sigma = 50, \gamma_o = 25$ and $k_\perp c/\Omega_e = 0.1, k_\parallel > 0.$

Outline

- Kinetic evaluation of the RE susceptibility near Cherenkov resonance
- Growth rates of slow and fast wave instability in the hybrid frequency range
- Collisional damping and instability threshold

Growth rates of LH waves

electrostatic approximation

- Electrostatic dispersion equation $\boldsymbol{E} \propto e^{-i(\omega t-\boldsymbol{k}\cdot\boldsymbol{r})}$

$$\varepsilon(\omega,k_\parallel,k_\perp)=1+\chi_e+\chi_i+\chi_b=0$$

$$\chi_e \cong -\frac{k_\parallel^2}{k^2}\frac{\omega_{pe}^2}{\omega^2}-\frac{k_\perp^2}{k^2}\frac{\omega_{pe}^2}{\omega^2-\Omega_e^2}\,,\chi_i \cong -\frac{k_\parallel^2}{k^2}\frac{\omega_{pi}^2}{\omega^2}-\frac{k_\perp^2}{k^2}\frac{\omega_{pe}^2}{\omega^2-\Omega_i^2}$$

Cold plasma approximation

- Approximate Solution $\omega=\omega_r+i\Gamma$

$$\varepsilon_r(\omega_r,k_\parallel,k_\perp)=1+\chi_e+\chi_i=0 \quad \rightarrow \quad k_\perp(\omega_r,k_\parallel)$$

$$\Gamma \cong -Im(\chi_b(\omega_r,k_\parallel,k_\perp))/(\partial\varepsilon_r/\partial\omega_r)$$

$$+$$

$\sigma = 50, \gamma_o = 25$ in pure Deuterium plasma with the ratio $\omega_{pe}/|\Omega_{ce}| = 0.5$.

LH growth and damping rates

electrostatic approximation

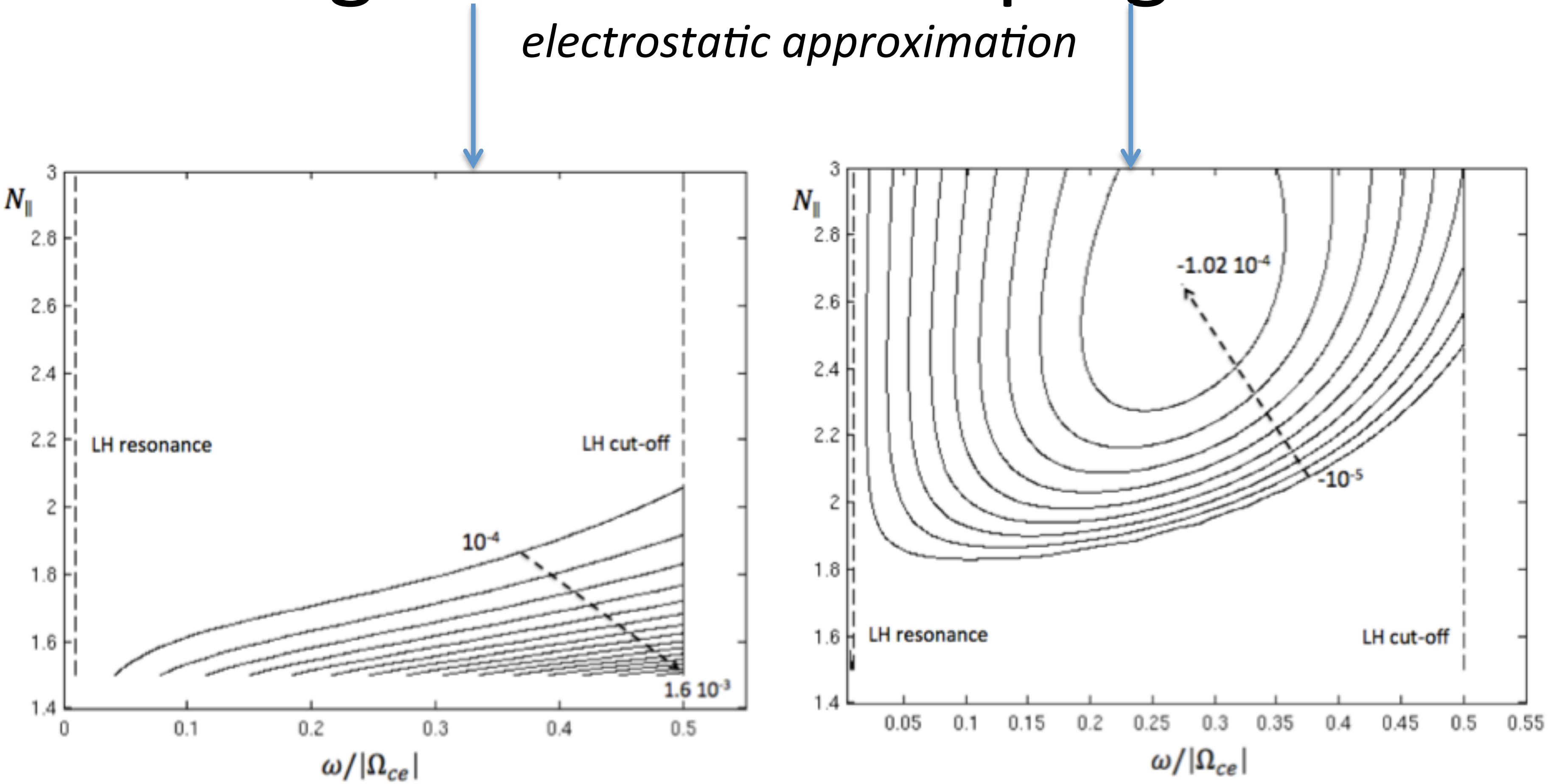


Figure 8. The growth rates (positive values, left) and the damping rates (negative values, right) of LHW are shown, in units $|\Omega_{ce}|n_b/n_e$, as contour plots in the 2D space of frequency and parallel refractive index. The growth rates increase along the direction of the arrow in steps of 10^{-4} towards the maximum ~1.6 10^{-3} at $N_\parallel \cong 1.5$ and $\omega \cong \omega_{pe}$ (cut-off). The damping rates decrease along the direction of the arrow in steps of 10^{-5} towards the minimum $-1.02\ 10^{-4}$ at $N_\parallel \cong 2.65$ and $\omega \cong 0.274\ |\Omega_{ce}|$. Here we consider RE distributions with $\sigma = 50, \gamma_o = 25$ in pure Deuterium plasma with the ratio $\omega_{pe}/|\Omega_{ce}| = 0.5$.

Growth rates

electromagnetic dispersion equation

$$\boldsymbol{k}\times(\boldsymbol{k}\times\boldsymbol{E}) + \frac{\omega_*^2}{c^2}\bar{\bar{\varepsilon}}\cdot\boldsymbol{E} = 0$$

$$\bar{\bar{\varepsilon}} = \bar{\bar{\varepsilon}}_o + \underset{\substack{\uparrow \\ \text{perturbation}}}{\bar{\bar{\chi}}_b}$$

$$\omega_* = \omega + i\Gamma$$

$$\left|\Lambda(\omega, n_\parallel, n_\perp)\right| = 0$$

$$\Lambda = \begin{pmatrix} S - N_\parallel^2 & -iD & N_\parallel N_\perp \\ iD & S - N^2 & 0 \\ N_\parallel N_\perp & 0 & P - N_\perp^2 \end{pmatrix}$$

(Stix reference frame)

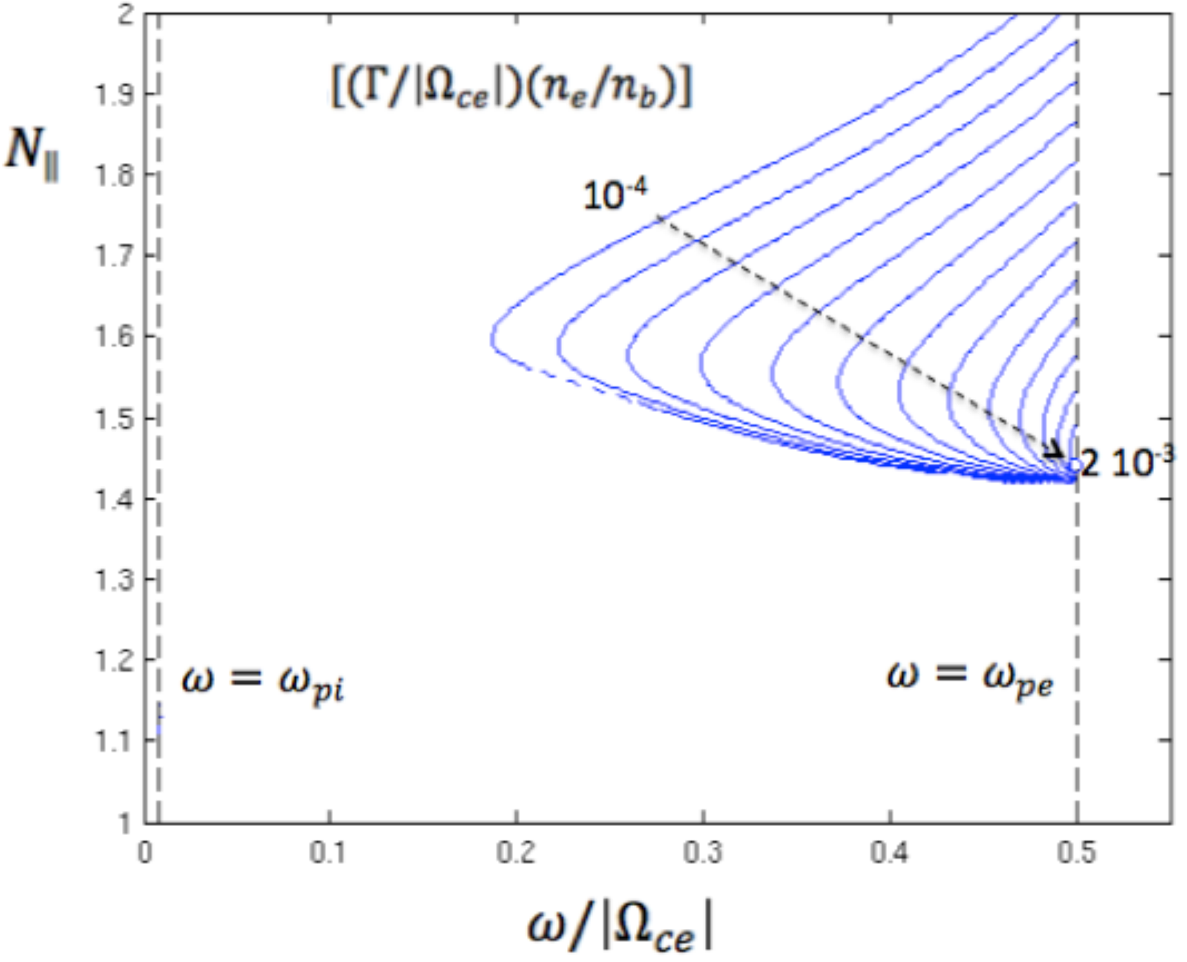
$$\Gamma \cong i\,\frac{\lambda_{ij}\omega^2\chi_{b,ij}^{A}}{\lambda_{ij}\dfrac{\partial\omega^2\varepsilon_{o,ij}}{\partial\omega}}$$

$$\lambda_{ij} = (1/2)\varepsilon_{ikl}\varepsilon_{jmn}\Lambda_{km}\Lambda_{ln}$$

LH Growth rates

electromagnetic dispersion equation

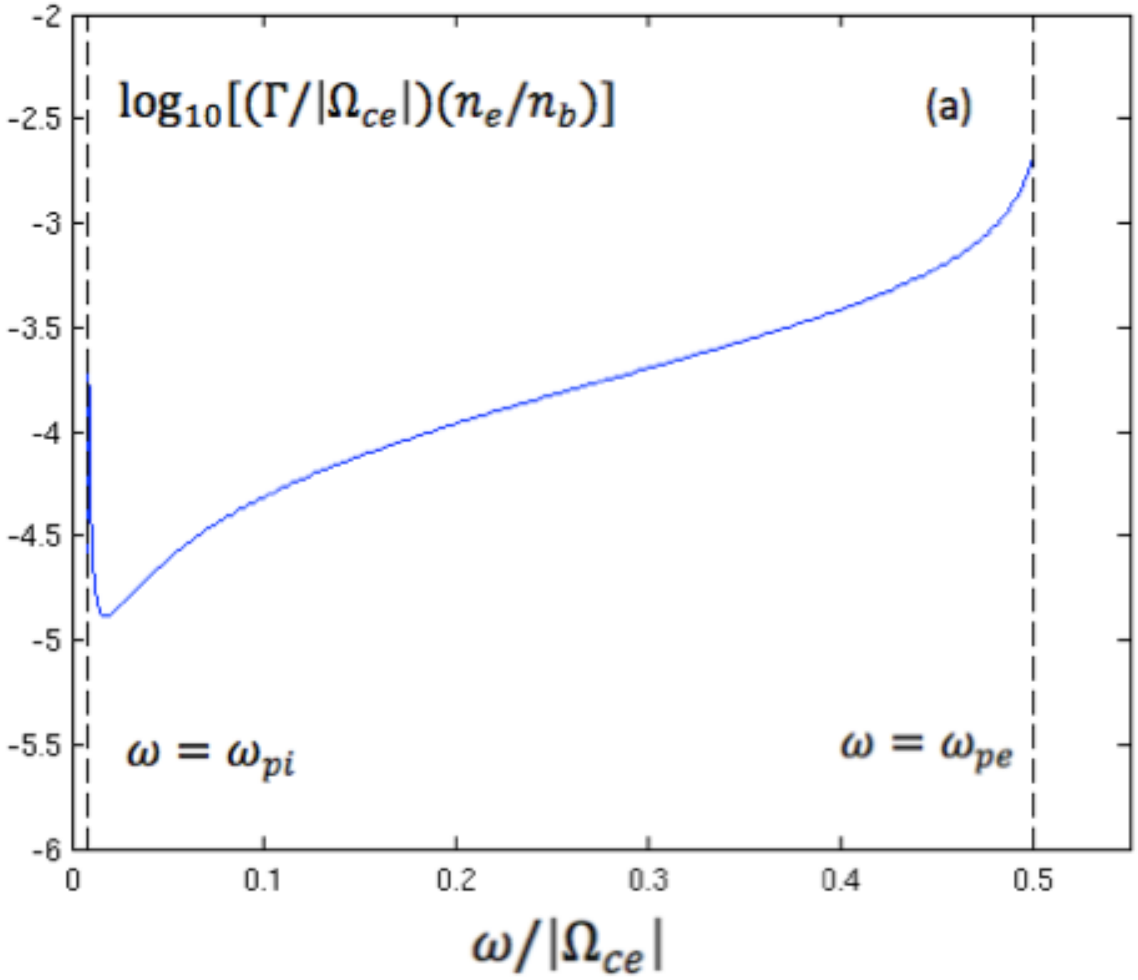
$\sigma = 50, \gamma_o = 25$ in pure Deuterium plasma with the ratio $\omega_{pe}/|\Omega_{ce}| = 0.5$.



LH Growth rates MAX($N_{//}$)

electromagnetic dispersion equation

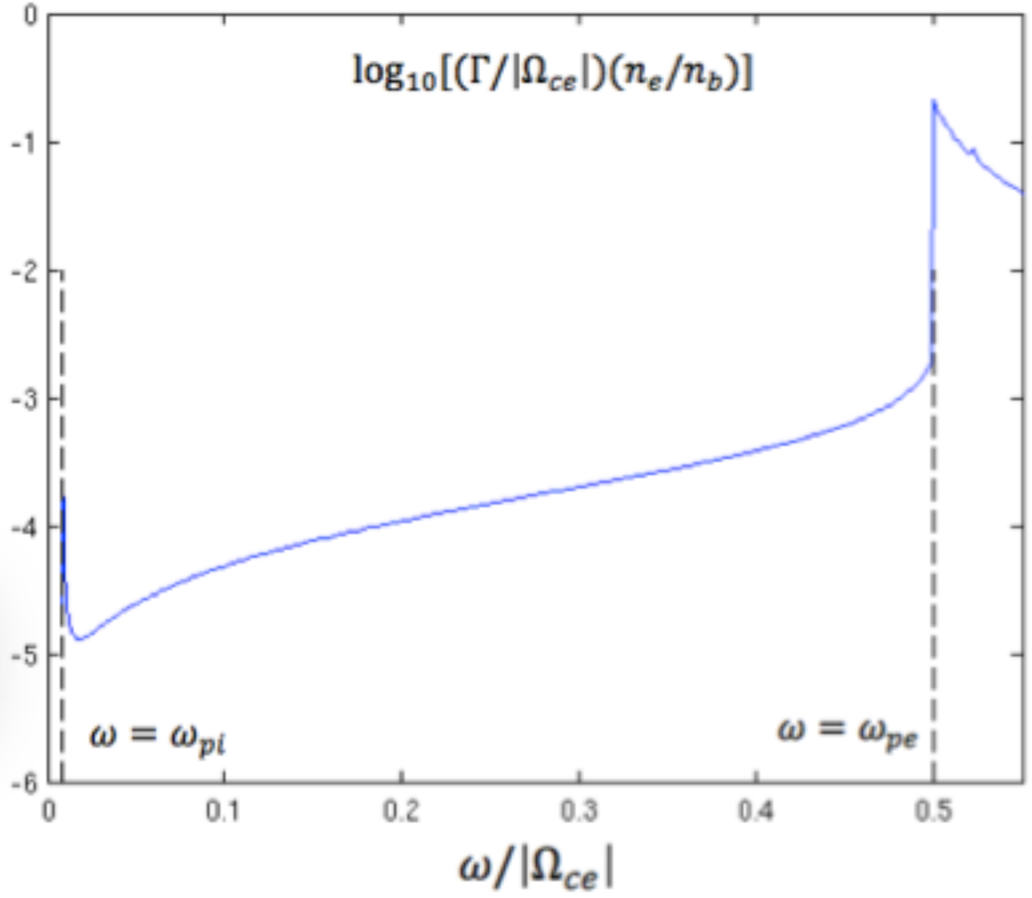
$\sigma = 50, \gamma_o = 25$ in pure Deuterium plasma with the ratio $\omega_{pe}/|\Omega_{ce}| = 0.5$.



Growth rates MAX($N_{//}$) SW

electromagnetic dispersion equation

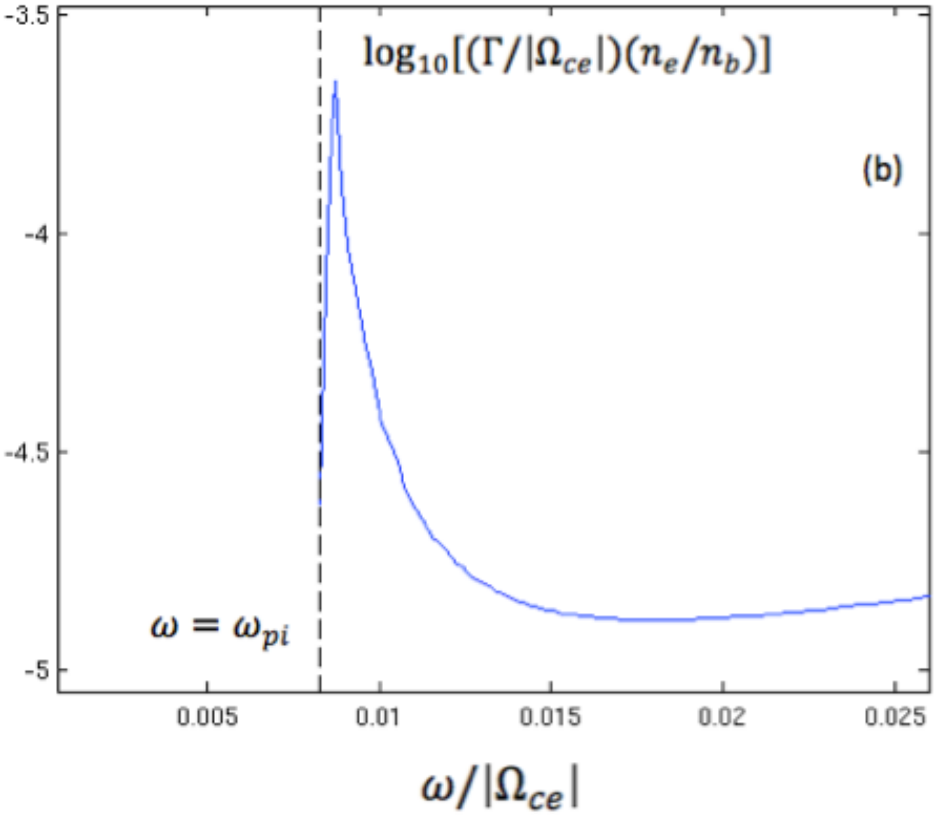
$\sigma = 50, \gamma_o = 25$ in pure Deuterium plasma with the ratio $\omega_{pe}/|\Omega_{ce}| = 0.5$.



LH Growth rates MAX($N_{//}$)

electromagnetic dispersion equation

$\sigma = 50, \gamma_o = 25$ in pure Deuterium plasma with the ratio $\omega_{pe}/|\Omega_{ce}| = 0.5$.



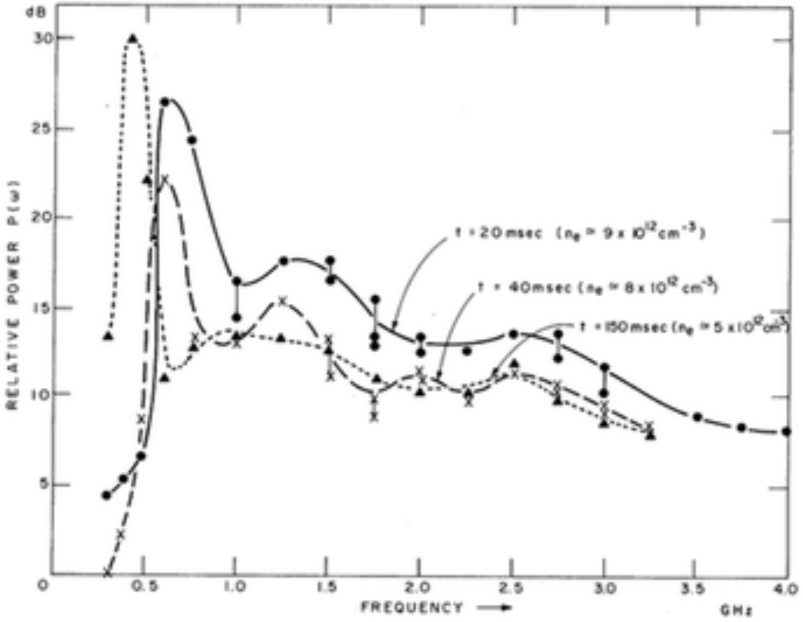
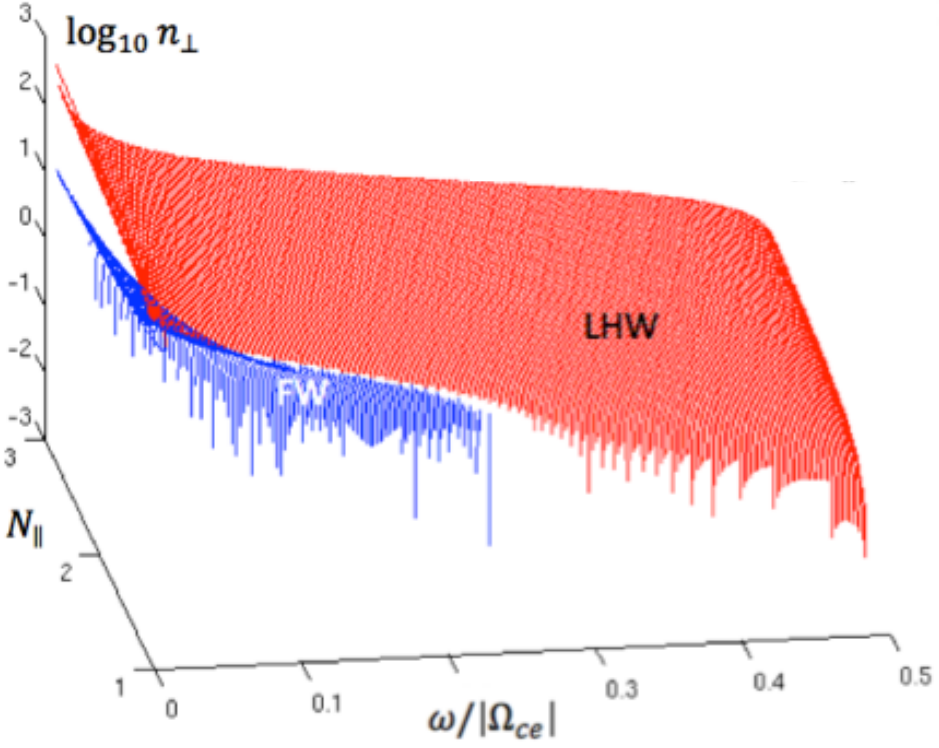
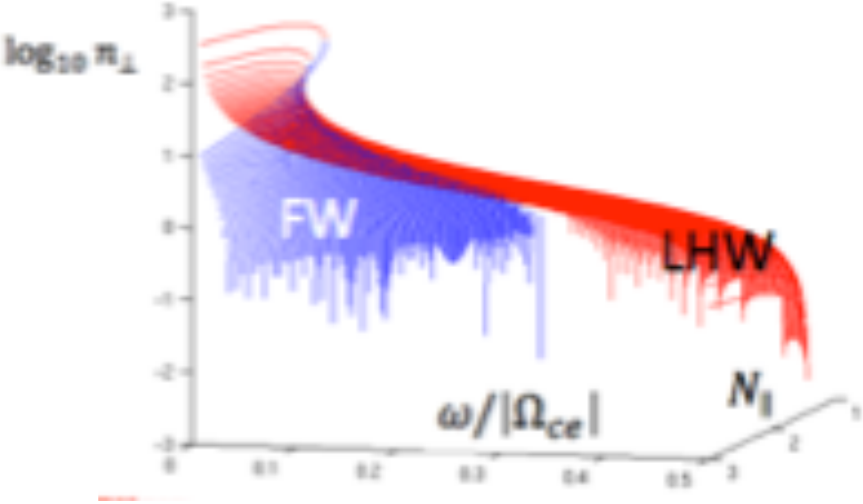


FIG. 2. Spectral emission from a discharge in hydrogen; shot-to-shot measurements were taken at $B_\varphi = 40$ kG, $I = 100$ kA.

Oomens et al. PRL 1976

Electromagnetic dispersion equation: Fast waves (WW) and slow waves (LH)

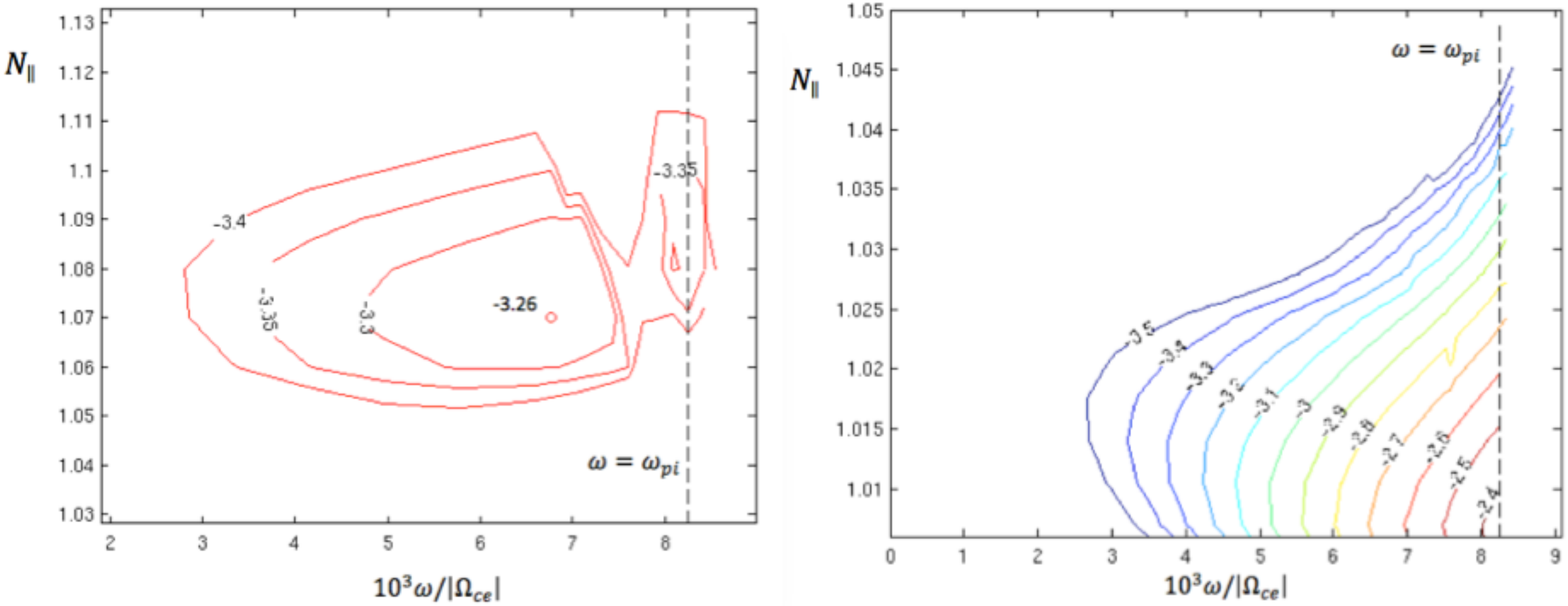




WW growth and damping rates

electromagnetic dispersion equation

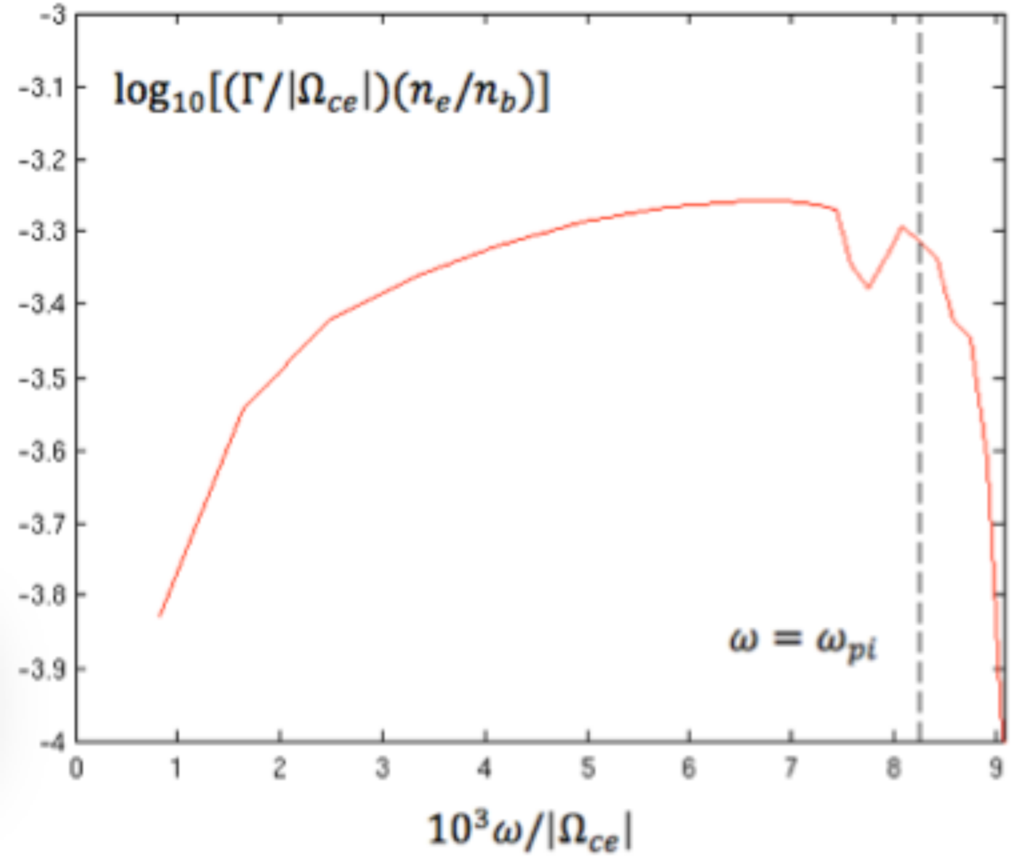
$\sigma = 50, \gamma_o = 25$ in pure Deuterium plasma with the ratio $\omega_{pe}/|\Omega_{ce}| = 0.5$.


$\log_{10}[(\Gamma/|\Omega_{ce}|)(n_e/n_b)]$

WW Growth rates MAX($N_{//}$)

electromagnetic dispersion equation

$\sigma = 50, \gamma_o = 25$ in pure Deuterium plasma with the ratio $\omega_{pe}/|\Omega_{ce}| = 0.5$.



Outline

- Kinetic evaluation of the RE susceptibility near Cherenkov resonance
- Growth rates of slow and fast wave instability in the hybrid frequency range
- Collisional damping and instability threshold

Collisional damping

- Within the fluid theory the effect of the e-i (Z=1) collisions is decribed by the force

$$\boldsymbol{F} = -m_e \nu_e \mathbf{v}, \qquad \nu_e = \frac{4\sqrt{2\pi}\Lambda_c e^4 n_e}{3 m_e^{1/2} T_e^{3/2}}$$

- The conductivity tensor is modified:

$$\overline{\overline{\sigma_o}}(\omega) \longrightarrow \overline{\overline{\sigma_o}}(\omega + i\nu_e)$$

- Taylor expansion gives the dielectric tensor

$$\bar{\bar{\varepsilon}} \cong \bar{\bar{\varepsilon}}_o + \overline{\overline{\varepsilon_\nu^A}}, \qquad \overline{\overline{\varepsilon_\nu^A}} = 4\pi i (\nu_e/\omega) d\overline{\overline{\sigma_o}}/d\omega$$

Collisional damping

$$\Gamma_\nu \cong i\,\frac{\lambda_{ij}\omega^2\varepsilon^A_{\nu,ij}}{\lambda_{ij}\,\dfrac{\partial\omega^2\varepsilon_{o,ij}}{\partial\omega}}$$

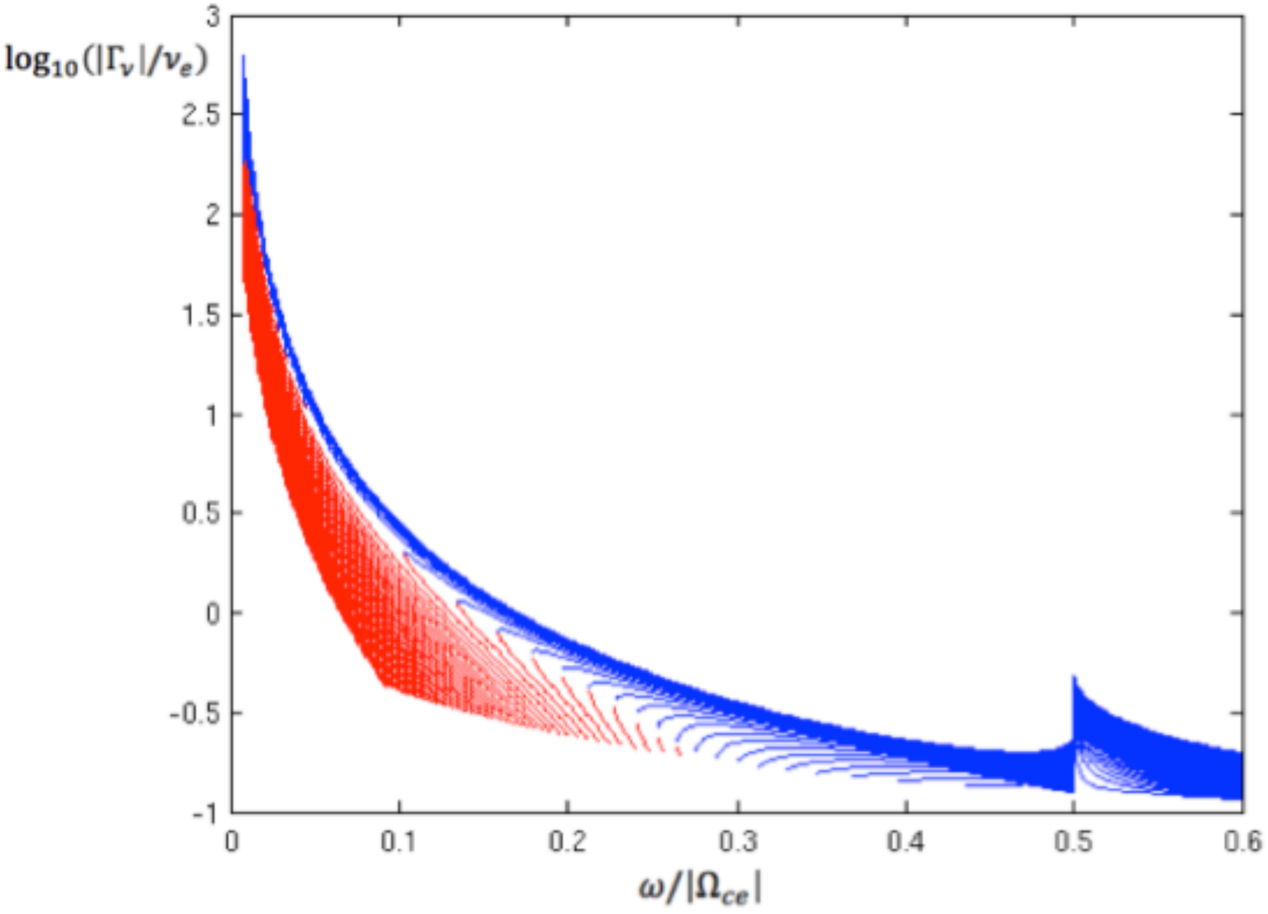
$$\varepsilon^A_{\nu,xy} = -\varepsilon^A_{\nu,yx} = -2\nu_e\,\frac{\omega_{pe}^2\Omega_{ce}}{(\omega^2-\Omega_{ce}^2)^2}$$

$$\varepsilon^A_{\nu,xx} = \varepsilon^A_{\nu,yy} = i\,\frac{\nu_e}{\omega}\,\frac{\omega_{pe}^2(\omega^2+\Omega_{ce}^2)}{(\omega^2-\Omega_{ce}^2)^2}$$

$$\varepsilon^A_{\nu,zz} = i\nu_e\,\frac{\omega_{pe}^2}{\omega^3}$$

Damping rate
Deuterium plasma

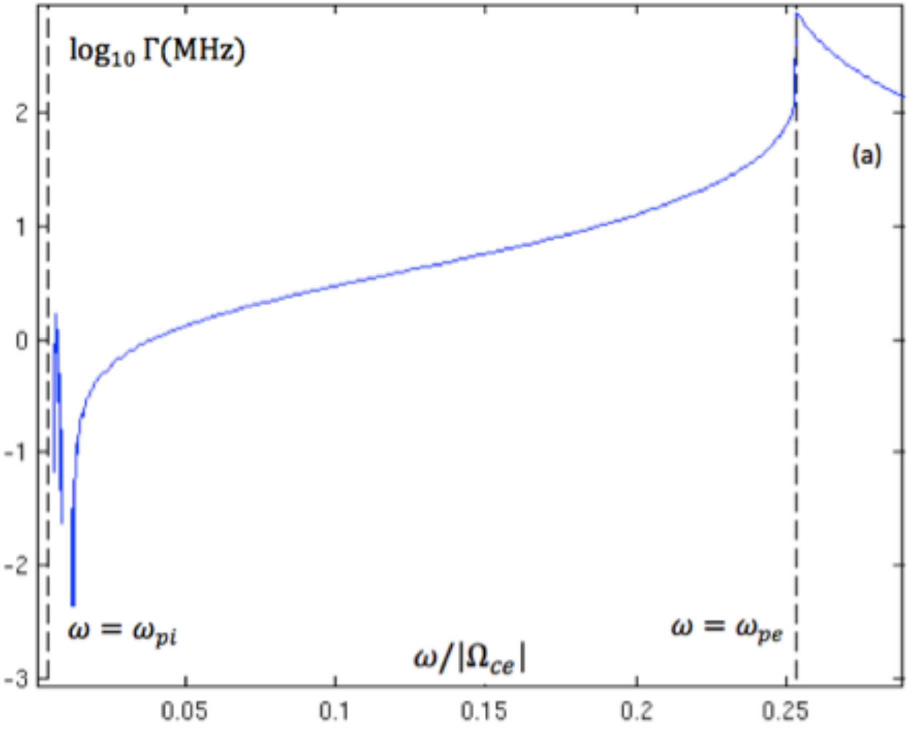
$\omega_{pe}/|\Omega_{ce}| = 0.5$



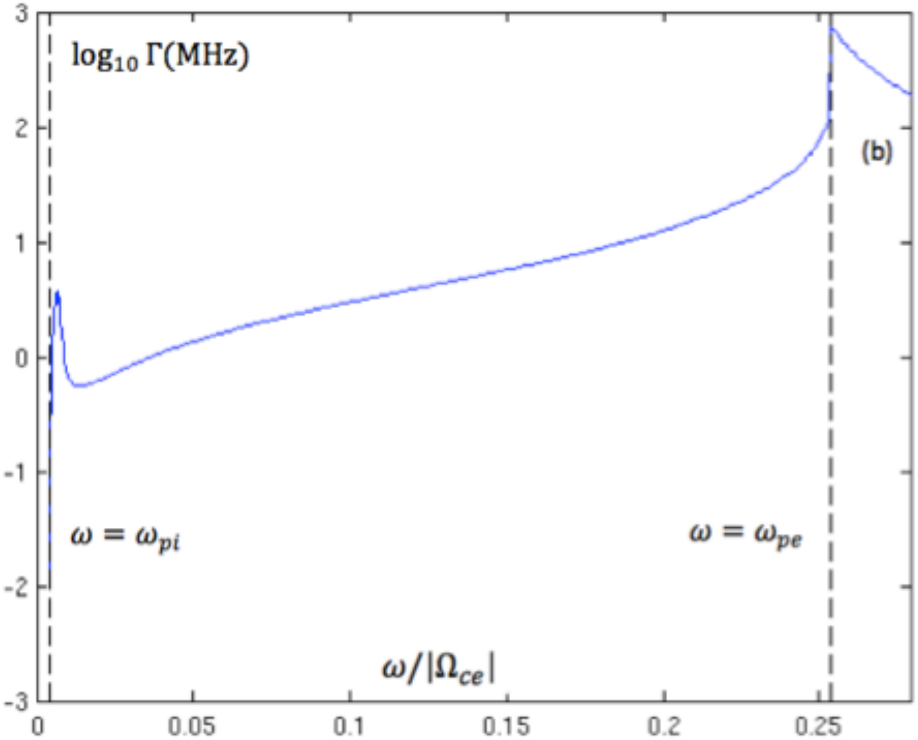
Growth rates MAX($N_{//}$) SW

$B_o = 4\text{T}, n_e = 10^{13}\text{cm}^{-3}, T_e = 1$ keV. The RE parameters are $n_b = 10^{11}\text{cm}^{-3}, \sigma = 50, \gamma_o = 25.$

Collisional

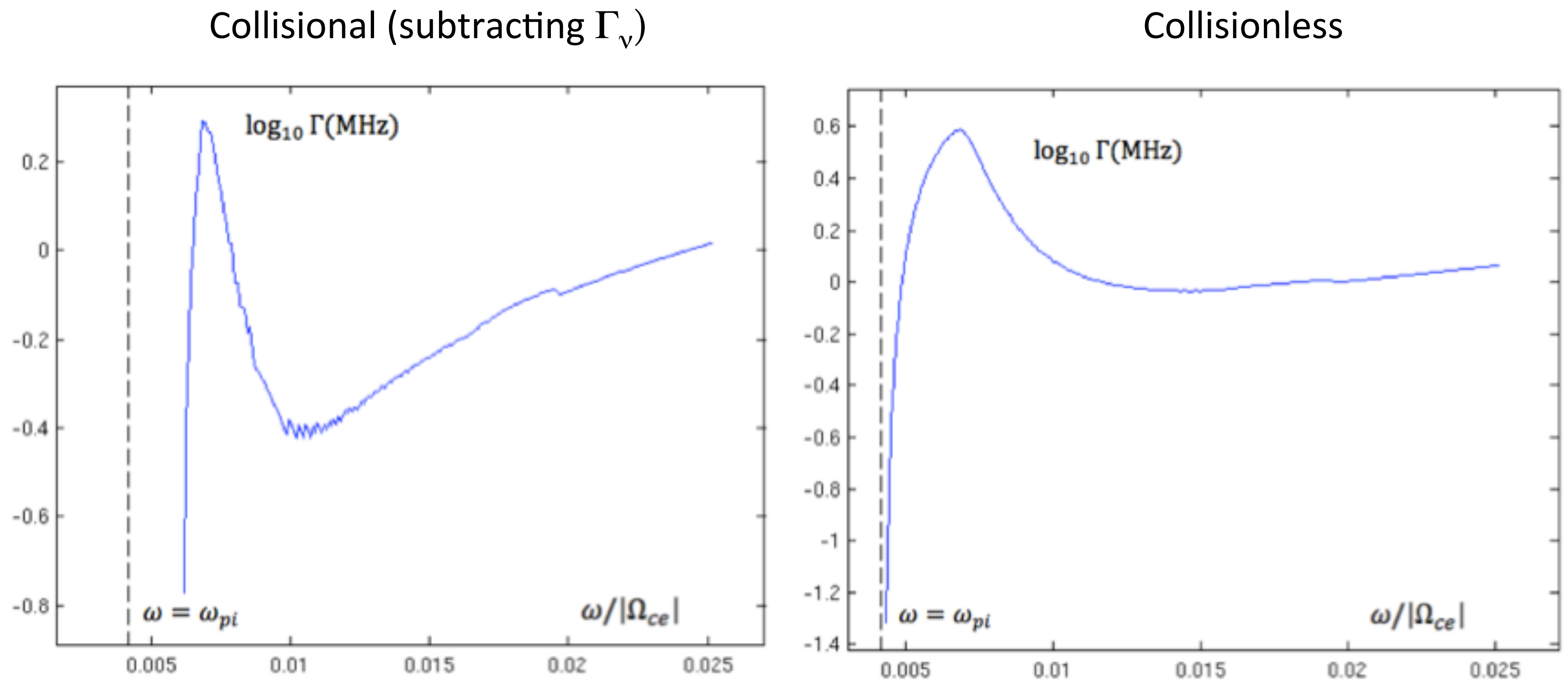


Collisionless



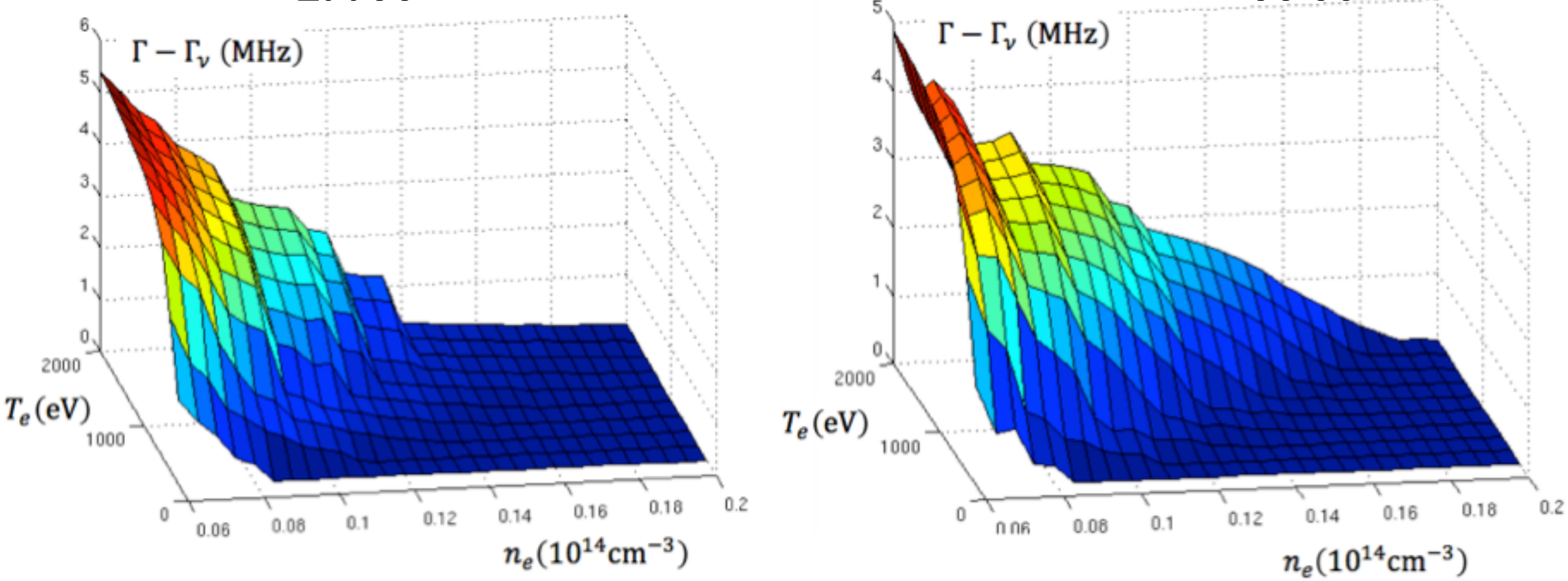
Growth rates MAX($N_{//}$) LHW

$B_o = 4\text{T}, n_e = 10^{13}\text{cm}^{-3}, T_e = 1$ keV. The RE parameters are $n_b = 10^{11}\text{cm}^{-3}, \sigma = 50, \gamma_o = 25$.


Collisional (subtracting Γ_ν)

Collisionless

Growth rates MAX($N_{//}$,ω) near ω_{pi}



LHW

WW

Figure 18. Maximum growth rates (subtracting the collisional damping) for LHW (left) and FW (right) as a function of the density and temperature for magnetic field $B_o = 4\text{T}$. The maximum is found within the range of frequencies $[0.96\,\omega_{pi}, 6\,\omega_{pi}]$and within the range [1,2] of the parallel refractive index. The RE parameters are $n_b = 10^{11}\text{cm}^{-3}, \sigma = 50, \gamma_o = 15$.

The temperature threshold for the onset of the instabilities is of the order of 200 eV for $n_e = 10^{13}$ cm^{-3}

Growth rates MAX($N_{//}$,ω) near ω_{pe}

SW FW

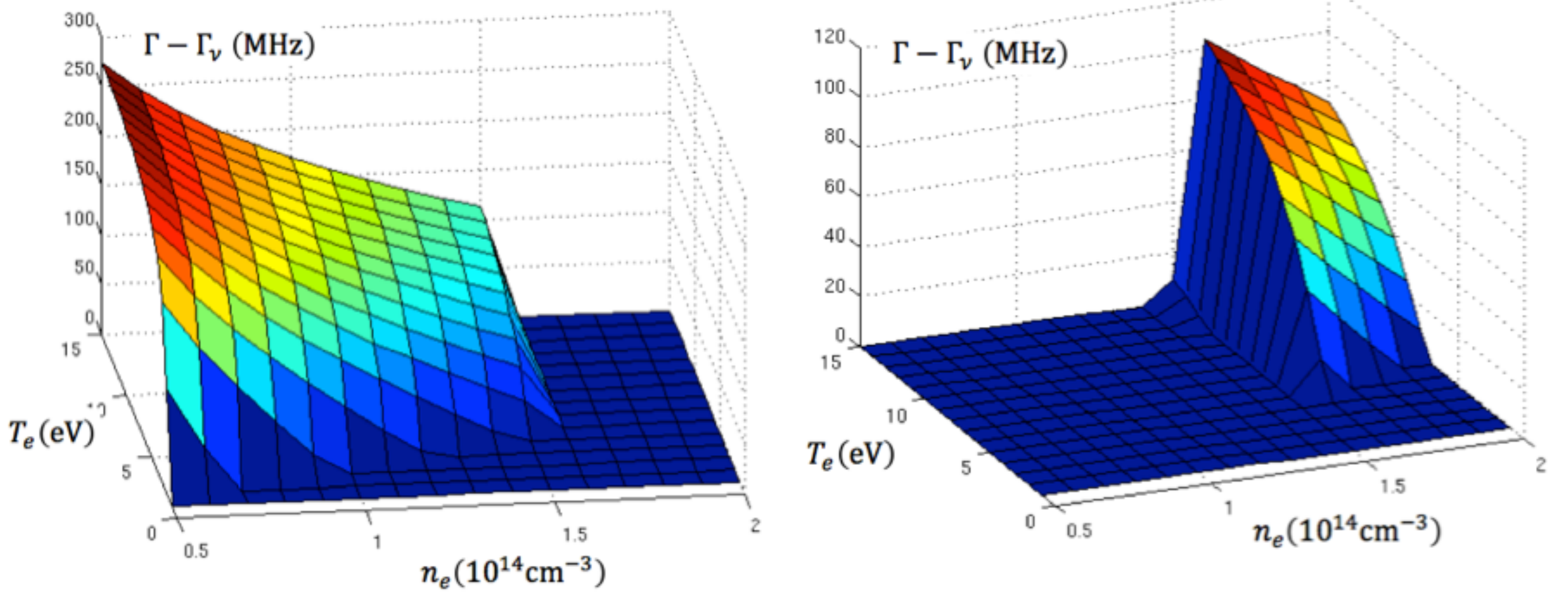


Figure 19. Maximum growth rates (subtracting the collisional damping) for SW (left) and FW (right) as a function of the density and temperature for magnetic field $B_o = 4\text{T}$. The maximum is found within the range of frequencies $[0.9\ \omega_{pe}, 1.1\ \omega_{pe}]$ and within the range [1.0,2.0] of the parallel refractive index. The RE parameters are $n_b = 10^{11}\text{cm}^{-3}, \sigma = 50, \gamma_o = 15$.

The temperature threshold for the onset of the instabilities is of the order of 2 eV for n_e =1.5 10^{14} cm^{-3}

Conclusions (I)

- LHW, WW can be driven unstable by RE via Cherenkov resonance. However WW can be subject to damping for 5% variation of $N_{//}$
- For isotropic or Dirac delta pitch angle distributions the modes are stable
- The growth rates are larger for larger kinetic energy of RE, this effect saturates at ~10 MeV
- The SW growth rates (maximized vs. $N_{//}$) vs the angular frequency peak at ω_{pi} and ω_{pe} and exhibit a minimum at $\sim 3\omega_{pi}$

Conclusions (II)

- At 1 keV and 10^{13} cm^{-3} 4T, with 12 MeV RE with 0.2 rad pitch angle spread
  - Near ω_{pi} the growth rates are ~1 MHz
  - Near ω_{pe} the growth rates are ~1 GHz

- Near ω_{pi} the temperature threshold for unstable waves is ~ 200 eV at 10^{13} cm^{-3}
- Near ω_{pe} the temperature threshold for unstable waves is ~ 2 eV at 1.5 10^{14} cm^{-3}