Validation of gyrokinetic simulations in NSTX including comparisons with a synthetic diagnostic for high-k scattering

J. Ruiz Ruiz\textsuperscript{1,2}

W. Guttenfelder\textsuperscript{3}, A. E. White\textsuperscript{1}, N. Howard\textsuperscript{1}, N. F. Loureiro\textsuperscript{1}, J. Candy\textsuperscript{8}, Y. Ren\textsuperscript{3}, S.M. Kaye\textsuperscript{3}, B. P. LeBlanc\textsuperscript{3}, E. Mazzucato\textsuperscript{3}, K.C. Lee\textsuperscript{4}, C.W. Domier\textsuperscript{5}, D. R. Smith\textsuperscript{6}, H. Yuh\textsuperscript{7}

\textsuperscript{1}. MIT \textsuperscript{2}. Oxford \textsuperscript{3}. PPPL \textsuperscript{4}. NFRI \textsuperscript{5}. UC Davis \textsuperscript{6}. U Wisconsin \textsuperscript{7}. Nova Photonics, Inc. \textsuperscript{8}. General Atomics

20\textsuperscript{th} International Spherical Tokamak Workshop
ENEA, Frascati, Oct 28-31, 2019
Spherical tokamaks (STs) minimize time-spent by plasma particles in the ‘unstable’, bad-curvature side

Standard tokamak: 
e.g: JET, Alcator C-Mod, DIII-D, AUG, etc.

Spherical tokamak: 
e.g: NSTX (PPPL), MAST (CCFE)
Spherical tokamaks:

- Small aspect ratio \( A \)
- High-beta \( \beta \)
- High shaping of magnetic surfaces
- High toroidal rotation (if neutral beam driven)

This talk will focus on the Spherical Torus (ST) NSTX

Can improve macro & micro stability [*]

ST H-modes have reported neoclassical levels of ion thermal transport, transport dominated by electron channel

- Ion thermal transport ($P_i$) observed close to neoclassical levels in NSTX NBI heated H-modes, due to *suppression of ion scale turbulence by ExB shear, beta, strong plasma shaping* [Rewoldt PoP 1996, Kaye NF 2007].
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• Electron thermal transport is always anomalous.
ST H-modes have reported neoclassical levels of ion thermal transport, transport dominated by electron channel

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- Electron thermal transport is always anomalous.

- This work will compare predictions of electron-scale turbulence and transport to experimental measurements at NSTX:

Ion scale (ITG, TEM, ...)  

Electron Scale (ETG)  

$\rho_s$ ion sound gyro radius

$k_{\perp}\rho_s$
ST H-modes have reported neoclassical levels of ion thermal transport, transport dominated by electron channel

- Ion thermal transport ($P_i$) observed close to neoclassical levels in NSTX NBI heated H-modes, due to **suppression of ion scale turbulence by ExB shear, beta, strong plasma shaping** [Rewoldt PoP 1996, Kaye NF 2007].

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- This work will compare predictions of **electron-scale turbulence** and transport to experimental measurements at NSTX:
  - Electron thermal power $P_e$ [MW]: ➔ using gyrokinetic simulation (GYRO).

Ion scale (ITG, TEM, …)

Electron Scale (ETG)

$\rho_s$ ion sound gyro radius

High-k scattering: $k_\perp \rho_s \sim 10-20$ ($k_\perp \rho_e \sim 0.2-0.3$)
Outline

• Turbulence fluctuation measurement (high-k scattering).
• GYRO simulation details.
• NSTX H-mode discharge under study.
• Electron thermal transport comparisons.
• Electron-scale turbulence comparisons:
  – Synthetic diagnostic description
  – f-spectra comparisons
  – k-spectra comparisons
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Use a high-k scattering diagnostic to probe electron-scale turbulence on NSTX

- Scattered power density \( P_s \propto \left| \frac{\delta n_e}{n_e} \right|^2 \), \( \delta n_e \) electron density
Use a high-k scattering diagnostic to probe electron-scale turbulence on NSTX

- Scattered power density\[P_s \propto \left| \frac{\delta n_e}{n_e} \right|^2, \delta n_e \text{ electron density}\]
- Gaussian microwave probe beam
  \[- f = 280 \text{ GHz (} \gg f_{pe}, f_{ce})\]
- Ray tracing to determine \( \vec{k}_{turb} \)

\[\vec{k}_s = \vec{k}_{turb} + \vec{k}_i\]

\[\omega_s = \omega_{turb} + \omega_i\]
Use a high-k scattering diagnostic to probe electron-scale turbulence on NSTX

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- Map experimental $\vec{k}_{turb}$ to $\vec{k}_{turb} = (k_r, k_\phi, k_\theta)^{\text{sim}}$

![Image of NSTX-U tokamak with probe beam and turbulence map]
Use a high-k scattering diagnostic to probe electron-scale turbulence on NSTX

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$$\begin{align*}
\vec{k}_s &= \vec{k}_{turb} + \vec{k}_i \\
\omega_s &= \omega_{turb} + \omega_i
\end{align*}$$

- Map experimental $\vec{k}_{turb}$ to $\vec{k}_{turb} = (k_r, k_\phi, k_\theta)^{\text{sim}}$
- Scattering system is toroidally localized [*]
  ➔ We model a 2D synthetic diagnostic

- **Preview**: Synthetic high-k diagnostic will require use of ‘big-box’ electron-scale simulations (Traditional e-scale simulations lack numerical k-resolution)

[*] Mazzucato PoP 2003, PPCF 2006
High-k scattering provides measurements of frequency and wavenumber spectra of electron-scale turbulence.
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**Frequency spectra:**
\[ S(f) \propto |\delta n|^2 \]

**Wavenumber spectra:**
\[ S(k) \propto |\delta n|^2 \]

Ch 1

Ch 2

Ch 3

Strong ETG

Weak ETG

Experimental noise at \( f = 0 \)

Turbulence fluctuations
High-k scattering provides measurements of frequency and wavenumber spectra of electron-scale turbulence.

Frequency spectra:
\[ S(f) \propto |\delta n|^2 \]

Wavenumber spectra:
\[ S(k) \propto |\delta n|^2 \]

Ch 1
Outline

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Accurate high-\(k\) turbulence comparisons require ‘big-box’ electron-scale simulation

- **Ion-scale** turbulence simulation \((k_\theta \rho_s \leq 1)\).
- **Traditional e-scale** sim. \((k_\theta \rho_s \geq 1)\) has too coarse wavenumber resolution for synthetic diagnostic deployment.

![Diagram showing ion-scale (ITG, TEM, ...) and traditional electron-scale (ETG) simulations with a high-\(k\) diagnostic region highlighted.]
Accurate high-k turbulence comparisons require ‘big-box’ electron-scale simulation

- **Ion-scale** turbulence simulation \((k_\theta \rho_s \leq 1)\).
- **Traditional e-scale** sim. \((k_\theta \rho_s \geq 1)\) has too coarse wavenumber resolution for synthetic diagnostic deployment.
- ‘**Big-box’ electron-scale** sim. contains same physics (ETG), but finer wavenumber grid for synthetic diagnostic deployment \((k_\theta \rho_s \geq 0.3)\).
Accurate high-k turbulence comparisons require ‘big-box’ electron-scale simulation

- **Ion-scale** turbulence simulation \((k_\theta \rho_s \leq 1)\).
- **Traditional e-scale** sim. \((k_\theta \rho_s \gtrsim 1)\) has too coarse wavenumber resolution for synthetic diagnostic deployment.
- ‘Big-box’ **electron-scale** sim. contains same physics (ETG), but finer wavenumber grid for synthetic diagnostic deployment \((k_\theta \rho_s \gtrsim 0.3)\).

- Experimental profiles used as input to GYRO
  - Local simulations performed at scattering location \((r/a \sim 0.7, R\sim135 \text{ cm})\).
  - 3 kinetic species, D, C, e- \((Z_{\text{eff}}\sim1.85-1.95)\)
  - Electromagnetic: \(A_{||}+B_{||} (\beta_e \sim 0.3\%)\).
  - Collisions \((v_{ei} \sim 1 \text{ } c_s/a)\).
  - ExB shear \((\gamma_E \sim 0.13-0.16 \text{ } c_s/a)\) + parallel flow shear \((\gamma_p \sim 1-1.2 \text{ } c_s/a)\)
  - Fixed boundary conditions (radial buffer region).

\[ \begin{align*}
\text{Ion-scale} & \quad \text{Big eddies} \quad 0.1 \\
\text{‘Big-box’ electron-scale (ETG)} & \quad 1 \quad \text{high-k diagnostic} \\
& \quad 10 \quad \text{Small eddies}
\end{align*} \]
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Performed an extensive validation effort to study electron thermal transport in a *modest-beta* NSTX H-mode

- NBI heated H-mode with controlled current ramp-down; two steady discharge phases, little MHD activity.
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- NBI heated H-mode with controlled current ramp-down; two steady discharge phases, little MHD activity.
- Local increase in $|\nabla n|$ $\Rightarrow$ ETG stabilization [*], observed in high-k fluctuation spectra.

[*] $\nabla n$ stabilization of ETG: Ren PRL 2011, Ruiz Ruiz PoP 2015
Performed an extensive validation effort to study electron thermal transport in a *modest-beta* NSTX H-mode

- NBI heated H-mode with controlled current ramp-down; two steady discharge phases, little MHD activity.
- Local increase in $|\nabla n|$ $\rightarrow$ ETG stabilization $[^*]$, observed in high-$k$ fluctuation spectra.
- In this work, perform sensitivity scans in $\{\nabla T_e, \nabla n_e, q, \hat{s}\}$ to compare:
  - Electron thermal power $P_e$ (TRANSP) via sensitivity scans of GYRO sims.
  - High-$k$ turbulence freq. and $k$-spectra via synthetic diagnostic for GYRO.
- Details in Ruiz Ruiz PPCF 2019.

$[^*]$ $\nabla n$ stabilization of ETG: Ren PRL 2011, Ruiz Ruiz PoP 2015
Regime diagrams from previous NSTX linear gyrokinetic sim suggest ETG could be relevant in present discharge.
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Regime diagrams (Guttenfelder, NF 2013):

![Regime Diagram](image)

**Dominant linear instability:**

(a) Low $\beta_e$: Electrostatic ITG/TEM/ETG.
   High $\beta_e$: Kinetic ballooning mode (KBM), micro-tearing mode (MT).
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(b) High $\beta_e \cdot a/L_{Te}$: MT.
High $\alpha_{MHD} (\propto p')$: KBM.
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(c) $a/L_{Te} > a/L_{Te, crit}$: ETG.
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This talk: **Strong ETG**
This talk: **Weak ETG**
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Ion-scale turbulence is predicted to play a negligible role in both conditions

**Strong ETG condition**

- Ion-scale sim. predicts turbulence is nonlinearly suppressed by ExB shear.
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- Ion-scale sim. predicts turbulence is nonlinearly suppressed by ExB shear.

**Weak ETG condition**
- Ion-scale sim. shows turbulence can be destabilized within uncertainty in drive terms.
- **BUT** ion thermal transport is close to neoclassical levels
  - ion-scale turbulence plays a negligible role
Ion-scale turbulence is predicted to play a negligible role in both conditions

**Strong ETG condition**
- Ion-scale sim. predicts turbulence is nonlinearly suppressed by ExB shear.

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- Ion-scale sim. shows turbulence can be destabilized within uncertainty in drive terms.
- **BUT** ion thermal transport is close to neoclassical levels
  - Ion-scale turbulence plays a negligible role

**Ion-scale turbulence contributions can be neglected in the strong and weak ETG conditions**
Strong ETG condition: electron-scale turbulence can match $P_e$ within experimental uncertainty

Perform sensitivity scans maximizing turbulence drive in 5 ‘big-box’ e-scale sims.

- $1-\sigma(\nabla T, \nabla n)$ max. uncertainty.
- 10% $q$, 20% $\dot{s}$

Pe comparisons using ‘Big-box’ electron-scale sim.

(Strong ETG)

Simulation:
Strong ETG condition: electron-scale turbulence can match $P_e$ within experimental uncertainty

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‘Big-box’ electron-scale sim
- Base (exp parameters): underpredict $P_e$

Simulation: base
Perform sensitivity scans maximizing turbulence drive in 5 ‘big-box’ e-scale sims.

- 1-σ(∇T,∇n) max. uncertainty.
- 10% q, 20% s

‘Big-box’ electron-scale sim
- Base (exp parameters): underpredict P_e
- σ(∇T,∇n): underpredict P_e

Strong ETG condition: electron-scale turbulence can match P_e within experimental uncertainty
Perform sensitivity scans maximizing turbulence drive in 5 ‘big-box’ e- scale sims.

- 1-σ(∇T,∇n) max. uncertainty.
- 10% q, 20% ŝ

‘Big-box’ electron-scale sim
- Base (exp parameters): underpredict P_e
- σ(∇T,∇n): underpredict P_e
- σ(∇T), q, ŝ: match P_e
Strong ETG condition: electron-scale turbulence can match $P_e$ within experimental uncertainty

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‘Big-box’ electron-scale sim

- Base (exp parameters): underpredict $P_e$
- $\sigma(\nabla T, \nabla n)$: underpredict $P_e$
- $\sigma(\nabla T)$, $q$, $\hat{s}$: match $P_e$
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‘Big-box’ electron-scale sim

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- $\sigma(\nabla n)$, $q$, $\hat{s}$: match $P_e$
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Validation metric $R_{Pe}$

<table>
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<th>$R_{Pe}$</th>
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<tbody>
<tr>
<td><strong>Base</strong></td>
<td>1</td>
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<tr>
<td>$\sigma(\nabla T, \nabla n)$</td>
<td>0.29</td>
</tr>
<tr>
<td>$\sigma(\nabla T)$, $q$, $\hat{s}$</td>
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</tr>
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<td>$\sigma(\nabla n)$, $q$, $\hat{s}$</td>
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<td>1</td>
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1 = bad
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[*] Ricci PoP 2011
Strong ETG condition: electron-scale turbulence can match $P_e$ within experimental uncertainty

Perform sensitivity scans maximizing turbulence drive in 5 ‘big-box’ e-scale sims.
- 1-$\sigma$($\nabla T, \nabla n$) max. uncertainty.
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- $\sigma(\nabla T, \nabla n)$, $q$, $\hat{s}$: overpredict $P_e$

**Strong ETG condition**
- Electron-scale turbulence can explain $P_e$.
- Scanning $q$ and $\hat{s}$ is needed for matching $P_e$. 
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• Turbulence fluctuation measurement (High-k scattering).
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• Electron-scale turbulence comparisons:
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  – f-spectra comparisons
  – k-spectra comparisons
Two synthetic diagnostics are implemented for quantitative comparisons of e-scale turbulence [*]

**Real space formulation:** $\psi_R(\vec{r})$

**$k$-space formulation:** $\psi_K(\vec{k} - \vec{k}_0)$

[*] Ruiz-Ruiz to be submitted
Two synthetic diagnostics are implemented for quantitative comparisons of e-scale turbulence [*]

Real space formulation: $\Psi_R(\vec{r})$

$k$-space formulation: $\Psi_K(\vec{k} - \vec{k}_0)$

$\delta n_e(\vec{r}, t) = \int \delta n_e(\vec{r}, t) \Psi_R(\vec{r}) e^{-\vec{k}_0 \cdot \vec{r}} d^3\vec{r}$

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**$k$-space formulation:** $\psi_K(\vec{k} - \vec{k}_0)$

$$\delta n_e(\vec{k}, t) = \frac{1}{(2\pi)^3} \int \delta n_e(\vec{k}, t) \psi_K(\vec{k} - \vec{k}_0) d^3\vec{k}$$

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Two synthetic diagnostics are implemented for quantitative comparisons of e-scale turbulence [*]

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$k$-space formulation: $\psi_K(\vec{k} - \vec{k}_0)$

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$k$-space formulation:

$\delta n_e^{syn}(t) = \frac{1}{(2\pi)^3} \int \delta n_e(\vec{k}, t) \psi_K(\vec{k} - \vec{k}_0) d^3\vec{k}$

Obtain a time series of turbulent density fluctuations $\delta \hat{n}_e^{syn}(t)$

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Synthetic $f$-spectrum reproduces spectral peak and spectral width $W_f$

![Spectral Density comparisons: ch1](image)

- Diagnostic (Strong ETG)
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Spectral Density comparisons: ch1

Diagnostic (Strong ETG)
Synthetic (Strong ETG)
Diagnostic (weak ETG)
Synthetic (weak ETG)
Synthetic $f$-spectrum reproduces spectral peak and spectral width $W_f$

**Spectral Density comparisons: ch1**

**Frequency spectra comparisons**
- Good agreement is achieved in the frequency spectra.
- Synthetic frequency spectrum should match experiment as a test of simulation and synthetic diagnostic.
- Cannot be used to differentiate between simulations.
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  – $k$-spectra comparisons
    ▪ 5 ‘big-box’ e- scale sims for the strong ETG case
    ▪ 1 ‘big-box’ e- scale sim for the weak ETG case
$k$-spectra comparisons isolate the importance of $q$ and $\hat{s}$ in determining the shape of the $k$-spectrum.

$k$-spectra shape comparisons for the strong ETG condition
5 ‘big-box’ e- scale sims.

- Synthetic spectra are scaled to compare the shape of the $k$-spectrum $S(k)$.

$k$-spectra comparisons isolate the importance of $q$ and $\hat{s}$ in determining the *shape* of the $k$-spectrum.

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\(k\)-spectra shape comparisons for the strong ETG condition
5 ‘big-box’ e- scale sims.

- Synthetic spectra are scaled to compare the shape of the \(k\)-spectrum \(S(k)\).
- Simulations run with scaled-\(q\), \(\hat{s}\) (c, d, e) best match the shape of the \(k\)-spectrum.

\[\star\] Ricci PoP 2011, Holland PoP 2016
$k$-spectra comparisons isolate the importance of $q$ and $\hat{s}$ in determining the shape of the $k$-spectrum

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- Validation metric $R_{shape} \in [0, 1]$ [*]

$\sigma(\nabla T, \nabla n)$
0.99

$\sigma(\nabla T), q, \hat{s}$
0.98

$\sigma(\nabla n), q, \hat{s}$
0.53

$\sigma(\nabla T, \nabla n), q, \hat{s}$
0.76

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<td>0.01</td>
<td>0.53</td>
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Fluctuation level ratio between strong and weak ETG conditions can be quantitatively compared to experiment.

\[
\frac{\langle S \rangle_{\text{strong ETG}}}{\langle S \rangle_{\text{weak ETG}}} \text{syn}
\]

- GYRO sim. for the *weak ETG* condition matched \( P_e \).

**Fluctuation power level ratio**

- **Sim:**
  - base
  - \((\nabla T, \nabla n)\)
  - \((\nabla T), \& q, \hat{s}\)
  - \((\nabla n), \& q, \hat{s}\)
  - \((\nabla T, \nabla n), \& q, \hat{s}\)

**Experimental range**

1 = bad
0 = good
Fluctuation level ratio between strong and weak ETG conditions can be quantitatively compared to experiment.

- GYRO sim. for the weak ETG condition matched $P_e$.
- Validation metric $R_{ratio} \in [0, 1]$

<table>
<thead>
<tr>
<th>Simulation</th>
<th>$R_{ratio}$</th>
</tr>
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<tbody>
<tr>
<td>Base</td>
<td>0.18</td>
</tr>
<tr>
<td>$\sigma(\nabla T, \nabla n)$</td>
<td>0.05</td>
</tr>
<tr>
<td>$\sigma(\nabla T), q, \hat{s}$</td>
<td>0.01</td>
</tr>
<tr>
<td>$\sigma(\nabla n), q, \hat{s}$</td>
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</tr>
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<td>$\sigma(\nabla T, \nabla n), q, \hat{s}$</td>
<td>0.72</td>
</tr>
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</table>

1 = bad
0 = good
Fluctuation level ratio between strong and weak ETG conditions can be quantitatively compared to experiment.

- GYRO sim. for the *weak ETG* condition matched $P_e$.
- Validation metric $R_{\text{ratio}} \in [0, 1]$

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<th>Sim:</th>
<th>$R_{Pe}$</th>
<th>$R_{\text{shape}}$</th>
<th>$R_{\text{ratio}}$</th>
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<tbody>
<tr>
<td>Base</td>
<td>1</td>
<td>1</td>
<td>0.18</td>
</tr>
<tr>
<td>$\sigma(\nabla T, \nabla n)$</td>
<td>0.29</td>
<td>0.99</td>
<td>0.05</td>
</tr>
<tr>
<td>$\sigma(\nabla T), q, \hat{s}$</td>
<td>0.006</td>
<td>0.98</td>
<td>0.01</td>
</tr>
<tr>
<td>$\sigma(\nabla n), q, \hat{s}$</td>
<td>0.01</td>
<td>0.53</td>
<td>0.47</td>
</tr>
<tr>
<td>$\sigma(\nabla T, \nabla n), q, \hat{s}$</td>
<td>1</td>
<td>0.76</td>
<td>0.72</td>
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1 = bad
0 = good
All comparisons are condensed via a composite metric for discrimination between simulations.

### Composite metric

The composite metric $M$ is defined as:

$$M = \frac{\sum_i h_i R_i}{\sum h_i}$$

where $h_i$ and $R_i$ are weighting and ratio terms, respectively.

#### Base

- $M_{\text{base}} = 0.67$

#### Fluctuation Level Ratio Comparisons

- $M_{\sigma(\nabla T, \nabla n)} = 0.47$
- $M_{\sigma(\nabla T), q, \hat{s}} = 0.40$
- $M_{\sigma(\nabla n), q, \hat{s}} = 0.40$
- $M_{\sigma(\nabla T, \nabla n), q, \hat{s}} = 0.76$

#### Reference for $M$

- $0$ → perfect agreement
- $0.04$ → error $\sim 1\sigma$
- $0.5$ → error $\sim 2\sigma$
- $0.9$ → error $\sim 3\sigma$

---

**Main outcome**: Validated e-scale GK simulations in the NSTX core using high-k turbulence measurements for the 1\textsuperscript{st} time.

[*] ST-FNSF, Brown FST 2017
[**] Sorbom FED 2015
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Summary

- 1\textsuperscript{st} simultaneous agreement between exp. & sim. of $P_e$, fluct. level ratio and $k$-spectra shape of e- scale turbulence in a tokamak $\Rightarrow$ ETG-driven turbulence can dominate in core-gradient region of modest beta NSTX H-modes.

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- 1st simultaneous agreement between exp. & sim. of $P_e$, fluct. level ratio and $k$-spectra shape of e-scale turbulence in a tokamak $\Rightarrow$ ETG-driven turbulence can dominate in core-gradient region of modest beta NSTX H-modes.
- Implemented two equivalent synthetic high-$k$ diagnostics $\Rightarrow$ novel ‘big-box’ e-scale is required for quantitative e-scale turb. comparisons.
- High-$k$ $f$-spectrum not a critical constraint to discriminate between simulations.
- Importance of $(q, \hat{s})$ in matching $P_e$ and determining shape of $k$-spectrum.

Main outcome: Validated e-scale GK simulations in the NSTX core using high-k turbulence measurements for the 1st time.

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- Importance of $(q, \$)$ in matching $P_e$ and determining shape of $k$-spectrum.

**Broader Impact:**
- A new framework applicable to additional coherent scattering turbulence measurements, like Doppler Backscattering, reflectometry, etc.
- Improved confidence in turbulent-transport models for prediction and optimization of future fusion reactors (ITER, ST-FNSF [*], ARC[**]).
- This work motivates further work in higher $\beta$ and lower collisionality conditions in NSTX-U and MAST-U.
- This work has demonstrated a multi-level validation methodology to enable future validation efforts of turbulent transport models.

[*] ST-FNSF, Brown FST 2017
[**] Sorbom FED 2015
Thank you

MIT PhD Thesis Committee
Prof. Anne White
Dr. Nathan Howard
Prof. Nuno Loureiro

Princeton Plasma Physics Lab (PPPL)
Dr. Walter Guttenfelder
Dr. Yang Ren
Backup
Wavenumber grid from standard e-scale simulation is too coarse to resolve measured $k$.

<table>
<thead>
<tr>
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<th>$k_{\theta\rho_s}$ [min, max]</th>
<th>$k_{r\rho_s}$ [min, max]</th>
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<tr>
<td><strong>e-scale</strong></td>
<td>[1.5, 65 or 86]$^*$</td>
<td>[1, 47 or 32]$^*$</td>
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<tr>
<td><strong>'Big-box' e-scale</strong></td>
<td>[0.3, 65 or 88]$^*$</td>
<td>[0.3, 32]</td>
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* max $k_{\theta\rho_s}$ different in two exp. conditions

*Computationally intensive*

$\sim$ 1-2 M CPU h/sim
Compare total power $P_{\text{tot}}$, spectral peak $<f>$ and spectral width $W_f$ in a prescribed frequency band.

**Simulation**

$$\log[S(f)] \text{ [m.s]}$$

- SYN: hyb scale scan
- SYN: fit
- $f_{\text{cut}}$
- $f_{\text{cut}}$

**Experiment**

$$\log[S(f)] \text{ [a.u.]}$$

- experimental noise at $f = 0$
Synthetic $f$-spectrum reproduces spectral peak $\langle f \rangle$, close to match spectral width $W_f$

**Spectral Density comparisons: ch1**

**STRONG ETG ch1**

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**WEAK ETG ch1**

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$f$-spectrum agreement is achieved for all channels

**Spectral density $S(f)$ comparisons**

### STRONG ETG ch1

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\( f \)-spectrum is determined by combination of turbulence characteristics, \( k \)-resolution and Doppler shift

- **Spectral peak** \( \langle f \rangle \) is dominated by Doppler Shift
  \[
  f_{\text{turb}} \ll f_{\text{Dop}}
  \]
  - Not a critical constraint on simulation model

- **Spectral width** \( W_f \) determined by combination of:
  - Turbulence spectrum in plasma frame
  - \( k \)-resolution of the high-\( k \) diagnostic
  - \( k \)-grid resolution of the simulation
  - Doppler shift

- \( f \)-spectrum does not provide critical constraints to discriminate between models.

- We find the wavenumber spectrum more useful for selection of simulations.

\[
\begin{align*}
  f_{\text{Dop}} &= \vec{k} \cdot \vec{v} \sim 1\text{MHz} \\
  f_{\text{turb}} &\sim 50 - 100 \text{kHz}
\end{align*}
\]
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\end{align*}
\]

**Frequency spectra comparisons**
- Good agreement is achieved in the frequency spectra.
- Synthetic frequency spectrum should match experiment as a test of simulation and synthetic diagnostic.
- Cannot be used to differentiate between simulations.
Synthetic diagnostic revealed high-k measurement is closer to ‘streamer’ peak than ‘naïve’ mapping suggests.

Spectral density $S(k_r, k_\theta) \propto |\delta n_e|^2$.
Synthetic diagnostic revealed high-k measurement is closer to ‘streamer’ peak than ‘naïve’ mapping suggests

Spectral density $S(k_r, k_\theta) \propto |\delta n_e|^2$

‘Naïve’ mapping does not take into account field-aligned geometry.

Geometric effects ($B_{ref}, \kappa, |\nabla r|,\ldots$) bring the measured $k$ close to peak in fluctuation spectrum.

Streamers (★) : predicted to dominate ETG transport in low-beta ST parameters [*].

⇒ Suggests high-k measurement is more relevant to ETG transport than previously thought.

Discharge conditions

Shot 141767

- $P_{NB}$ (MW)
- $I_p$ (MA)
- $D_\alpha$ (a.u.)
- Low-$f$ Mivov (G)
- Line-integrated $n_e$ ($10^{15} \text{cm}^{-2}$)

Time range of interest

$t(s)$

$0$, $0.1$, $0.2$, $0.3$, $0.4$, $0.5$, $0.6$
Spectrogram of high-k density fluctuations

(a) Spectrogram of high-k fluctuations

(b) Frequency spectrum (channel 1)

- Channel 1, $k_s \rho_s \sim 13-17$
- Channel 2, $k_s \rho_s \sim 10-14$
- Channel 3, $k_s \rho_s \sim 8-11$

- $t = 398$ ms
- $t = 448$ ms
- $t = 498$ ms
- $t = 565$ ms
High-k Density Fluctuations are Linearly Stabilized by Density Gradient through the Critical Gradient

- \( R/L_{ne} \) is a **linear stabilizing** mechanism when it dominates the Jenko critical gradient (Jenko PoP 2001).

\[
(R / L_{Te})_{crit} = \max \left\{ 0.8 R / L_{ne} \right. \\
\left. f(\tau, \hat{s} / q, \varepsilon, \varepsilon d\kappa / d\varepsilon) \right\}
\]

- \( R/L_{ne} \) increases and fluctuations decrease.

- \( R/L_{ne} \) increases at constant \((R/L_{Te}^{exp}) - (R/L_{Te})_{crit}\) suggests **nonlinearly** stabilizes turbulence.

\[
(R/L_{Te})_{crit} \text{ Jenko} \\
0.8 R / L_{ne} \text{ toroidal} \\
f(\tau, \hat{s} / q, \varepsilon, \varepsilon d\kappa / d\varepsilon) \text{ slab}
\]
Weak ETG Condition: electron-scale turbulence simulation can match $P_e$

Perform 2 ‘big-box’ e-scale sims.

‘Big-box’ electron-scale sim
- **Base** (exp parameters): $P_{e, \text{sim}} \sim 0$
- $\sigma(\nabla T, \nabla n)$ scan: Match $P_e$
- e-scale turbulence close to marginal
Compare fluctuation level ratio between 5 ‘big-box’ sims. for the strong ETG to the weak ETG condition

- Experiment since not absolutely calibrated.
- Synthetic spectra have absolute units.
Total thermal transport budget Strong ETG

Strong ETG: using exp. $\nabla n$

Strong ETG: using scaled $1-\sigma(\nabla n)$

$P_e [MW]$ vs. $a/L_{Te}$

$P_e [MW]$ vs. $a/L_{Te}$

$a/L_{Te}^{crit,ETG} \sim 0.66$

$\sigma(a/L_{Te}) \sim 25\%$, $\sigma(a/L_{ne}) \sim 50\%$

$q = 3.79$, $s = 1.8$ (toroidal)
Total thermal transport budget Weak ETG

**Part a)**
- Weak ETG: using exp. $\nabla n$
- EXP (TRANS) sim.
- ion-scale sim
- e-scale sim
- ‘big-box’ e-scale: base sim.

**Part b)**
- Weak ETG: using scaled $1-\sigma(\nabla n)$
- EXP (TRANS) sim.
- ion-scale sim
- e-scale sim
- ‘big-box’ e-scale: $\nabla T, \nabla n$-scan sim.

- $P_{e \text{,\, scale}}(a/L_T=5.1) \sim 8\text{-}10\text{MW}$

- $a/L_{\text{Te}}^{\text{crit,ETG}} \sim 3$
- $\sigma(a/L_{\text{Te}}) \sim 30\%$,
- $\sigma(a/L_{n_e}) \sim 30\%$,
- $a/L_{Te} = 4.5$,
- $a/L_{n_e} = 4.06$,
- $q = 3.07$, $s = 2.35$ (slab)