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Analytical studies of PROTO-SPHERA equilibria

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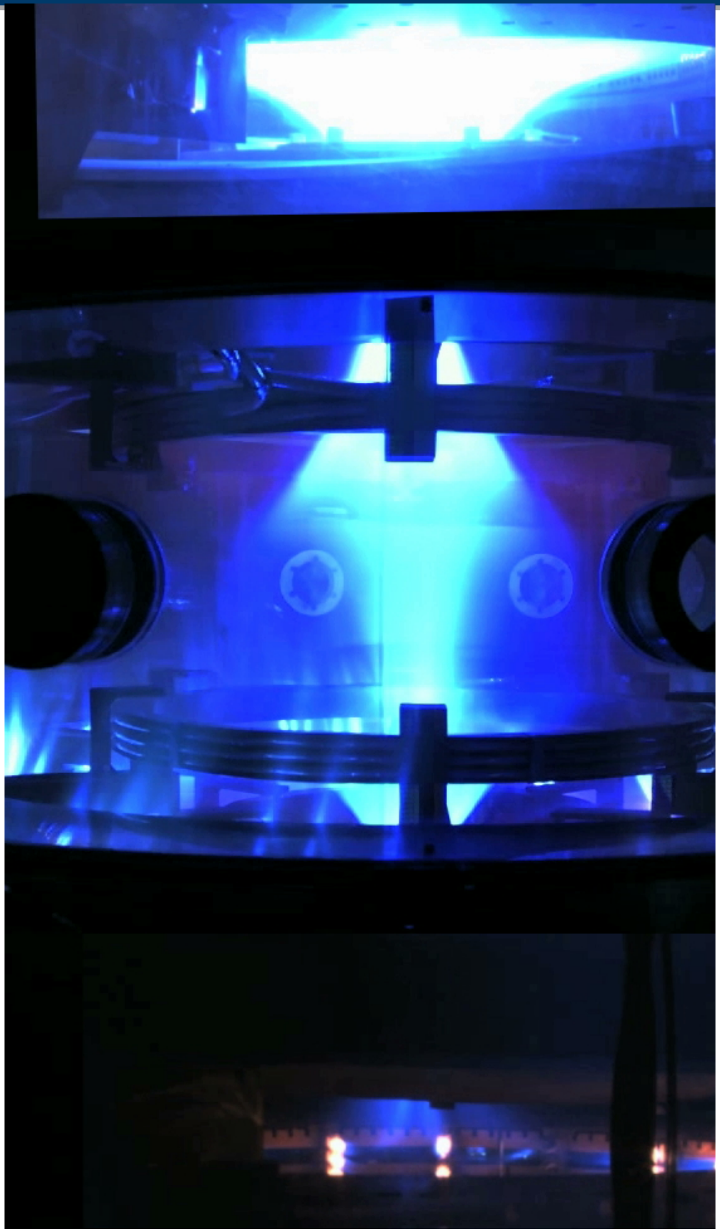
Experimental background (1)

Anode region

Core region
(in a pinch-only case)

Cathode region

Experimental background (2)



Motivation and outline

- Provide a simple geometric basis for transport studies
- Help diagnostics design and interpretation

- Model assumptions
- Equilibrium building bricks
- The CKF configuration
- The generalized bumpy pinch

Grad-Shafranov equation

- Axisymmetric plasma without inertia $\mathbf{j} \times \mathbf{B} = \nabla p$

$$\frac{1}{4\pi^2\mu_0}\nabla\cdot\left(\frac{1}{R^2}\nabla\psi\right) + \frac{\mu_0 I}{4\pi^2 R^2}\frac{\mathrm{d}I}{\mathrm{d}\psi} = -\frac{\mathrm{d}p}{\mathrm{d}\psi}$$

- where

ψ is poloidal magnetic flux
R is distance from axis
$I(\psi)$ is poloidal current
$p(\psi)$ is plasma pressure

Model assumptions

- Linear poloidal current $\mu_0 I = \mu \psi$

- Linear pressure $4\pi^2 \mu_0 p = \alpha \mu^2 \psi$

- With these, the G-S equation becomes linear

$$R^2 \nabla \cdot \left(\frac{1}{R^2} \nabla \psi \right) + \mu^2 \psi = -\alpha \mu^2 R^2$$

Comments on model assumptions

- Linear pressure $4\pi^2\mu_0 p = \alpha\mu^2\psi$ is also assumed in Solov'ev's tokamak equilibrium model.

- The second Solov'ev's assumption $I\dfrac{\mathrm{d}I}{\mathrm{d}\psi} = const.$ is replaced by $\mu_0 I = \mu\psi$ in order to avoid singularities at the axis.

- The μ parameter is related to the Kruskal-Shafranov limit.

- The α parameter is related to the plasma β.

Particular solutions

- Particular solution of the non-homogeneous equation

$$\psi_p(R) = -\alpha R^2$$

- Lowest degree Chandrasekhar-Kendall solution of the homogeneous equation

$$\psi_{CK}(R, Z; \mu)$$

- Furth “square-toroid” solution

$$\psi_F(R, Z; \mu, k)$$

- where *k* is a free parameter.

Details of particular solutions

- CK separated in spherical (r, θ, φ) coordinates

$$\psi_{CK}(r,\theta;\mu) = \left(\frac{\sin(\mu r)}{\mu r} - \cos(\mu r)\right)\sin^2\theta$$

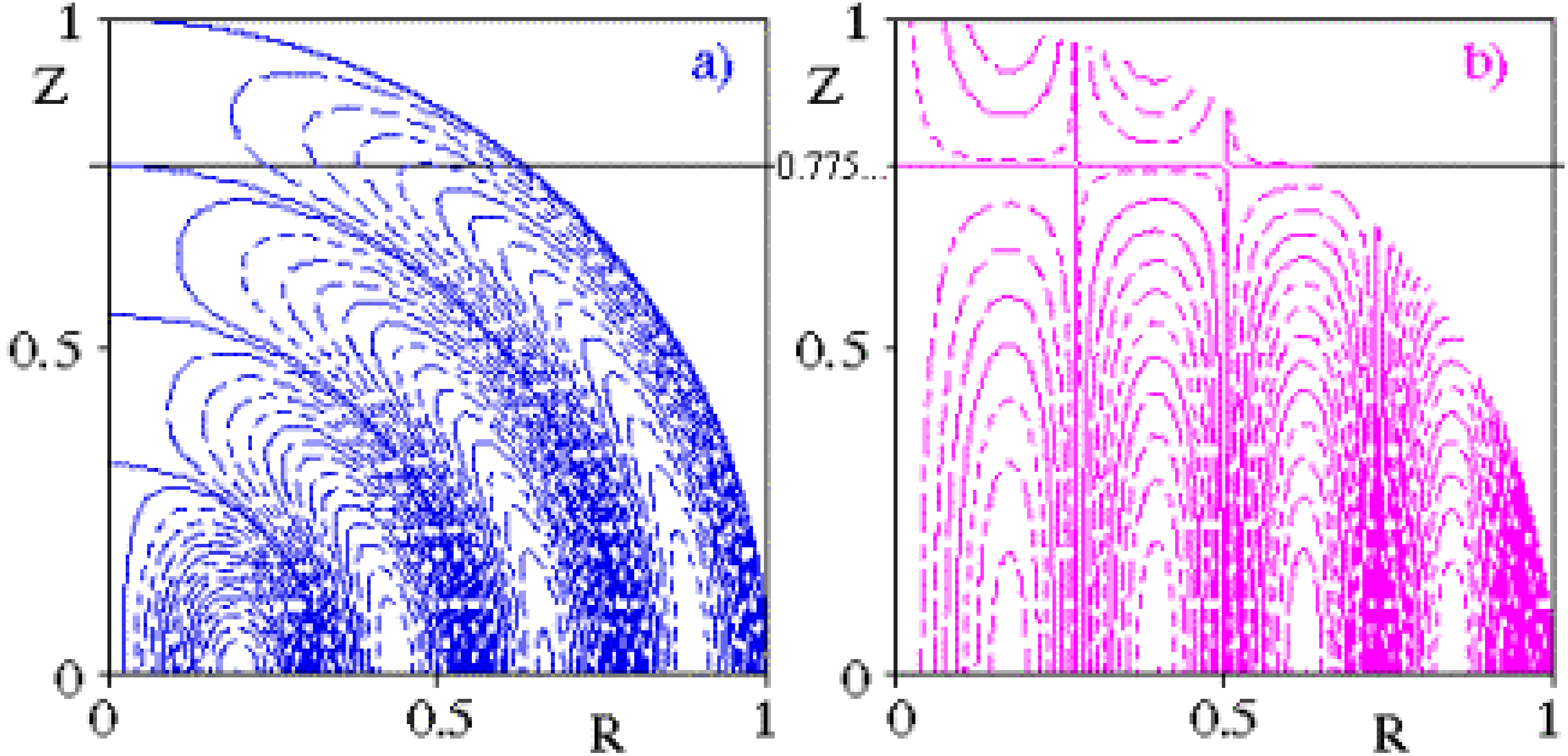
$$r = \sqrt{R^2+Z^2}; \quad \tan\theta = R/Z$$

- Furth separated in cylindrical (R, Z, φ) coordinates

$$\psi_F(R,Z;\mu,k) = \sqrt{\mu^2-k^2}\,R\,\mathrm{J}_1\left(\sqrt{\mu^2-k^2}\,R\right)\cos(kZ)$$

Details of particular solutions

$\psi_{CK}(R,Z;\mu)$ $\psi_F(R,Z;\mu,k)$



The CKF model

- Configuration with half length *L*
- Put the third radial zero of CK at *r* = *l*

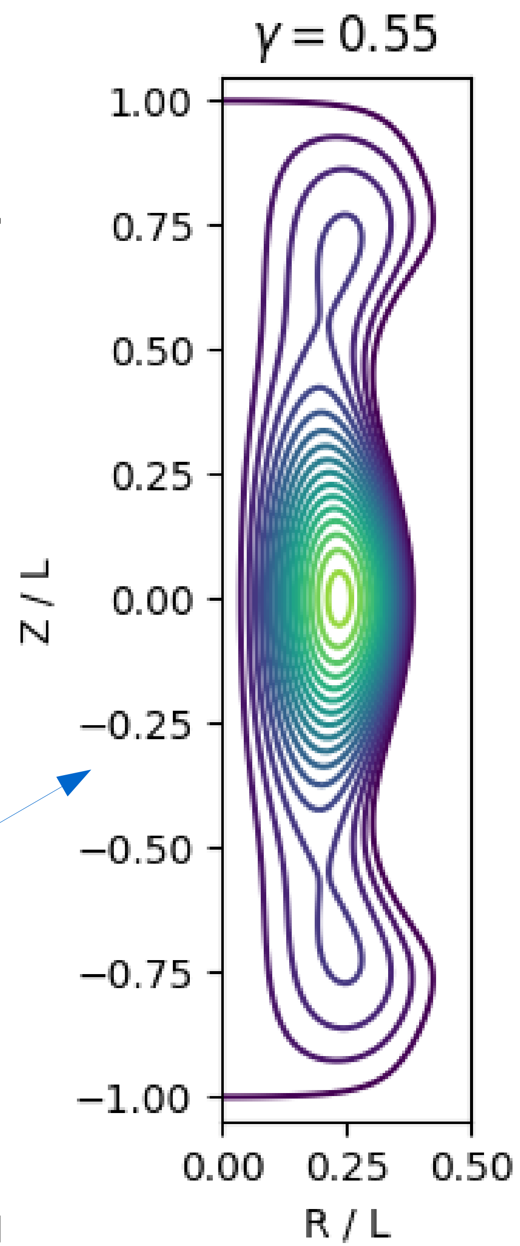
$$\mu = 10.904/L$$

- Put the first axial zero of F at *Z* = *L*

$$k = \pi/(2L).$$

- Mix:

$$\psi_{CKF} = \psi_{CK} + \gamma\psi_F$$



Topology of flux contours for 0.402 < γ < 0.69

Looking for a more flexible model (1)

- CKF can not reproduce the different observed plasma shapes: need something more flexible.
- Do without a global configuration:
  this is the price of flexibility.
- Restrict analysis to the core region, or to the region between x-points if present.
- Discard the CK component and use more F components.
- Augment the model including pressure.

Looking for a more flexible model (2)

- Use three solutions of the F type $f_n(R)\cos(k_n Z)$ with different scales:

$$k_0 = 0$$

$$k_1 = \frac{\pi}{L_x} \qquad\qquad f_n = \frac{k_1^2 R}{\sqrt{\mu^2 - k_n^2}} \mathrm{J}_1\left(\sqrt{\mu^2 - k_n^2} R\right)$$

$$k_2 > k_1$$

- where L_x is the half length of the considered region
- The model with two terms only is dubbed bumpy pinch

Normalized spatial scales

- Use k_1 to normalize spatial scales:
- Variables $\rho = k_1 R$ and $\zeta = k_1 Z$
- Parameters $\kappa = \mu / k_1$ and k_2 / k_1
- e.g.

$$f_2(\rho; \kappa, \frac{k_2}{k_1}) = \frac{\rho}{\sqrt{\kappa^2 - k_2^2/k_1^2}} J_1\left(\sqrt{\kappa^2 - k_2^2/k_1^2}\,\rho\right)$$

- The Kruskal-Shafranov limit in a cylinder is $\kappa = 2$

Extension to low-current

Extend F-solutions to $\mu < k_n$

- Just replace Bessel J with modified Bessel I
- e.g.

$$f_1 = \begin{cases} \rho\, \mathrm{I}_1(\sqrt{|\kappa^2 - 1|}\,\rho)/\sqrt{|\kappa^2 - 1|} & \text{if } \kappa < 1 \\ \rho\, \mathrm{J}_1(\sqrt{|\kappa^2 - 1|}\,\rho)/\sqrt{|\kappa^2 - 1|} & \text{if } \kappa > 1 \\ \rho^2/2 & \text{if } \kappa = 1 \end{cases}$$

Including pressure

Include pressure:

- Use the non-homogeneous solution

$$\psi_p(R) = -\alpha R^2$$

- a similar term is present in Solov'ev's equilibria

- Introduce a reference field B_0 and normalize to

$$\frac{2\pi B_0}{k_1^2}$$

The generalized bumpy pinch

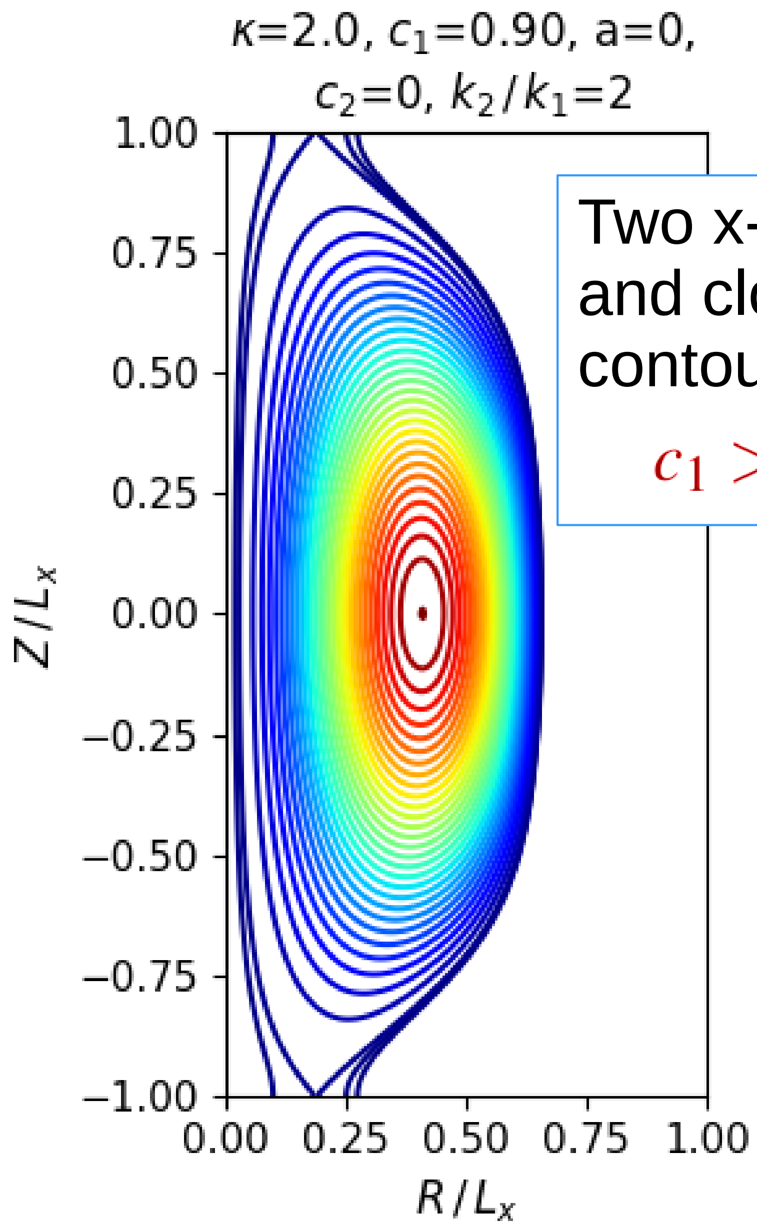
- Normalized poloidal flux $\hat{\psi} = \frac{k_1^2}{2\pi B_0}\psi$

$$\hat{\psi} = -\frac{\alpha}{2\pi B_0}\rho^2 + f_0 + c_1 f_1 \cos(\zeta) + c_2 f_2 \cos(\frac{k_2}{k_1}\zeta)$$

Flux contours depend on:

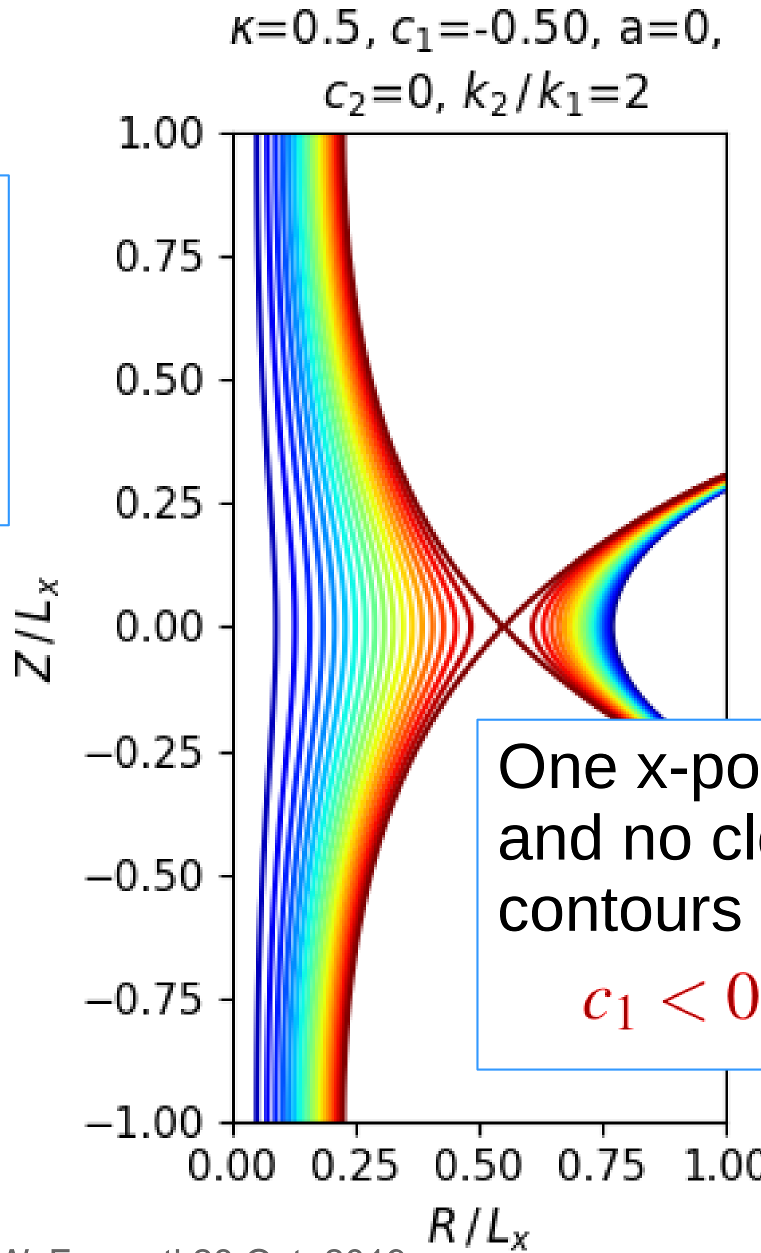
- The normalized current parameter $\kappa = \mu/k_1$
- the shaping constants c_1 and c_2 and k_2/k_1
- the pressure parameter α/B_0 $(\approx \text{maximum } \beta)$

Topology of flux contours



Two x-points and closed contours for

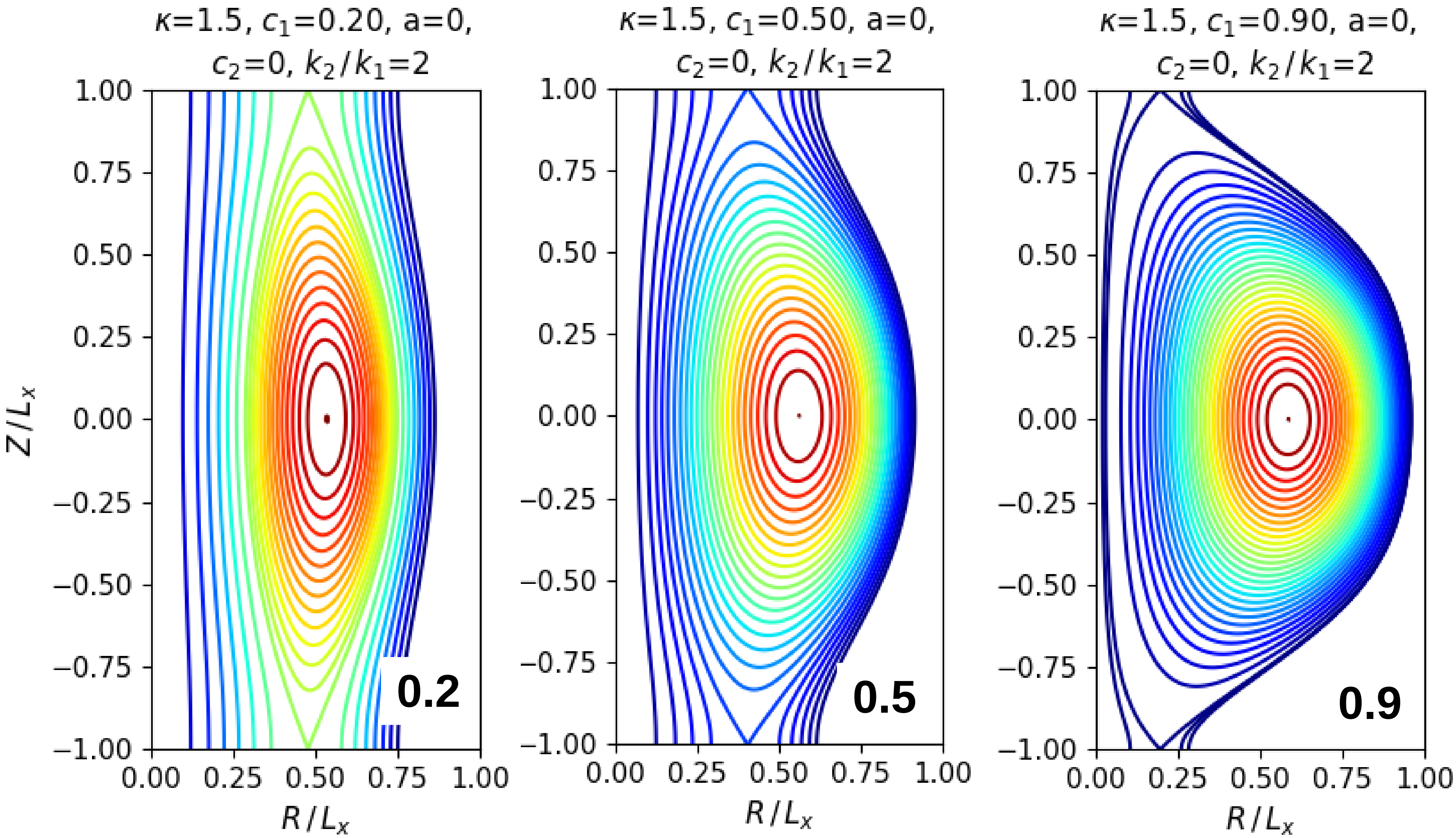
$c_1 > 0$



One x-point and no closed contours for

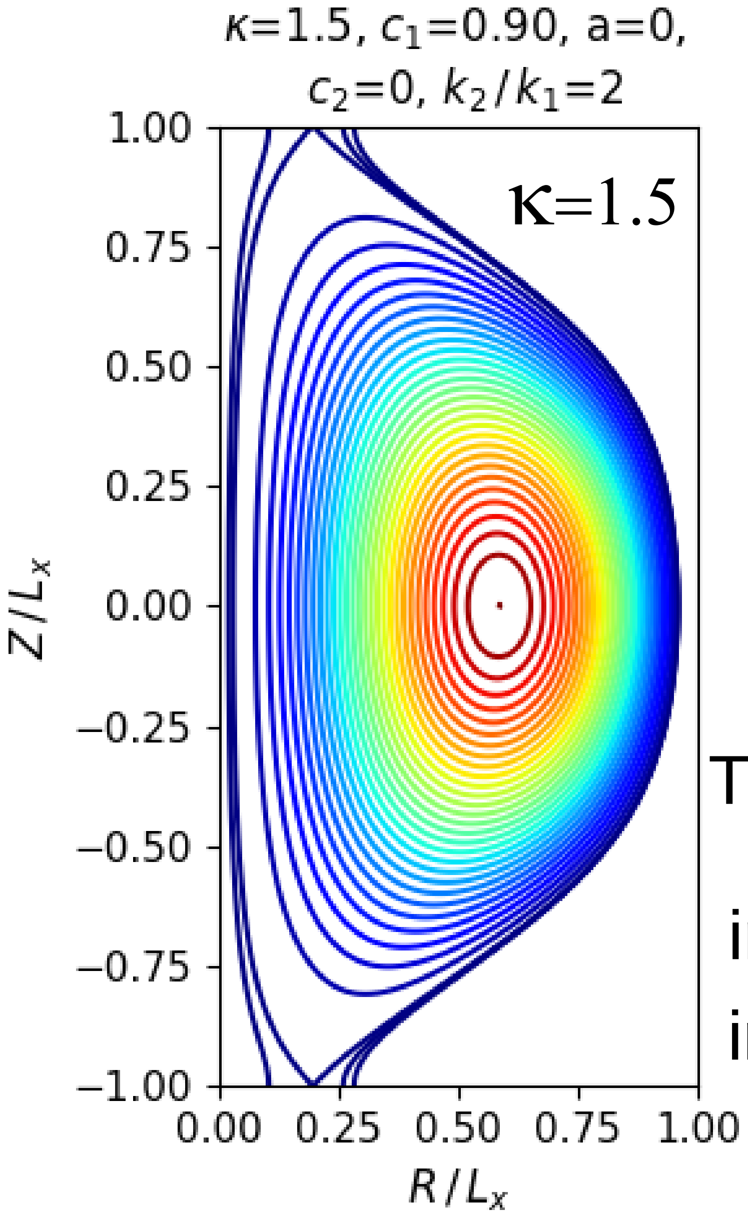
$c_1 < 0$

Role of (positive) c_1 value

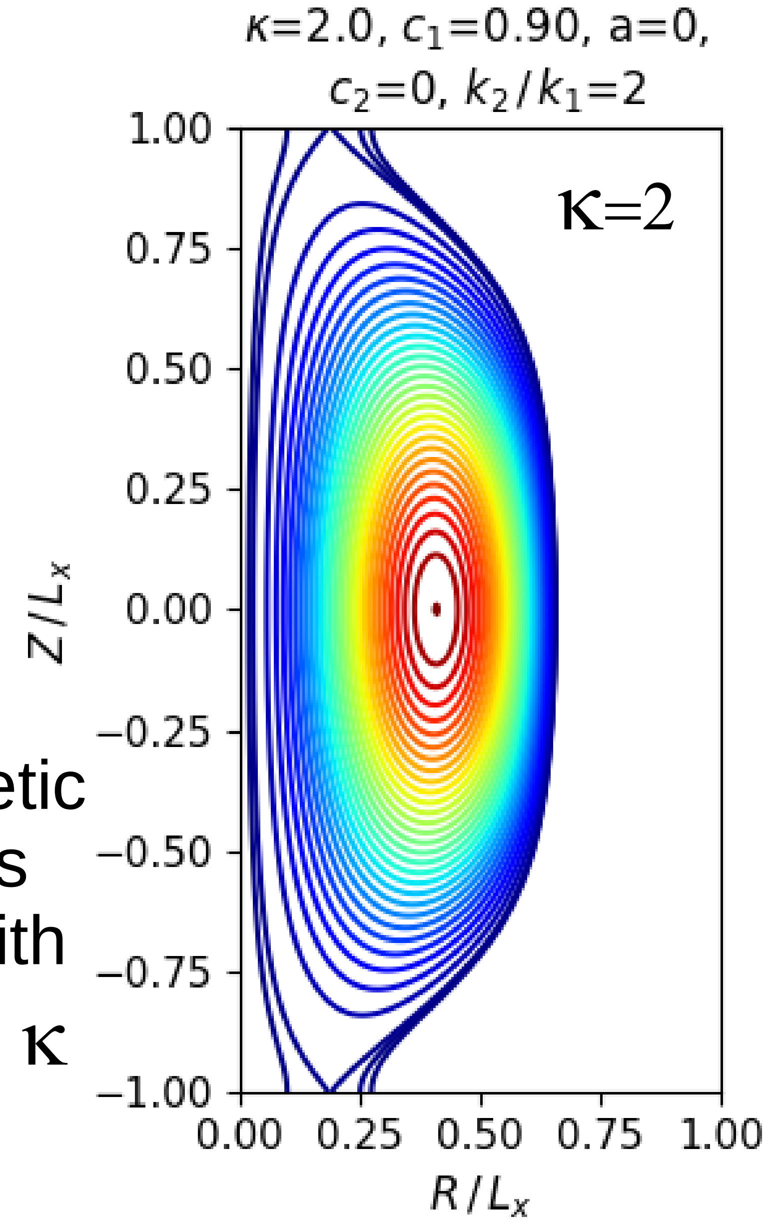


Separarix broadens and x-points shift-in with increasing c_1

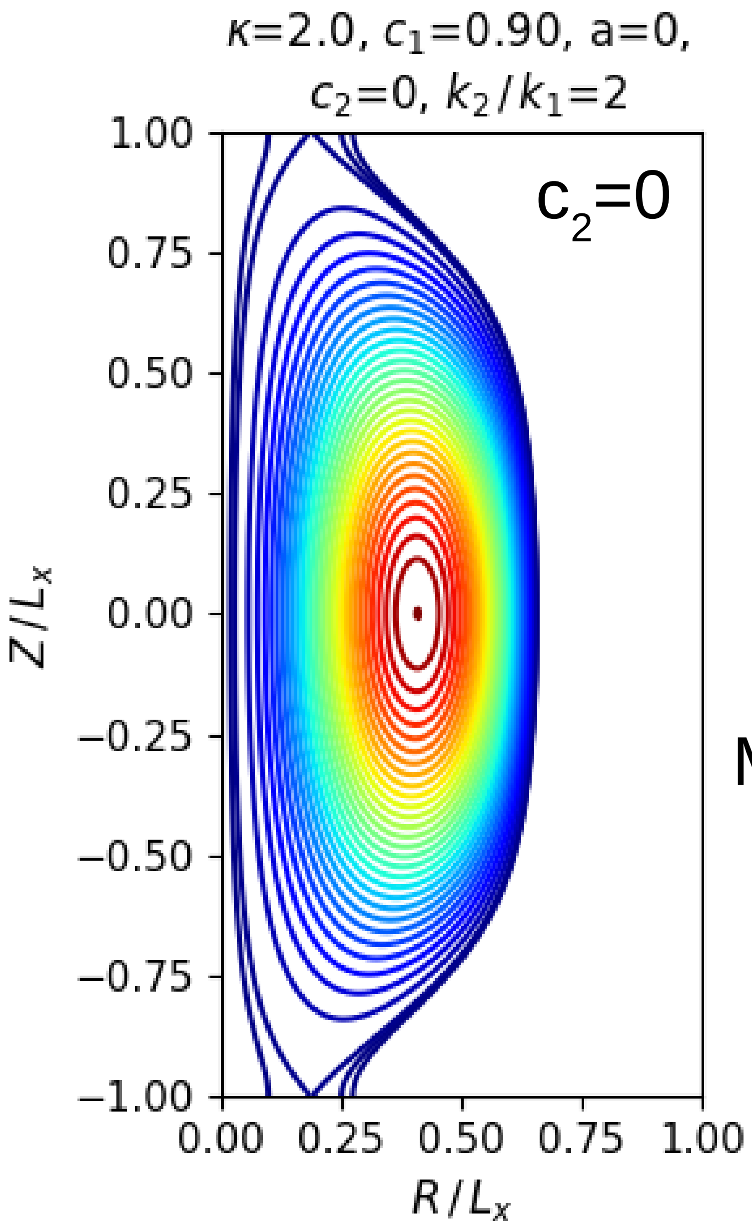
Role of the κ parameter



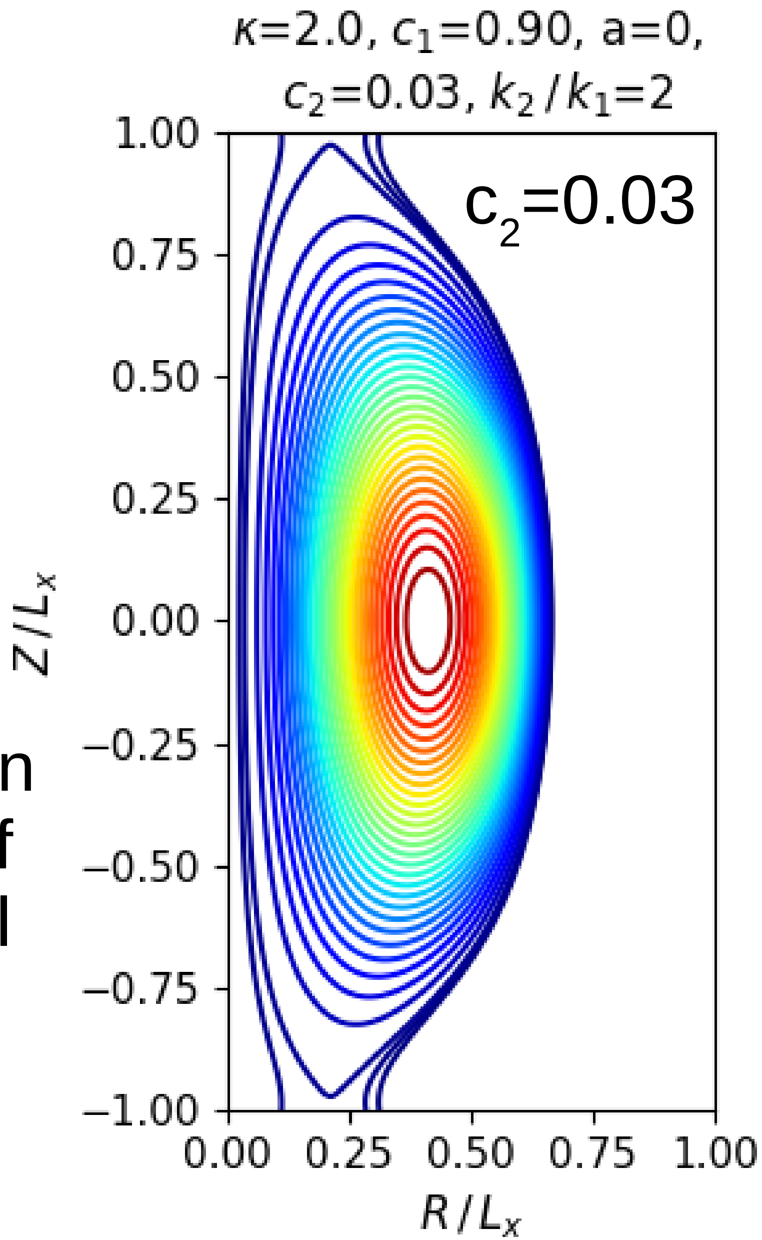
The magnetic axis shifts inwards with increasing κ



Role of c_2 with positive c_1

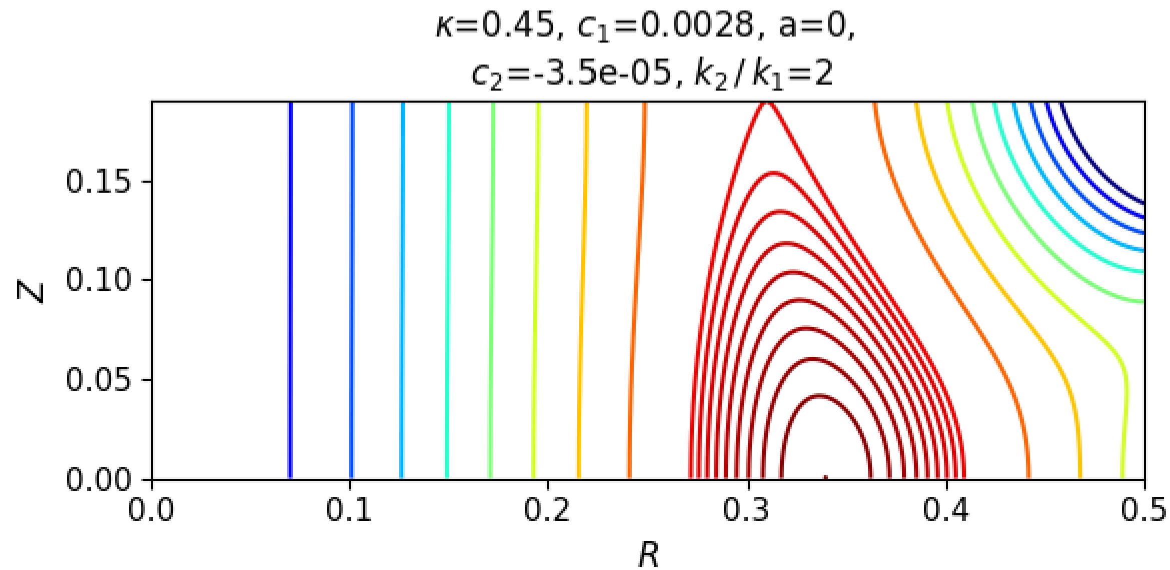


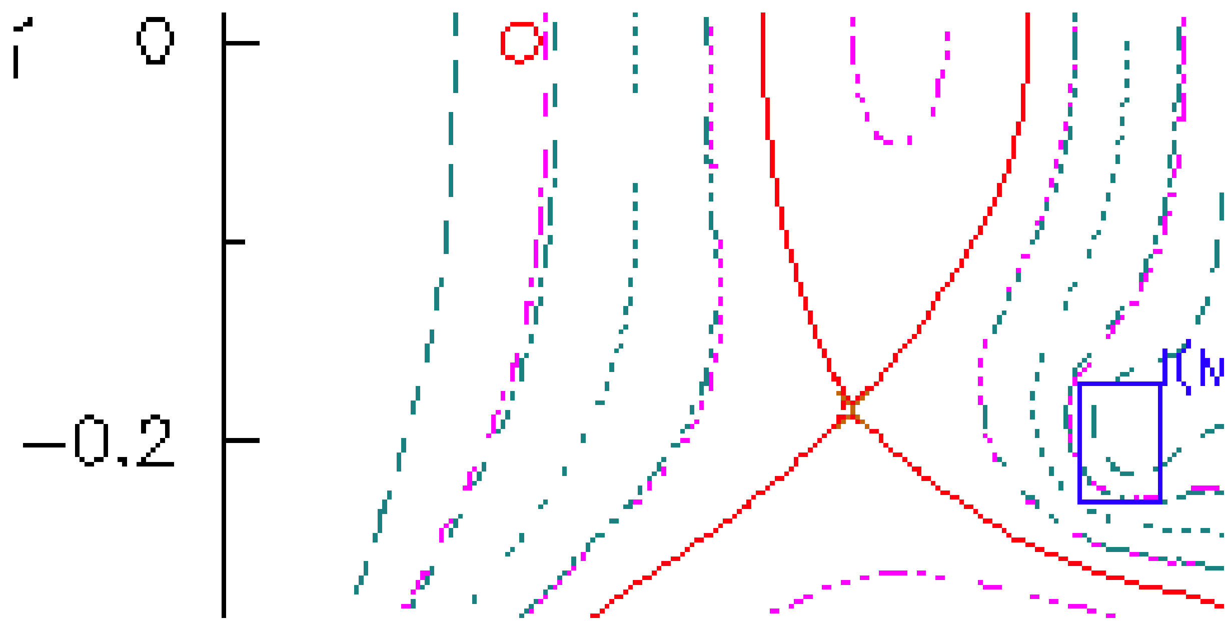
Mild effect on curvature of the external separatrix branch



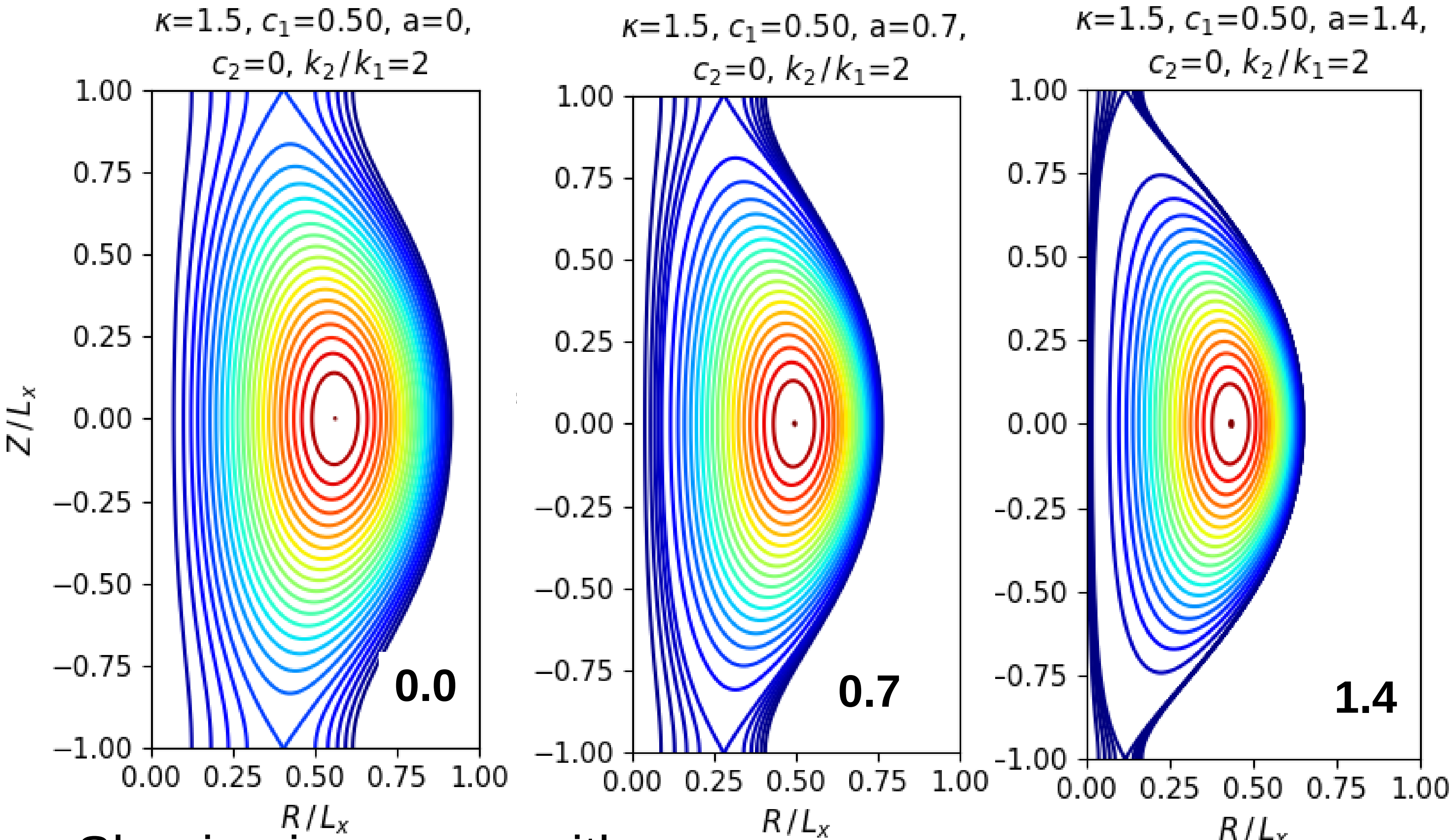
Slim tori

- Qualitatively reproduced
- Very sensitive to shaping parameters
- Not the best application of the model.





Role of pressure $\alpha/B_0 \ (\approx \text{maximum}\ \beta)$



- Shaping increases with pressure
- x-point collapses on the *Z*-axis at $\alpha/B_0 \approx \pi(1-c_1)$

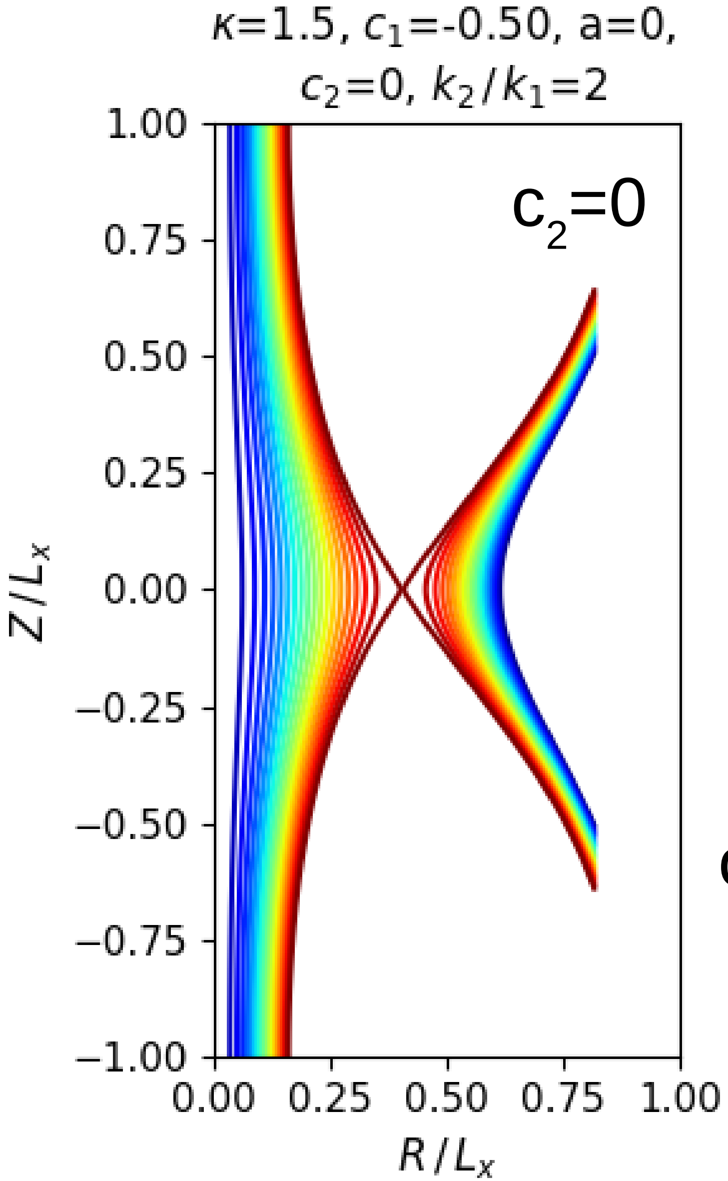
Summary

- Models can reproduce fat tori
- The generalized bumpy pinch can also reproduce the centerpost discharge without any surrounding torus
- A simple pressure term has been included

Perspectives

- Use the model field geometry for transport studies to estimate pinch resistivity at high current.
- Support image reconstruction and magnetic reconnection studies.

Role of c_2 with negative c_1



strong effect on the x-point region