

Deep Learning for data driven understanding and control of complex systems

A. Murari, M. Scarcella, R. Rossi, T. Craciunescu, M. Quadrini and M. Gelfusa
on behalf of JET Contributors



TOR VERGATA
UNIVERSITÀ DEGLI STUDI DI ROMA



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High temperature plasmas for the research on thermonuclear fusion cannot come in contact with material objects and the most promising technique to confine them relies on strong magnetic fields.

The most advanced configuration of the fields is the tokamak. Unfortunately magnetised high temperature plasmas of the tokamak type are very complex systems that have to be kept out of equilibrium to achieve the required performances.

They therefore feed and support a vast number of electrostatic and electromagnetic instabilities, which have not only to be understood but also properly controlled.

When confronted with signals generated by poorly known complex dynamical systems, the first steps are the determination of their dimensionality, the assessment of their cause-effect relationships and then the formulation of reduced models for control.





Outline

- **Practical definition of dimensionality: the Effective Dimension for COntrol (EDCO)**
- **Deep autoencoders of suitable architecture have been developed to calculate EDCO**
- **Causality detection with Physics Informed Interaction Autoencoders (PIAs)**
- **Deep autoencoders to implement data driven feedback schemes**
- **Conclusions**



Practical definition of dimensionality

The determination of a system dimensionality has been the subject of impressive amounts of work and the main conceptual tools to quantify it are the Intrinsic Dimension (ID) and the Fractal Dimension (FD).

Irrespective of their enormous relevance in many contexts, an important and often overlooked point is that by definition ID and FD are the dimensionality in the minimal manifold embedding the data. The techniques for ID and FD calculation have been developed from this mathematical and theoretical perspective.

This is the case of the paradigmatic dimensions with the algorithms of Grassberger and Procaccia (GP) and Krakovska (KR), and Levina-Bickel (LB) dimension.

However, in many practical applications neither the analyst nor the controller have direct access to the quantities defining such a minimal manifold but to those of a larger space containing the manifold.

Krakovská A., Chvosteková M., Simple correlation dimension estimator and its use to detect causality, Chaos, Solitons and Fractals 175 (2023) 113975.



Need for EDCO

For the purpose of the present work, what is important to appreciate is that for identification and control a more general definition of dimension is required. In the following, such a dimension is called the Effective Dimension for COntrol (EDCO) and can be loosely defined as:

Definition: the Effective Dimension for COntrol (EDCO) is the minimal number of variables needed to represent a system without loss of information, in the original space, in which it is embedded (and not on the minimal manifold).

By definition the EDCO provides only integer values that, as the ID, are the integer upper bound of the fractal dimension.

One crucial aspect is that we have developed techniques (autoencoders) that can calculate this dimension accurately and effectively.

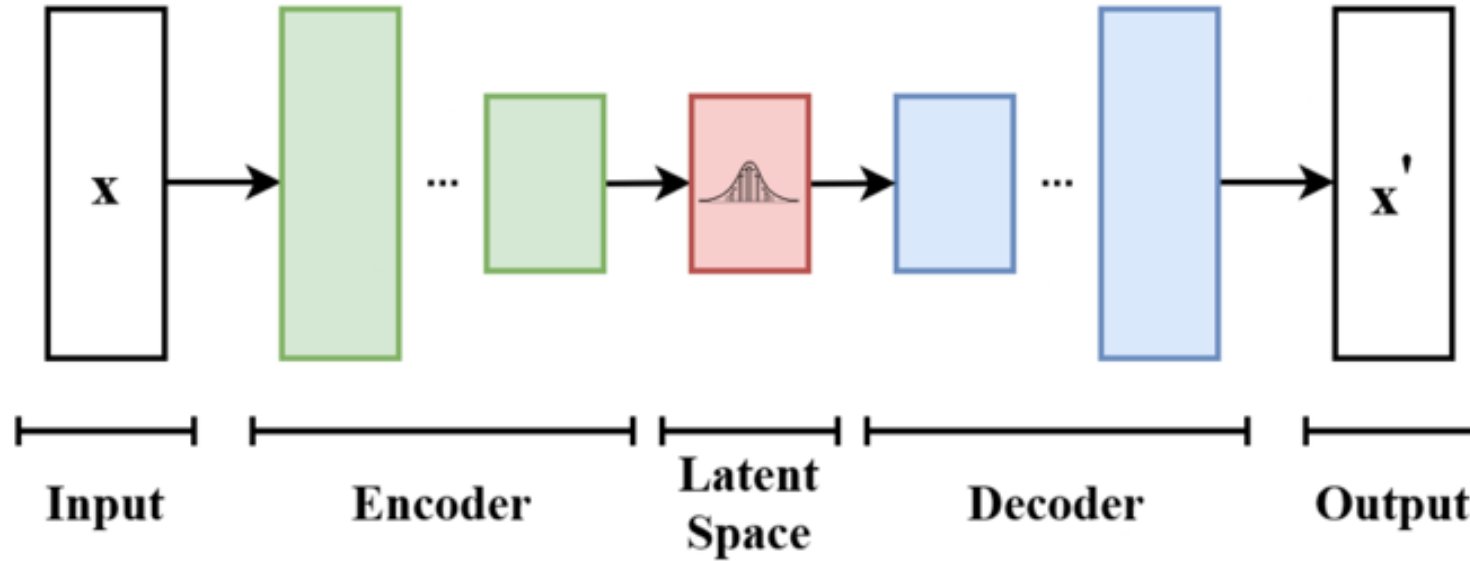


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EDCO and autoencoders



Key components of an autoencoder:

- **Encoder:** Compresses the input into a compact latent representation.
- **Code:** The compact latent representation.
- **Decoder:** Reconstructs the original input from the encoded representation, aiming to minimize the loss between the original and reconstructed data.

Autoencoders are a special class of unsupervised neural networks that aim to learn compressed, meaningful representations of input data.

These networks work by encoding the input into a lower-dimensional latent space and then decoding it back to reconstruct the original data.

This process forces the model to capture the most critical features of the data, which makes autoencoders highly effective for feature extraction, anomaly detection, denoising, data reconstruction and [dimensionality identification](#).

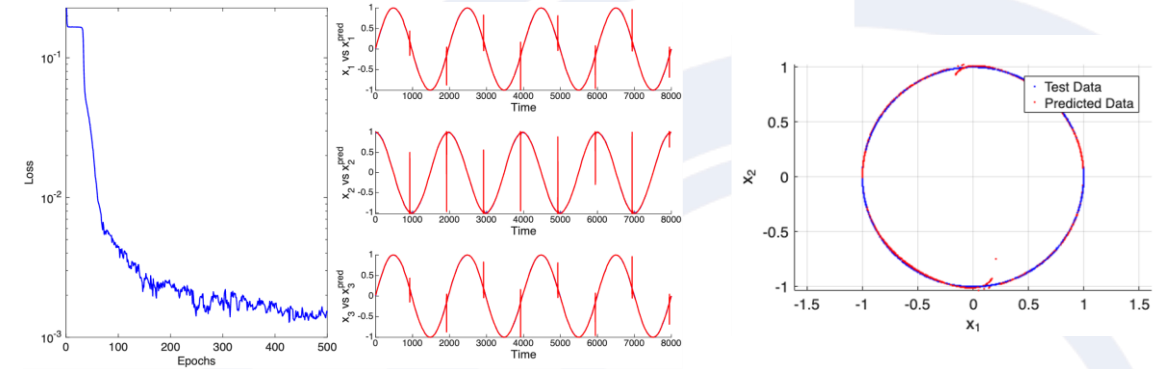


EDCO and autoencoders

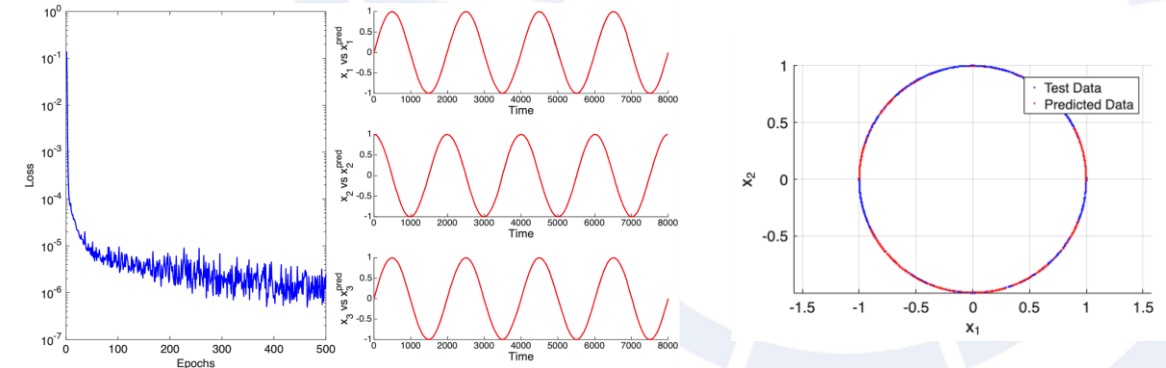
Let's consider the following parametric system of equations :

$$\begin{cases} x_1 = \sin(\omega t) \\ x_2 = \sin(\omega t + \phi) \\ x_3 = \sin(\omega t) \end{cases}$$

d=1



d=2

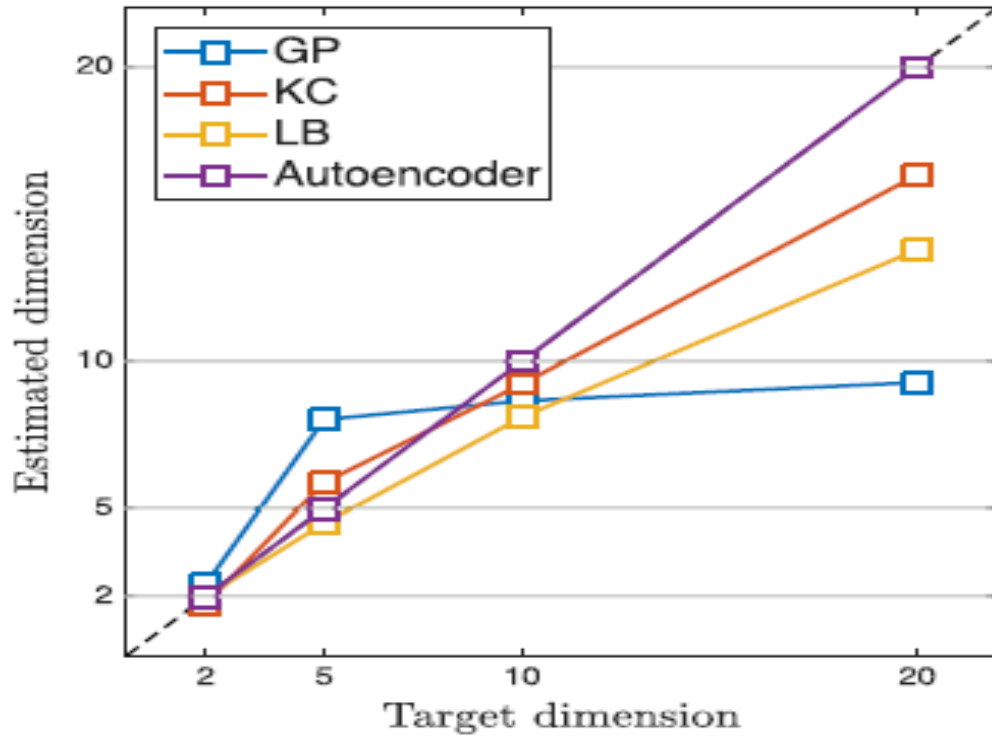


From a purely mathematical point of view the system depends on only the time variable t and it has therefore an ID of 1. Indeed for $\phi = 0$ and $\omega = 1$, the three equations become identical, the EDCO equals 1 and the autoencoder manages to perfectly reconstruct the input.

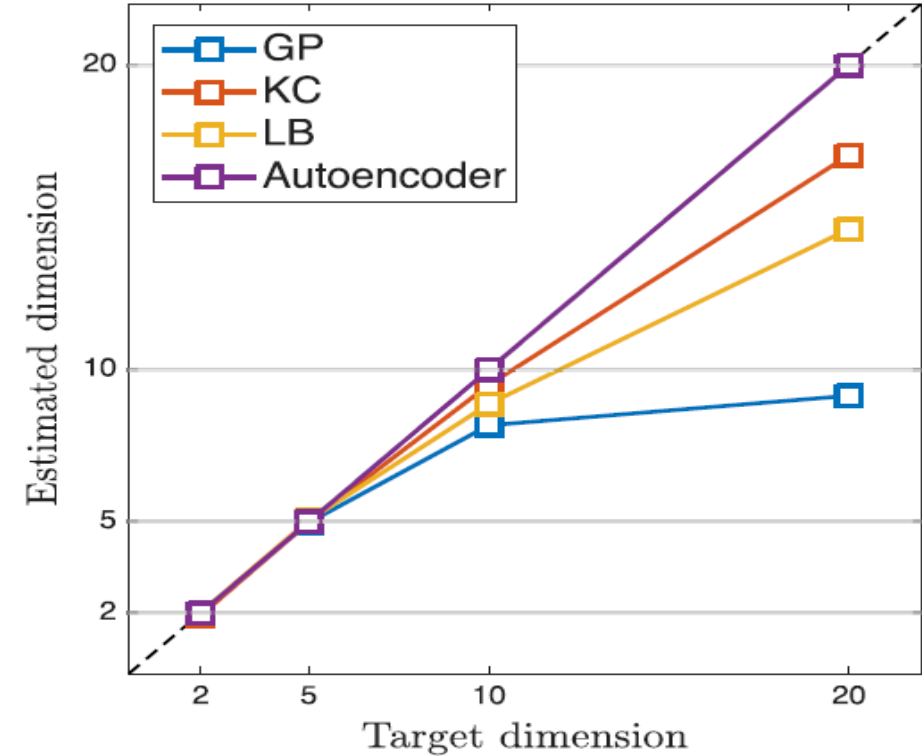
However example for $\phi = \pi/2$ and $\omega = 1$ the system describes a circle of unit radius. The value of EDCO becomes 2 because this is the minimum number of neurons required by the autoencoder to properly reproduce the three input time series.



Autoencoders and high dimensionality systems



Left: Identification of the data dimension for random points generated in spaces of different dimensions.



Right: Identification of the data dimension for white noise in spaces of different dimensions.

In these examples, the various techniques should provide a value of the dimension that is equal to the dimension of the space, in which they are embedded. Only the developed autoencoders do not underestimate the dimensionality of the data.



Autoencoders and noise

For the simulated Lorenz system in the chaotic regime, the theoretical FD is slightly above 2 and only the autoencoders, tasked to predict the next time point, identify the EDCO value of 3 irrespective of the noise level.

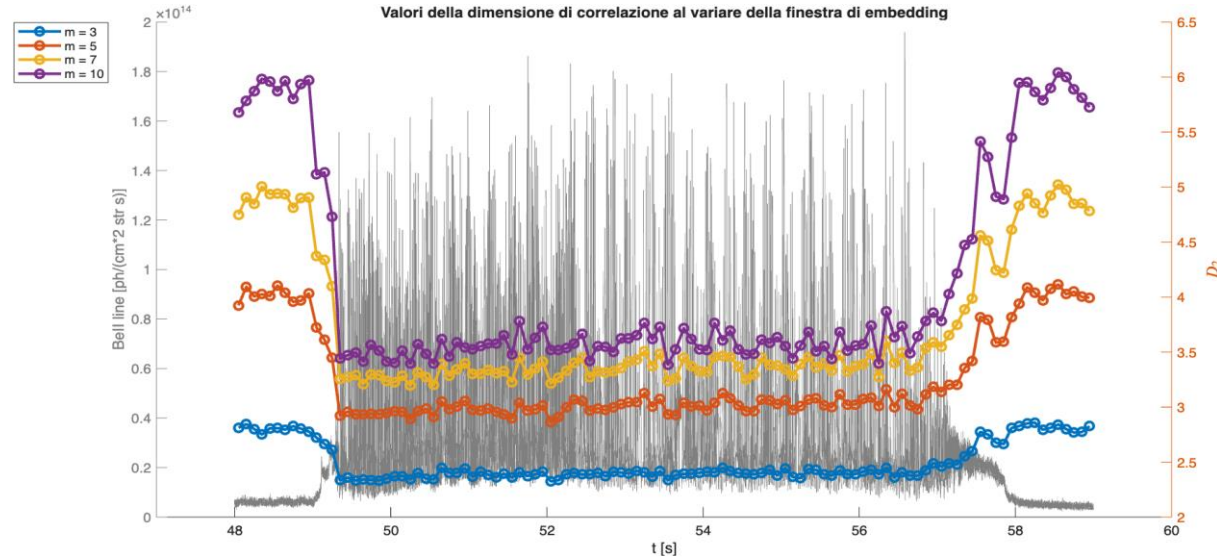
δ_η	d	p	CD	KC	LB	LPCA (95%)	Autoencoder
0 %	3	6	2.07	2.04	2.13	2.00	3.00
1 %	3	6	2.94	3.49	2.38	2.00	3.00
2 %	3	6	3.69	4.77	2.98	3.00	3.00
5 %	3	6	4.27	5.30	4.16	5.00	3.00
10 %	3	6	4.50	5.62	4.74	5.00	3.00
20 %	3	6	5.56	5.85	5.62	6.00	3.00
50 %	3	6	5.76	5.87	5.53	6.00	3.00

The noise level is indicated as the percentage of the signal standard deviation.

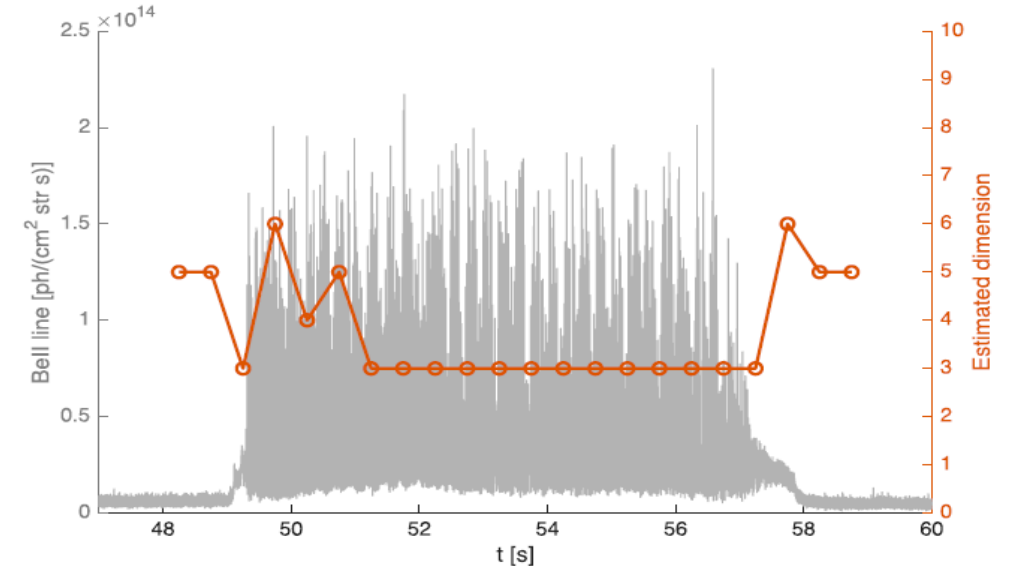


Examples of tokamak instabilities. ELMs

Gray: the time evolution of the Bell signal for JET discharge #96534.



Left. In the various colours the fractal dimension estimated by Krakovska-Chvostekova (KC) for various values of the embedding. At both ends of the interval the signal is stochastic and therefore the dimension is high. However in the deterministic chaotic central region, the technique provides values strongly dependent on the level of embedding and often completely unrealistic.



Right: the EDCO calculated with the autoencoder. At both ends of the interval the signal is stochastic and the EDCO is very high. In the central part of the discharge, characterised by the strong quasi periodic oscillations, the evolution is quasi deterministic and the dynamics is consistently estimated to have an effective dimension of 3.



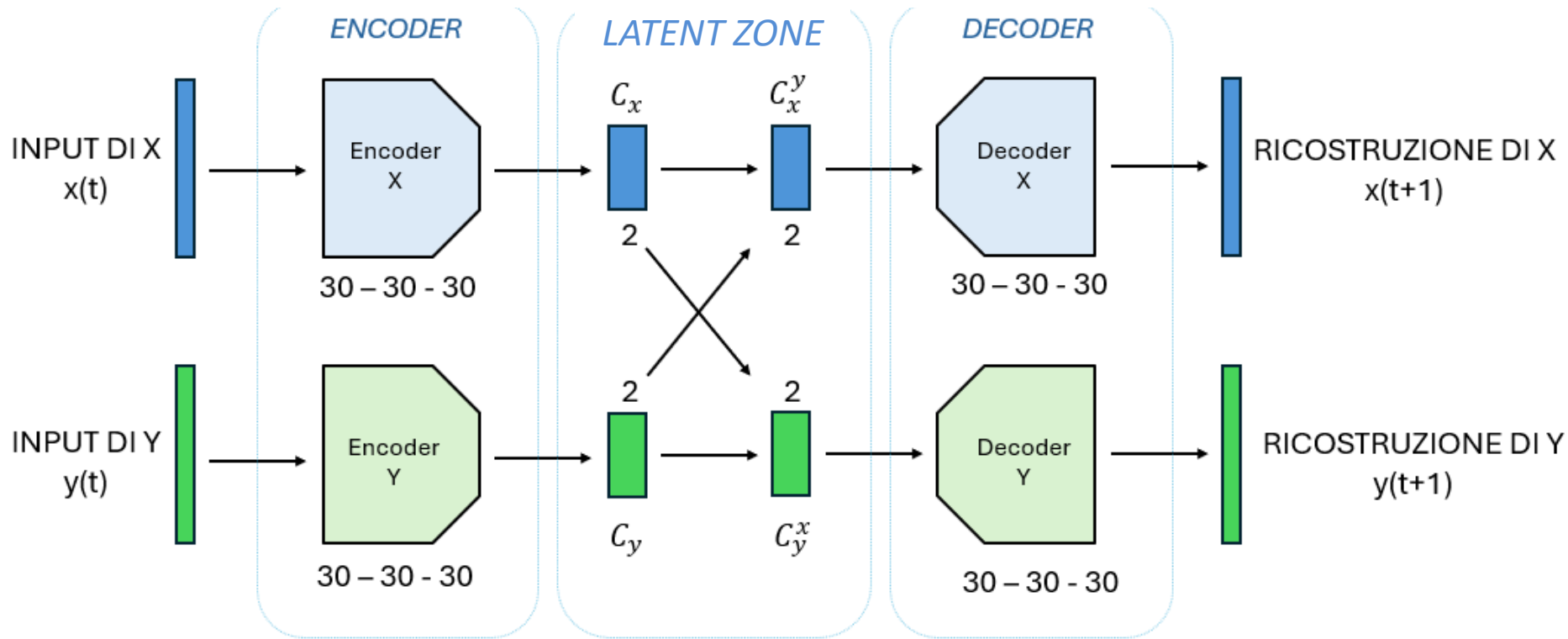
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Causality detection: physics based autoencoders

A Granger causality approach is adopted: a variable is considered to influence another if it helps predicting its future. The bivariate case is exemplified in the following: it has to be determined whether the time series x causes y or vice versa.



Physics based autoencoders have been developed: they are trained to reproduce the inputs with and without the candidate causal variable.

The quality of the two predictions are assessed with test statistics.

$$C_y^x = C_y + W_{yx} \cdot C_x \quad C_x^y = C_x + W_{xy} \cdot C_y$$

$$x \rightarrow y = \frac{1}{N} \sum_{i=1}^N (\log_{10}(\text{err}_X^{\text{alone}}) - \log_{10}(\text{err}_X^{\text{with } Y}))$$

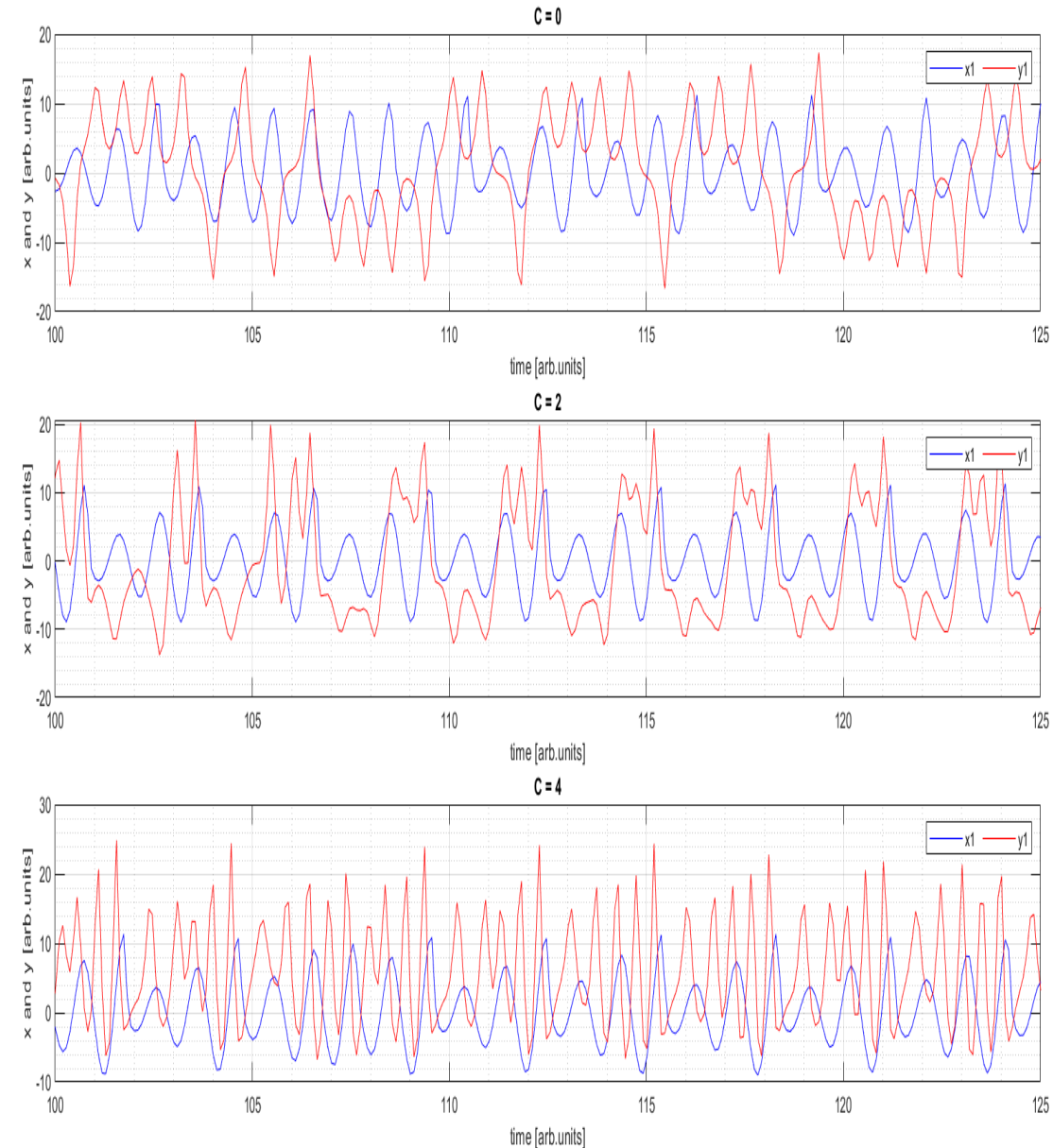
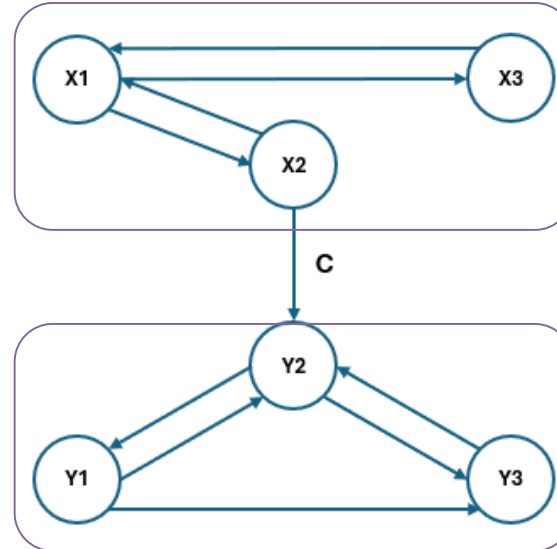


Rössler-Lorenz

The Rossler system influences the Lorenz one (both in the chaotic regime) through the parameter C .

$$\begin{aligned}\dot{x}_1 &= -6(x_2 + x_3) \\ \dot{x}_2 &= 6(x_1 + 0.2x_2) \\ \dot{x}_3 &= 6[0.2 + x_3(x_1 - 5.7)]\end{aligned}$$

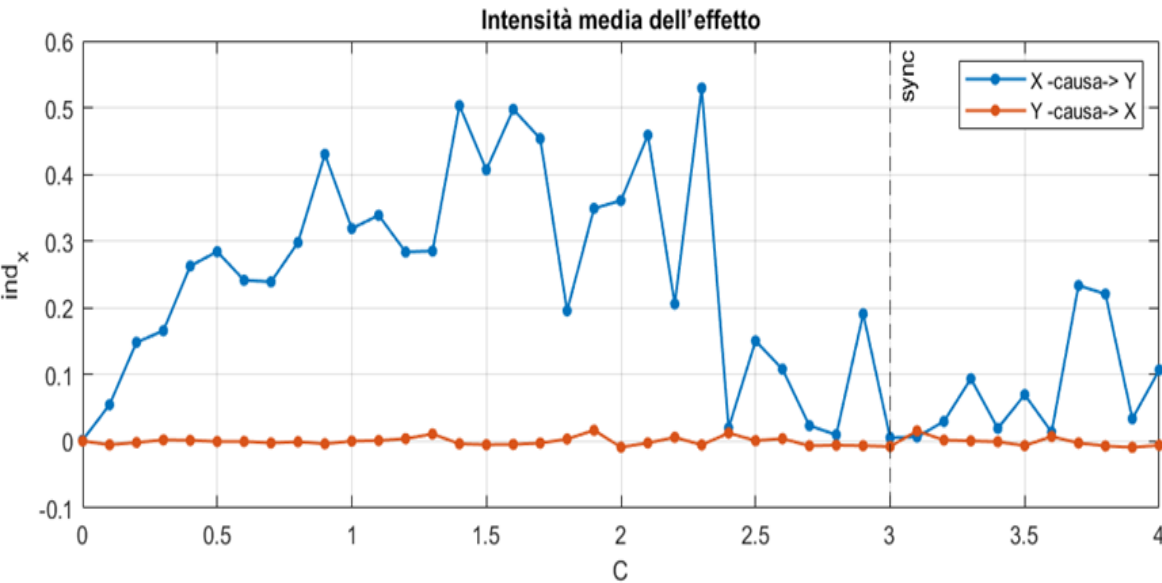
$$\begin{aligned}\dot{y}_1 &= 10(y_2 - y_1) \\ \dot{y}_2 &= 28y_1 - y_2 - y_1y_3 + Cx_2^2 \\ \dot{y}_3 &= y_1y_2 - \frac{8}{3}y_3\end{aligned}$$



The time traces show how difficult it is to determine the causal relationship between the systems.

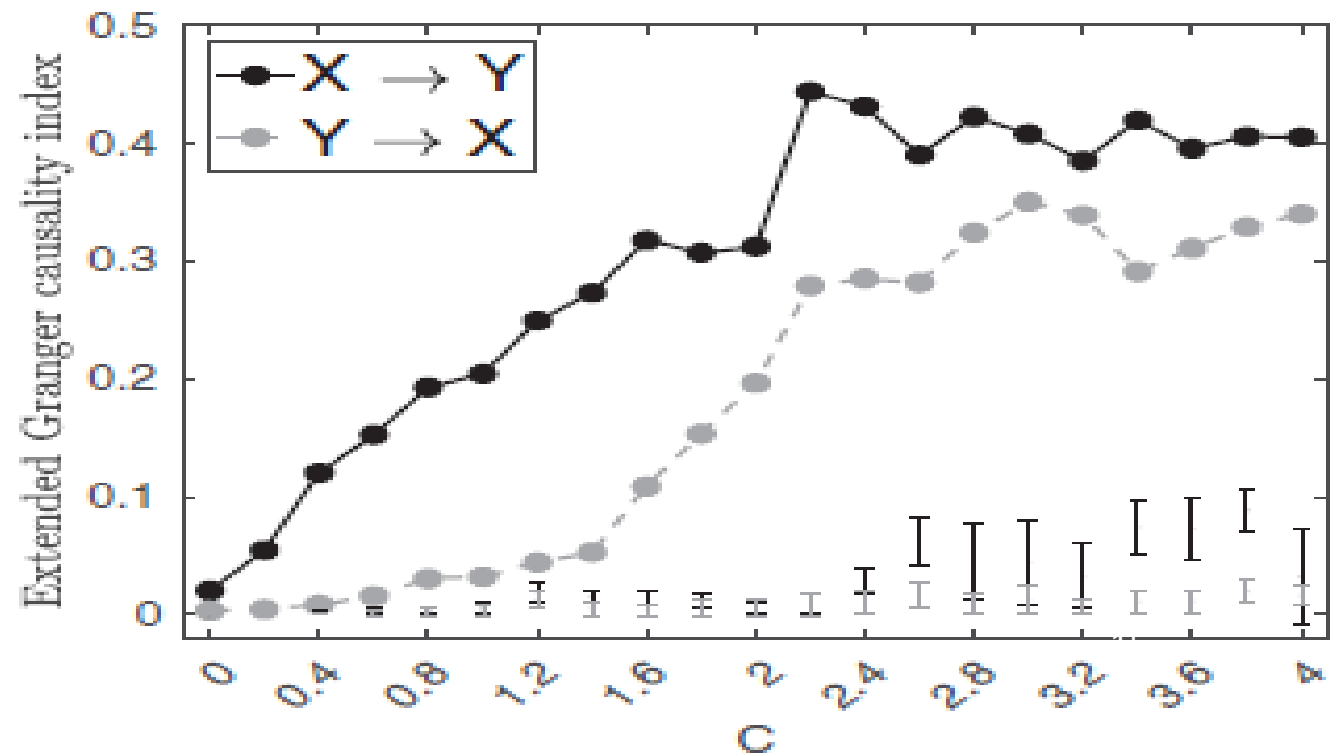


Rössler-Lorenz: results



The autoencoder manages to identify very well the direction of the causal influence. At C around 3 there is full synchronization and the concept of empirical causality is not applicable.

Even the quite sophisticated approach of Extended Granger Causality fares much more poorly: many false positives.



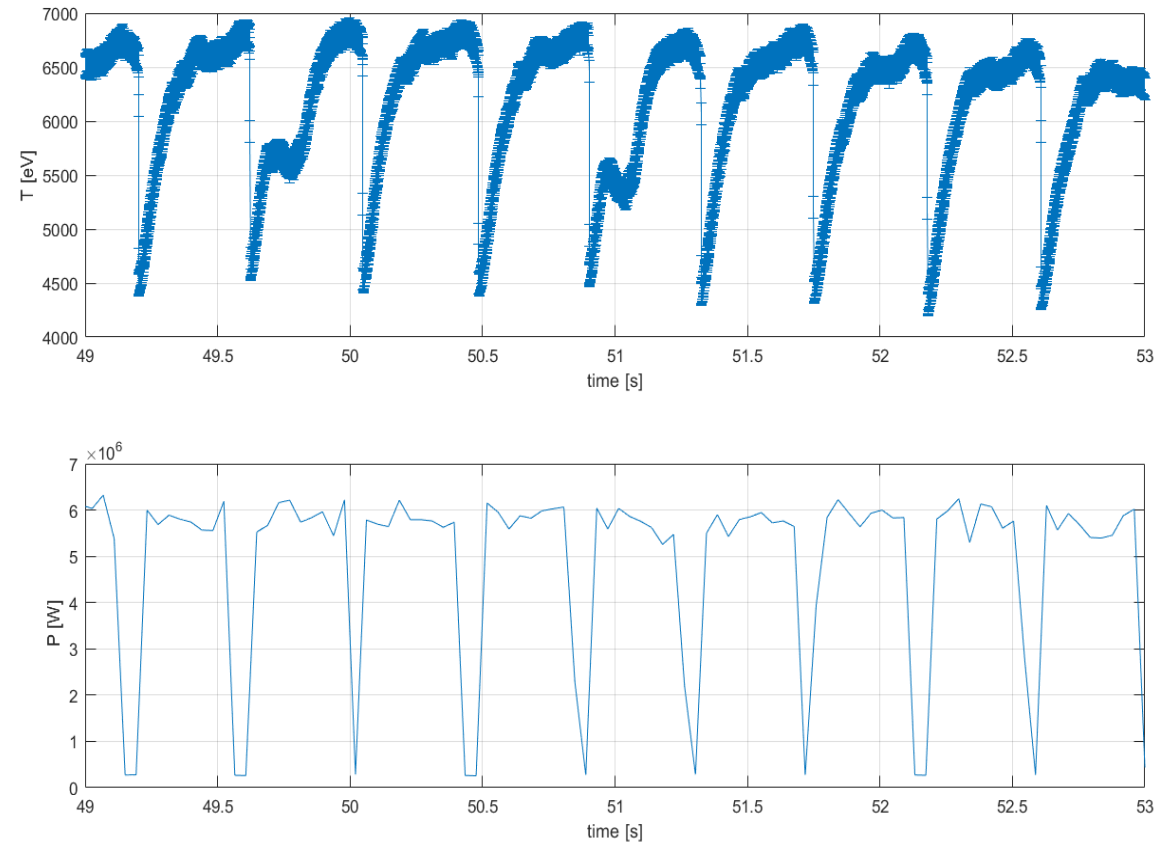


Sawteeth pacing with ICRH (1)

Sawteeth are core instabilities, whose main signature are quasi periodic oscillations in the central temperature as exemplified.

Their crash is triggered when the current density in the plasma centre becomes excessive.

On JET it was tried to pace them by modulating on additional heating power the ICRH, which can drive fast particles that have a stabilising effect on the instability.



Notches in the ICRH power reduces the amount of fast particles destabilising the instability and causing a crash.

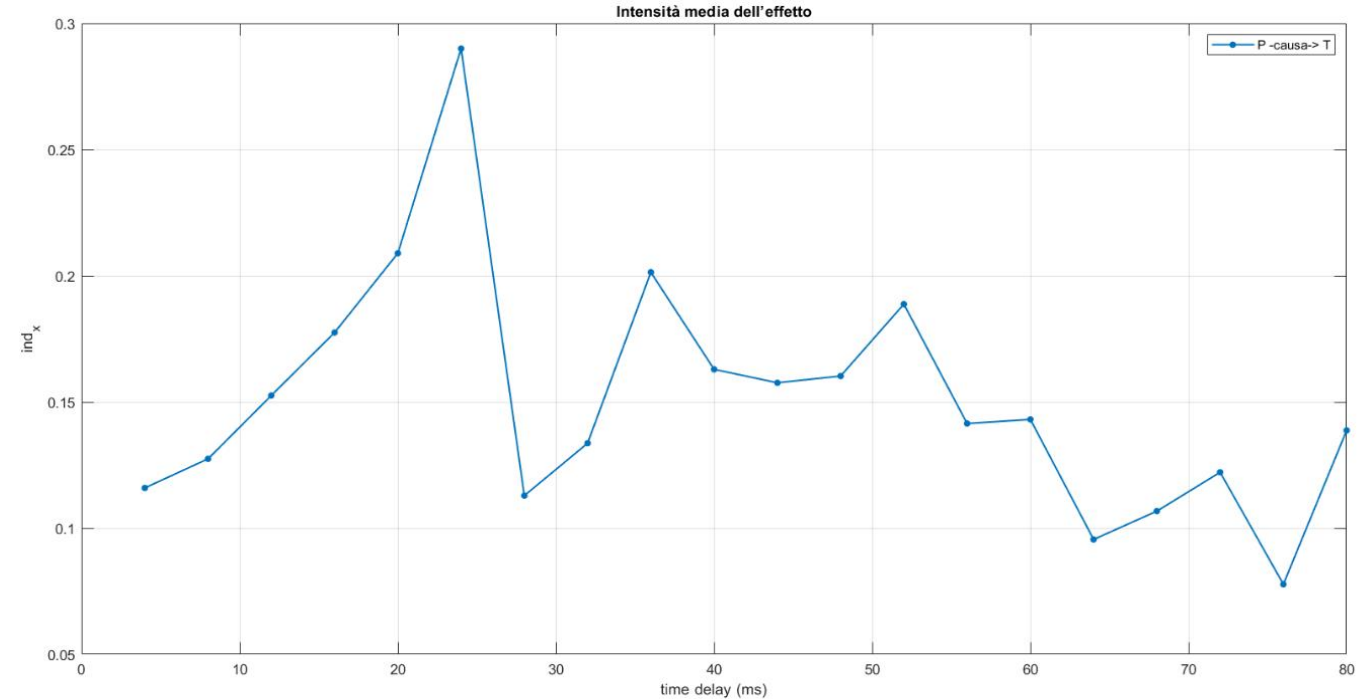


Sawteeth pacing with ICRH (2)

Sawteeth are quasi periodic instabilities and therefore even without a notch the crash would occur anyway.

So it is difficult to determine the efficiency of the pacing scheme.

The autoencoders for causality detection allow determining whether the main mechanism assumed for the pacing scheme is coherent with the data.



The causality indicator derived from the bivariate autoencoders. There is a clear peak in the causal effect on the sawteeth for a delay of about 25 ms after the switching off of the additional power, confirming the physical mechanism at the basis of the pacing technique, the slowing down time of the fast particles.



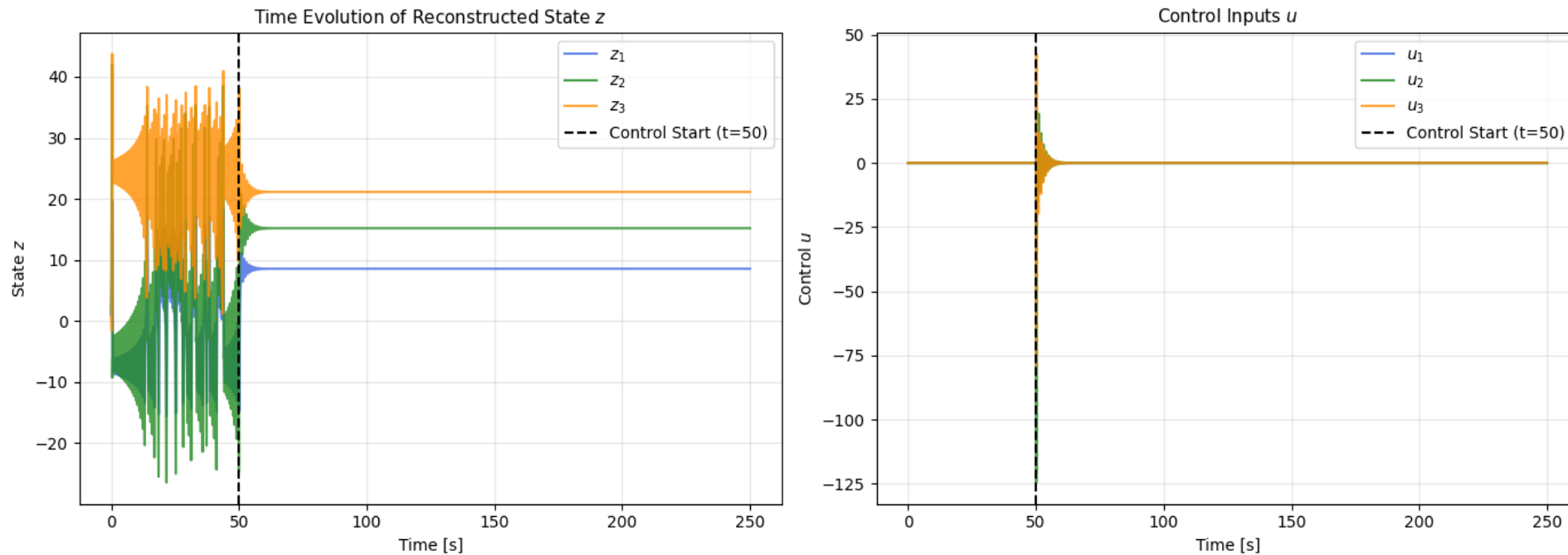
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Example of controlling a system in the chaotic regime

Once properly trained these autoencoders give the opportunity of designing data-driven controllers in a space of reduced dimensionality. Namely the code in the latent space is a model that can be utilised for feedback control of the original system (and not a simple number as ID or FD). Indeed the code provides a good observer, which absorbs the nonlinearities of the system to be controlled. Then traditional linear control theory (PID) can be applied.



Left example for a Lorenz system the chaotic regime with 10% of added random noise.



Conclusions

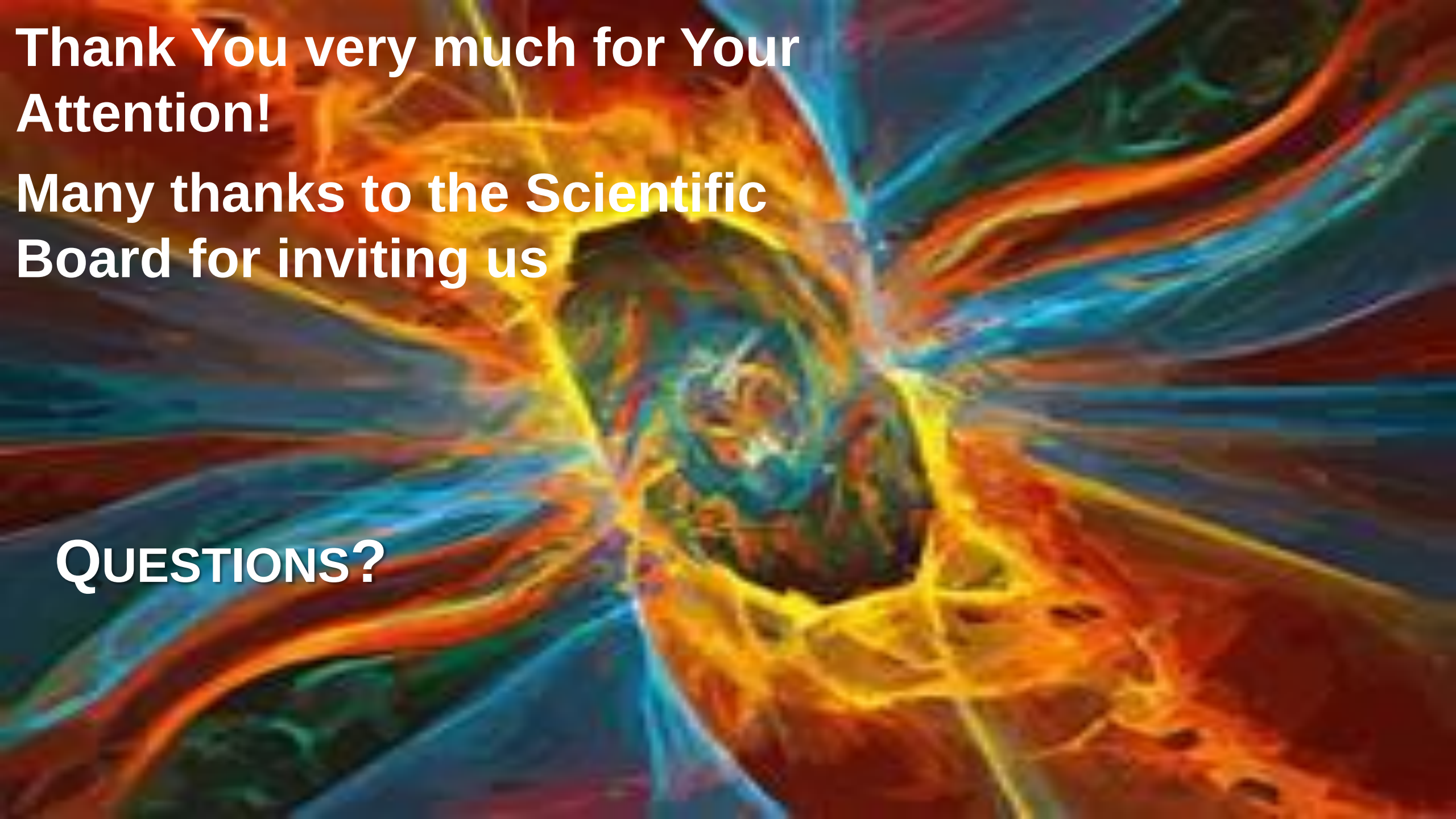
For control of dynamical systems the alternative dimension called the Effective Dimension for COntrol (EDCO) is more appropriate.

Overall when applied to time series for dimensionality and causality studies, deep autoencoders have proved to outperform alternative techniques.

They indeed tend to be more accurate in high dimensions, less affected by noise and highly flexible. They are also very straightforward to implement.

The autoencoders can also be trained to implement various forms of feedback control. Indeed they can be trained to predict the future evolution of the signals and therefore help in the design of data driven controllers by absorbing their non-linearities.

Another possible future promising line of research would be the development of physics based deep autoencoders, to design better interpretable reduced controllers so important in fields such as thermonuclear fusion.



**Thank You very much for Your
Attention!**

**Many thanks to the Scientific
Board for inviting us**

QUESTIONS?



Backup slides





Autoencoders for control

Once properly trained these autoencoders give the opportunity of designing data-driven controllers in a space of reduced dimensionality.

Namely the code in the latent space can be utilised for feedback control of the original system.

In particular it can be designed to provide a good observer which absorbs the non-linearities of the system to be controlled. Then traditional linear control theory can be applied.

