
PROPRIETÀ CINETICHE DEL PLASMA GRAVITAZIONALE E TURBOLENZA IN PLASMI DI LABORATORIO

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Timeline

- 2017 – Laurea Magistrale in Fisica presso Sapienza Università di Roma
Tesi: Onde gravitazionali in teoria macroscopica della gravità
- 2021 – Dottorato di Ricerca in Fisica presso Sapienza Università di Roma
Tesi: Morfologia delle onde gravitazionali in teoria macroscopica e teorie alternative di gravità
- 2022/2023 – Borsa di ricerca Angelo della Riccia per collaborazione con Università di Valencia
- 2024 - Assegno di Ricerca presso Enea Frascati

Outline

Passato

- Landau damping
- Sistema BP

Presente e futuro

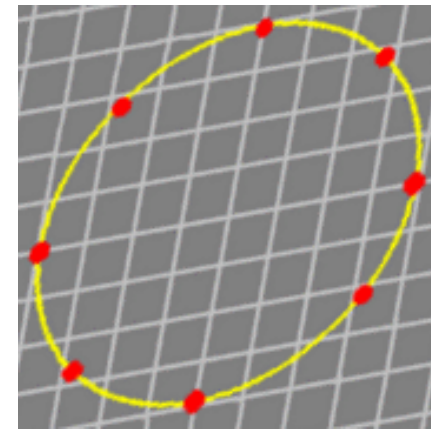
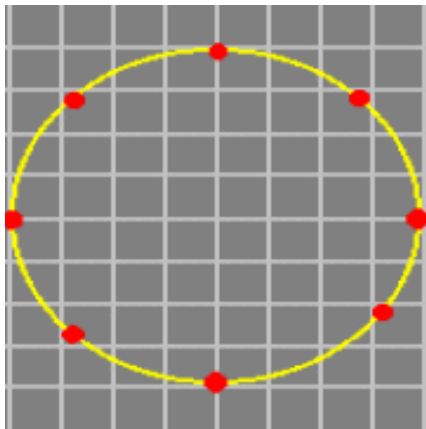
- Turbolenza al bordo di una macchina Tokamak: modello ridotto
- Evoluzione di tracers e trasporto

Parte I

GW in relatività generale

- Teoria della relatività generale: onde trasverse, velocità della luce
- Due polarizzazioni: plus e cross

$$\square \bar{h}_{ij} = -2\kappa T_{ij}$$



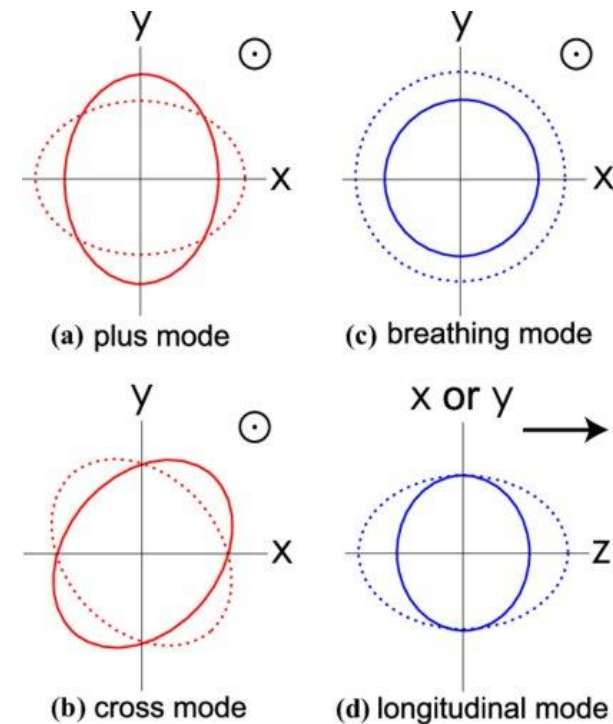
Moretti, Bombacigno, Montani, Gravitational Landau damping for massive scalar modes. Eur. Phys. J. C 80, 1203 (2020)

Gravità modificata

- Teorie di Horndeski: onde modificate
- Polarizzazione aggiuntiva: mix di breathing e longitudinale

$$\square \bar{h}_{ij} = -2\kappa' T_{ij}$$

$$(\square - m^2) \phi = \kappa'' T$$



Teoria cinetica

- Equazione di Vlasov

$$\frac{\partial f}{\partial t} + \frac{dx^m}{dt} \frac{\partial f}{\partial x^m} + \frac{dp_m}{dt} \frac{\partial f}{\partial p_m} = 0$$

- Forzante

$$\frac{du^\alpha}{d\tau} + \Gamma_{\mu\nu}^\alpha u^\mu u^\nu = 0 \quad \longrightarrow \quad \frac{dp_m}{dt} = \frac{1}{2p^0} \left(p_i p_j \frac{\partial \bar{h}_{ij}}{\partial x^m} + \alpha \mu^2 \frac{\partial \phi}{\partial x^m} \right)$$

- Equilibrio + perturbazione

$$f(\vec{x}, \vec{p}, t) = f_0(p) + \delta f(\vec{x}, \vec{p}, t)$$

$$\frac{\partial \delta f}{\partial t} + \frac{p^m}{p^0} \frac{\partial \delta f}{\partial x^m} - \frac{f'_0(p)}{2p} \left(p_i p_j \frac{\partial \bar{h}_{ij}}{\partial t} - \alpha p^2 \frac{\partial \phi}{\partial t} - \alpha p^0 p^m \frac{\partial \phi}{\partial x^m} \right) = 0$$

Sorgenti

$$T = -\mu^2 \int d^3p \frac{\delta f(\vec{x}, \vec{p}, t)}{p^0}$$
$$T_{ij} = \int d^3p \frac{p_i p_j}{p^0} \delta f(\vec{x}, \vec{p}, t),$$

Sistema autonomo

$$\square \bar{h}_{ij} = -2\kappa' T_{ij}$$

$$(\square - m^2) \phi = \kappa'' T$$

$$\bar{h}_{ij}^{(k,s)} = \frac{\left(s - \frac{\pi\kappa'}{2} \int d\rho dp_3 \rho^5 \frac{f'_0(p)}{p(p^0 s + ikp_3)} \right) \bar{h}_{ij}^{(k)}(0)}{(s^2 + k^2) \epsilon^{(h)}(k, s)}$$
$$\phi^{(k,s)} = \frac{\left(s + \alpha\pi\mu^2\kappa'' \int d\rho dp_3 \rho \frac{f'_0(p)p}{p^0 s + ikp_3} \right) \phi^{(k)}(0)}{(s^2 + k^2 + m^2) \epsilon^{(\phi)}(k, s)}$$

Funzioni dielettriche

$$\epsilon^{(\phi)}(k, s) = 1 + \frac{\alpha\pi\mu^2\kappa'}{s^2 + k^2 + m^2} \int d\rho dp_3 \rho \frac{f'_0(p)}{p} \frac{p^2 s + i k p^0 p_3}{p^0 s + i k p_3}$$
$$\epsilon^{(h)}(k, s) = 1 - \frac{\pi\kappa''}{2(s^2 + k^2)} \int d\rho dp_3 \rho^5 \frac{f'_0(p)}{p} \frac{s}{p^0 s + i k p_3}$$

$$\epsilon_r^{(\phi, h)}(k, \omega_r) = 0$$

Relazione di dispersione

$$\omega_i = - \left. \frac{\epsilon_i^{(\phi, h)}(k, \omega)}{\frac{\partial \epsilon_r^{(\phi, h)}(k, \omega)}{\partial \omega}} \right|_{\omega=\omega_r}$$

Coefficiente di smorzamento

Poli delle funzioni dielettriche

$$f_0(p) = \frac{n}{4\pi\mu^2\Theta K_2(\zeta)} e^{-\frac{\sqrt{\mu^2+p^2}}{\Theta}} \quad \zeta = \frac{\mu}{\Theta}$$

$$\epsilon^{(\phi)}(k, \omega_r) = 1 - \frac{\alpha\kappa'n}{2(k^2 + m^2 - \omega_r^2)\Theta^2 K_2(\zeta)} \int_0^{+\infty} d\rho \int_0^{+\infty} dp_3 \frac{\rho \left(p_3^2 - \frac{v_p^2}{1-v_p^2} \rho^2 \right)}{p_3^2 - \frac{v_p^2}{1-v_p^2} (\mu^2 + \rho^2)} e^{-\frac{\sqrt{\mu^2+\rho^2+p_3^2}}{\Theta}}$$
$$\epsilon^{(h)}(k, \omega_r) = 1 - \frac{\kappa''n}{4(k^2 - \omega_r^2)\mu^2\Theta^2 K_2(\zeta)} \frac{v_p^2}{1-v_p^2} \int_0^{+\infty} d\rho \int_0^{+\infty} dp_3 \frac{\rho^5}{p_3^2 - \frac{v_p^2}{1-v_p^2} (\mu^2 + \rho^2)} e^{-\frac{\sqrt{\mu^2+\rho^2+p_3^2}}{\Theta}}$$

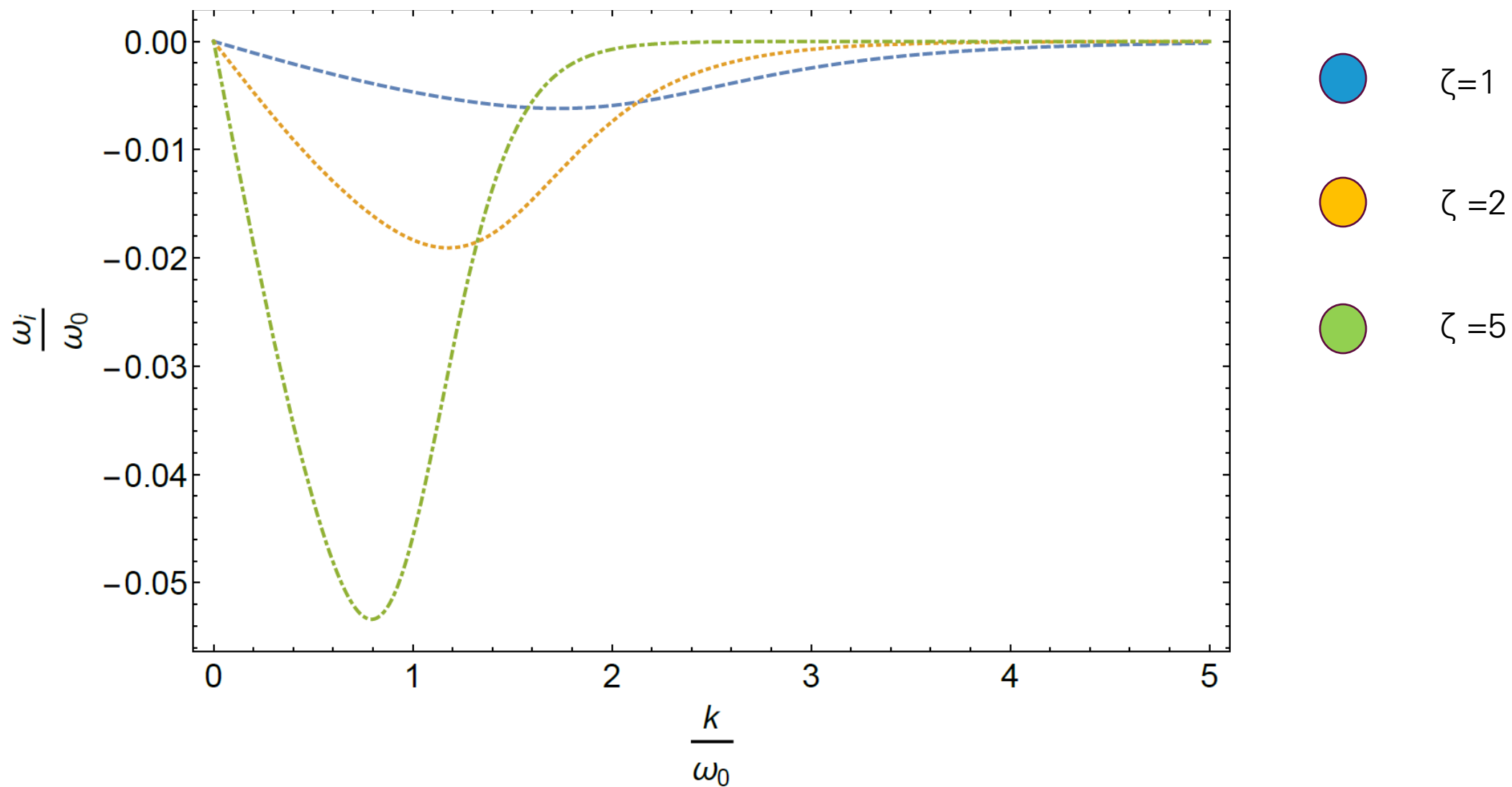
$$v_p < 1$$

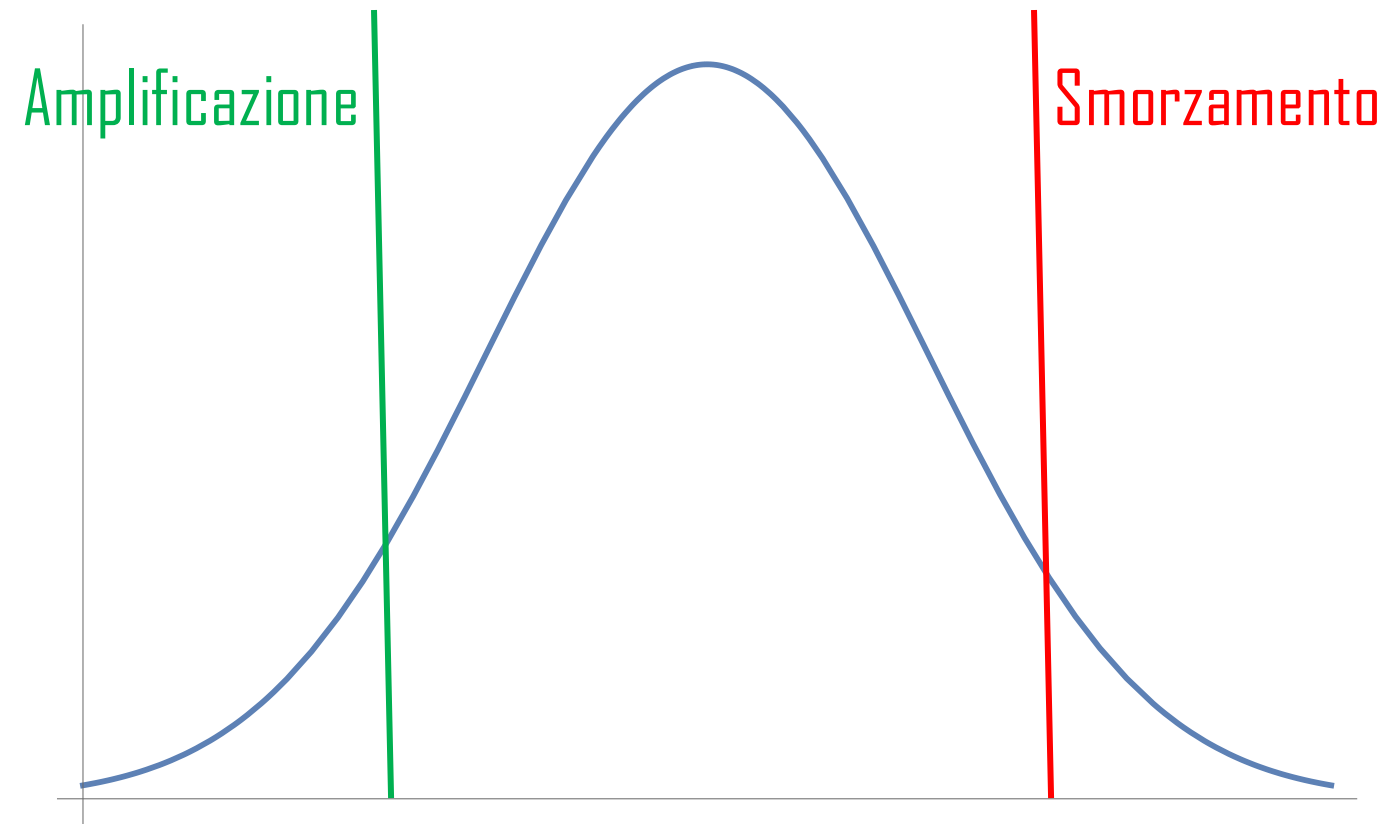
Relazioni di dispersione

$$\bar{h}_{\mu\nu} \quad \rightarrow \quad \omega_r^2 = \frac{k^2 + 2\omega_P^2 \frac{x - \gamma(\zeta)}{x^2} + \sqrt{\left(k^2 + 2\omega_P^2 \frac{x - \gamma(\zeta)}{x^2}\right)^2 + 8\omega_P^2 \frac{\gamma(\zeta)}{x^2}}}{2} \quad v_p > 1 \quad \forall k$$

$$\phi \quad \rightarrow \quad \omega_r^2 = \frac{k^2 + m^2 - \frac{3}{2}\gamma\omega_P^2 + \sqrt{\left(k^2 + m^2 - \frac{3}{2}\gamma\omega_P^2\right)^2 + 2\gamma\omega_P^2 k^2}}{2}$$

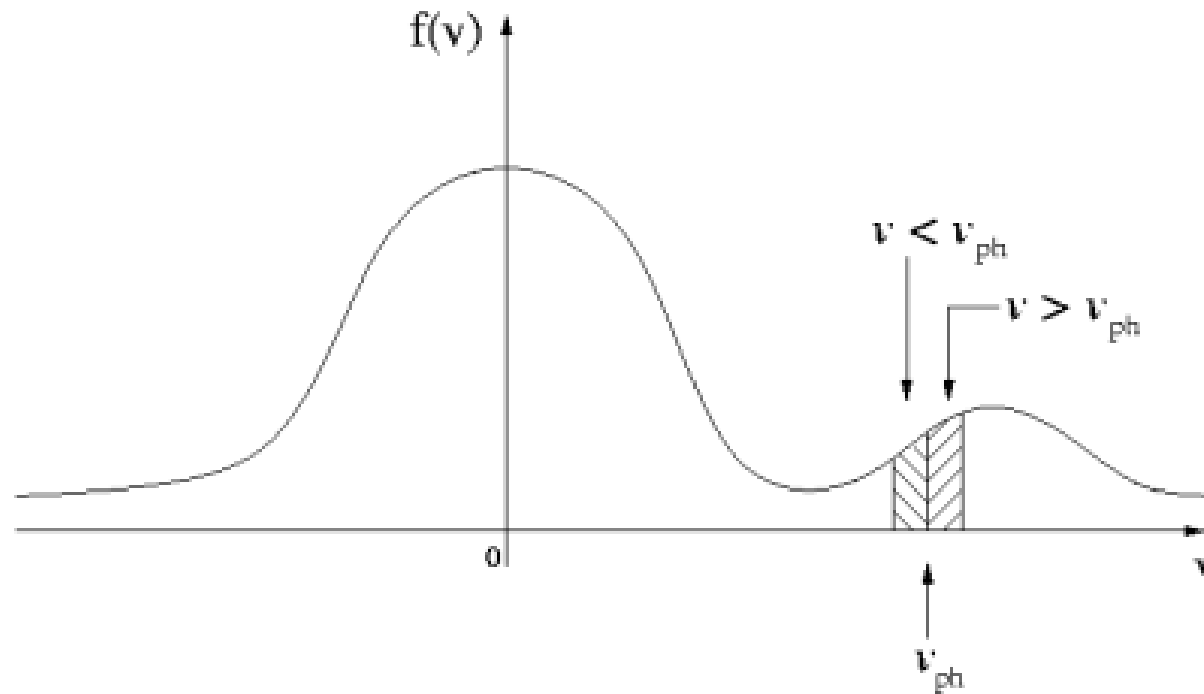
$$v_p < 1 \quad \Longleftrightarrow \quad m^2 < \gamma\omega_P^2 \quad \omega_P^2 = \kappa n \mu = \kappa \rho$$





Sistema beam-plasma

$$f_0^{TOT}(p_1, p_2, p_3) = f_0^P(p) + f_0^B(p_1, p_2, p_3)$$



Moretti, Del Prete, Montani, Linear analysis of the gravitational beam-plasma instability. Eur. Phys. J. C 83, 486 (2023)

Delta beam

$$f_0^B(p_1, p_2, p_3) = n_B \delta(p_1) \delta(p_2) \delta(p_3 - p_B)$$

- Funzione dielettrica totale

$$\epsilon(k, \omega) = 1 - \frac{3\omega_P^2}{4\pi(k^2 + m^2 - \omega^2)\mu\Theta^2 K_2\left(\frac{\mu}{\Theta}\right)} \int d^3p \frac{p_3^2 - \frac{\omega^2}{k^2 - \omega^2}(p_1^2 + p_2^2)}{p_3^2 - \frac{\omega^2}{k^2 - \omega^2}(\mu^2 + p_1^2 + p_2^2)} e^{-\frac{\sqrt{p^2 + \mu^2}}{\Theta}} +$$

$$+ \frac{3\omega_P^2}{k^2 + m^2 - \omega^2} \frac{n_B}{n_P} \frac{(1 - v_B^2)^{\frac{3}{2}} (1 + v_B^2 - 2\frac{\omega}{k}v_B)}{(v_B - \frac{\omega}{k})^2}$$

$$\epsilon^P(k, \omega) = 1 + 3\gamma(x)\omega_P^2 \frac{k^2 - 3\omega^2}{\omega^2(k^2 + m^2 - \omega^2)}$$

$$v_B = \frac{p_B}{\sqrt{p_B^2 + \mu^2}}$$

$$\epsilon^{TOT}(k, \omega) = 0 \longrightarrow \frac{\epsilon^P(k, \omega) (\omega^2 - k^2 - m^2) \left(v_B - \frac{\omega}{k}\right)^2}{(1 + v_B^2 - 2\frac{\omega}{k}v_B) (1 - v_B^2)^{\frac{3}{2}}} = 3\omega_P^2 \frac{n_B}{n_P} \quad (\star)$$

- Beam tenue $\eta \equiv \frac{n_B}{n_P} \ll 1$

$$\begin{aligned} \epsilon^P = 0 &\longrightarrow \omega_r^2 = \frac{k^2 + m^2 - \frac{3}{2}\gamma\omega_P^2 + \sqrt{(k^2 + m^2 - \frac{3}{2}\gamma\omega_P^2)^2 + 2\gamma\omega_0^2 k^2}}{2} \\ \epsilon^B = 0 &\longrightarrow \omega_r = kv_B \end{aligned}$$

- Punti degeneri $\epsilon^P(k_0, v_B k_0) = 0$

$$k_0 = \sqrt{\frac{\frac{1}{2}\gamma\omega_P^2 (3v_B^2 - 1) - m^2 v_B^2}{v_B^2 (1 - v_B^2)}} \xrightarrow{\text{Realtà}} v_B \geq \frac{1}{\sqrt{3 - \frac{2m^2}{\gamma\omega_P^2}}} = v_B^{\min}.$$

$$m^2 < \gamma\omega_P^2 \longrightarrow v_B^{\min} \in \left(3^{-\frac{1}{2}}, 1\right)$$

- Espansione al terzo ordine intorno al punto degenere

$$\left(\frac{\delta\omega}{\omega_0} - \frac{\delta k}{k_0}\right)^2 \left(P \frac{\delta\omega}{\omega_0} + Q \frac{\delta k}{k_0}\right) = C$$

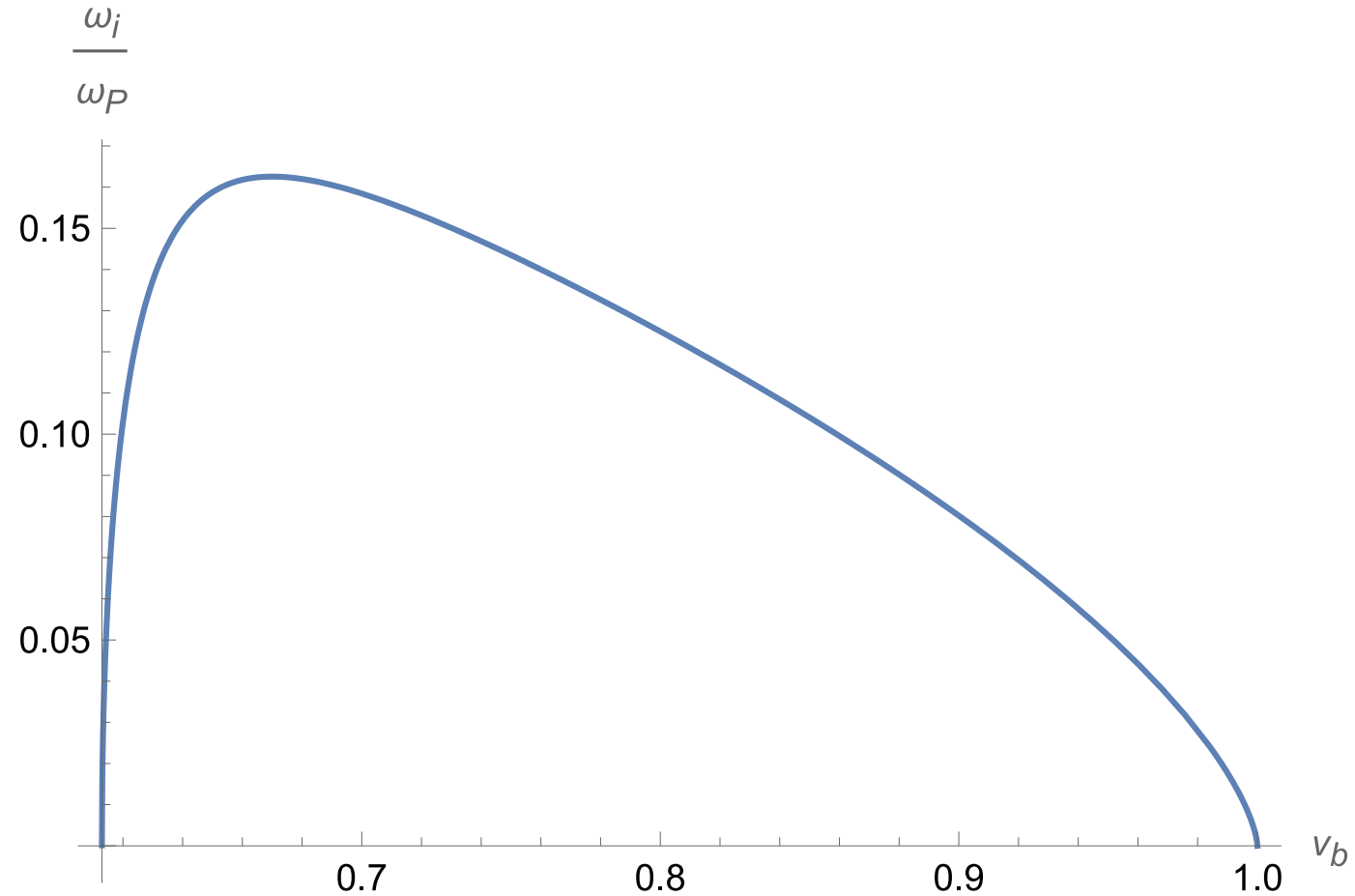
- Selezione della soluzione con parte immaginaria positiva: instabilità massima in k_0

$$\frac{\Im(\delta\omega^*(0))}{\omega_0} = \left(\frac{\sqrt{27}\eta (1 - v_B^2)^{\frac{7}{2}}}{16 \left(1 - 2v_B^2 + \left(\frac{v_B^2}{v_{\min}} \right)^2 \right)} \right)^{\frac{1}{3}}$$

Caso E.M.

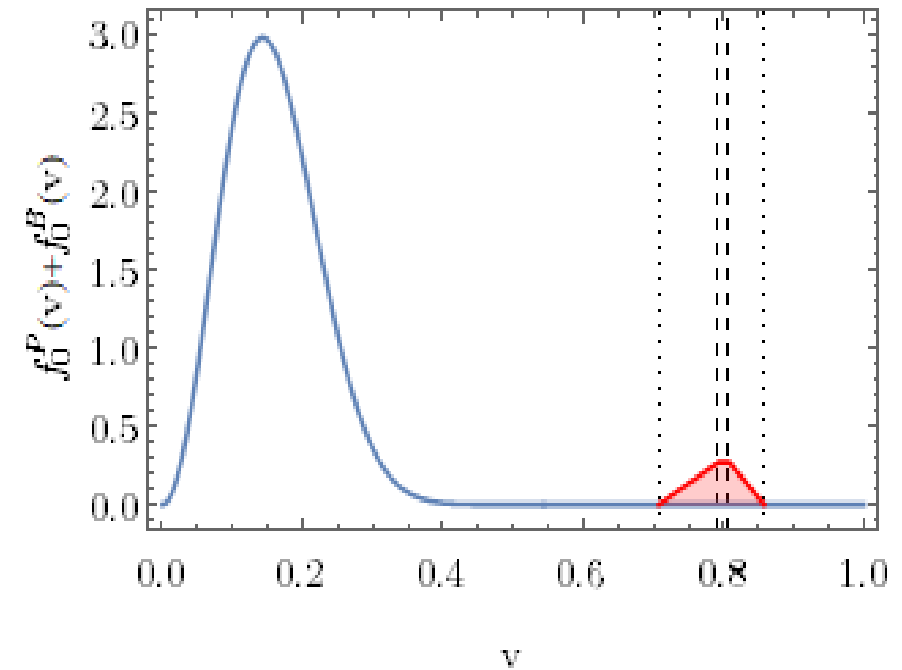
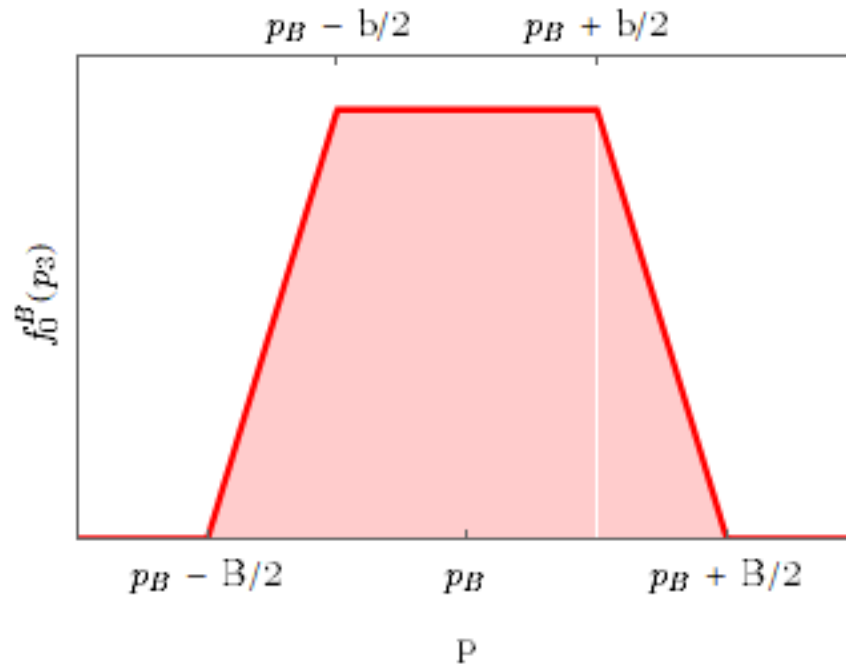
$$\frac{\Im(\delta\omega^*(0))}{\omega_0} = \left(\frac{\sqrt{27}\eta}{16} \right)^{\frac{1}{3}}$$

Andamento del fattore relativistico



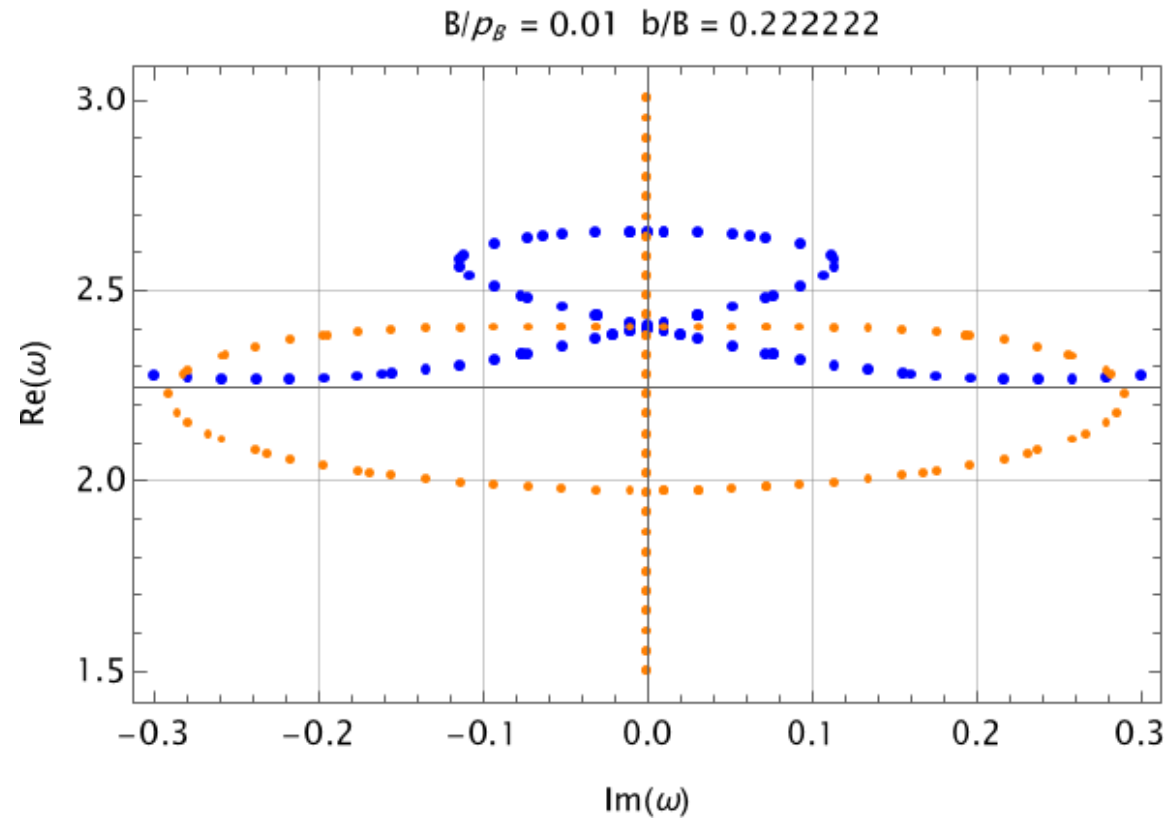
Analisi numerica

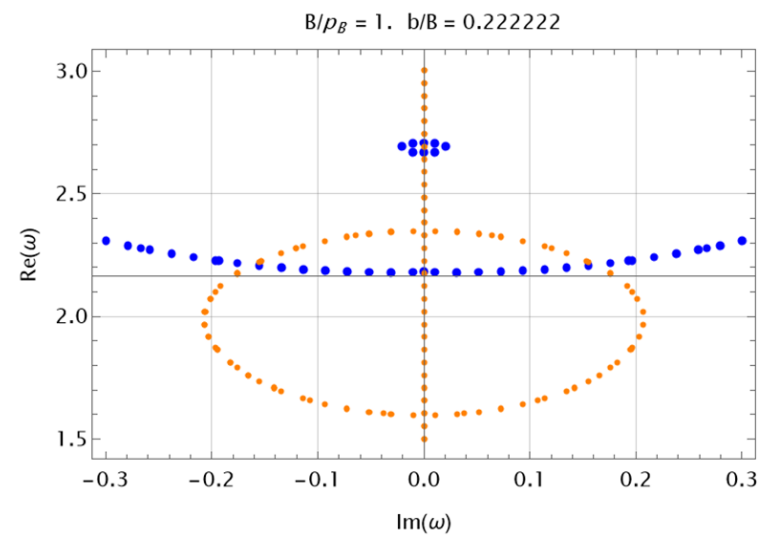
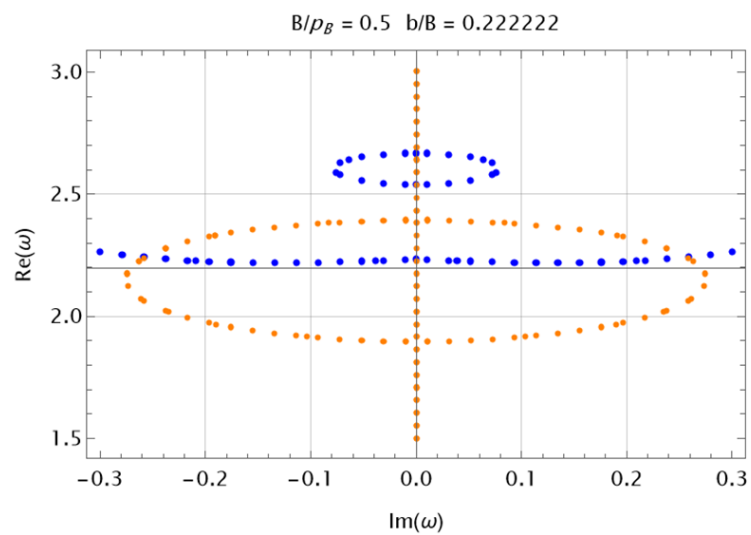
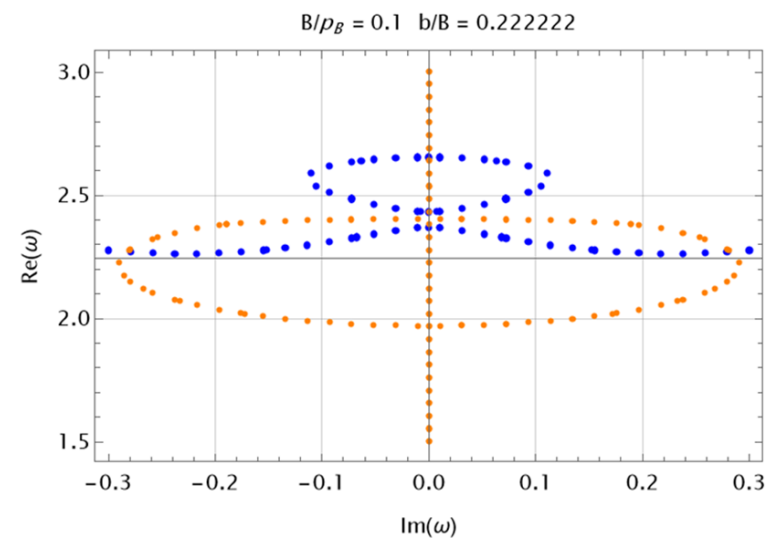
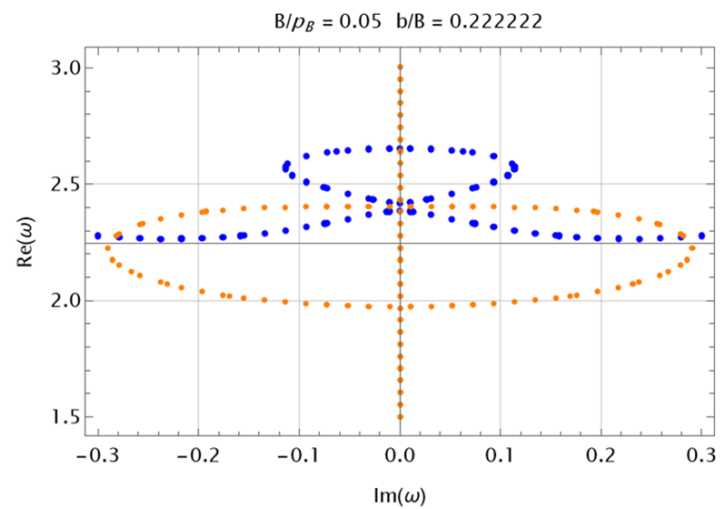
- Beam trapezoidale



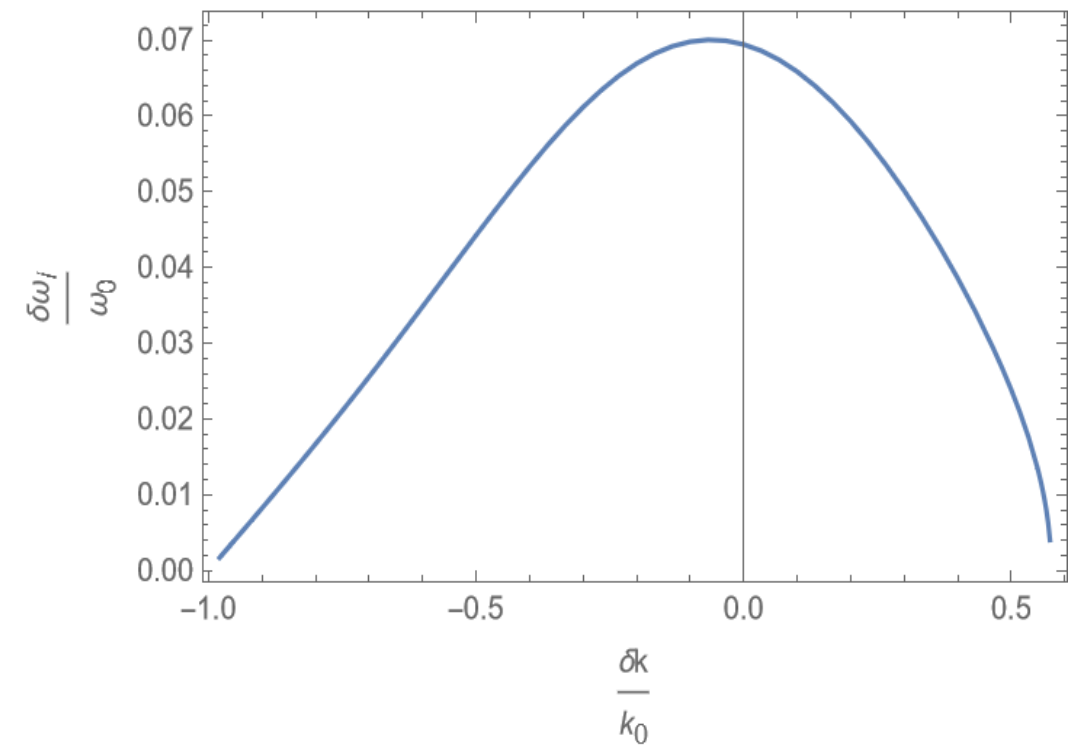
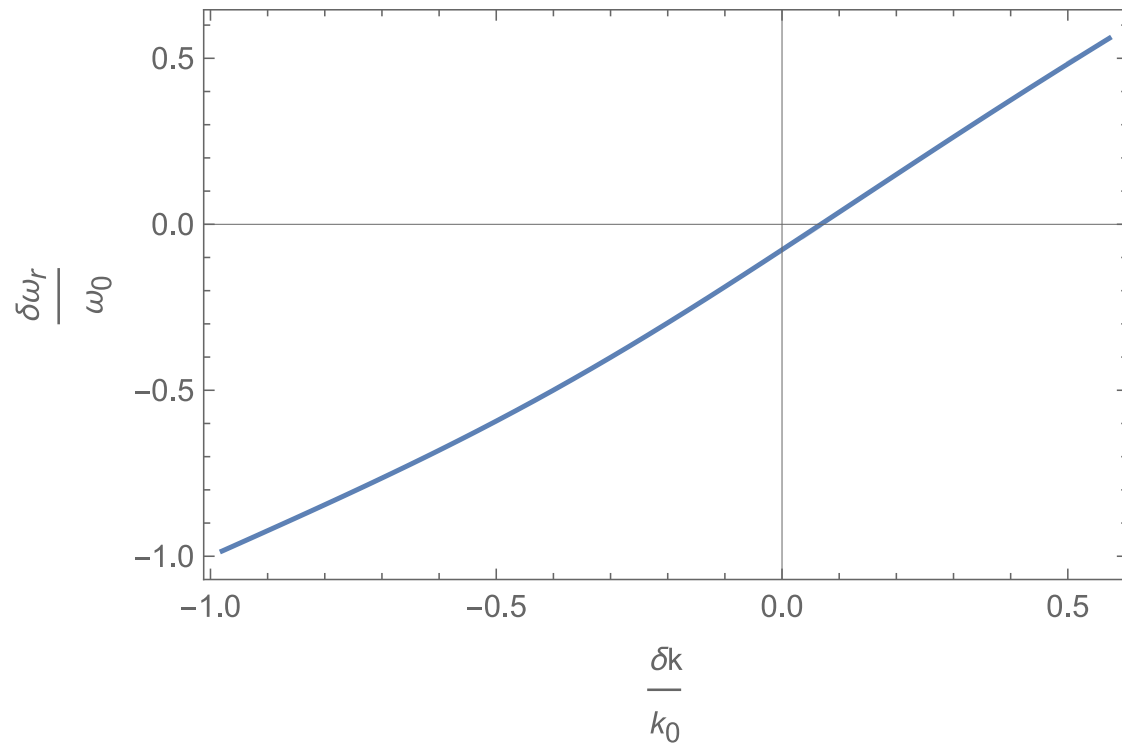
- Funzione dielettrica totale

$$\epsilon(k, \omega) = \epsilon^P(k, \omega) + \frac{3\omega_P^2}{k^2 + m^2 - \omega^2} \frac{4\eta\mu^2}{B^2 - b^2} \left(\int_{u_1}^{u_2} \frac{du}{\sqrt{1-u^2}} \frac{1}{u - \frac{\omega}{k}} - \int_{u_3}^{u_4} \frac{du}{\sqrt{1-u^2}} \frac{1}{u - \frac{\omega}{k}} \right)$$

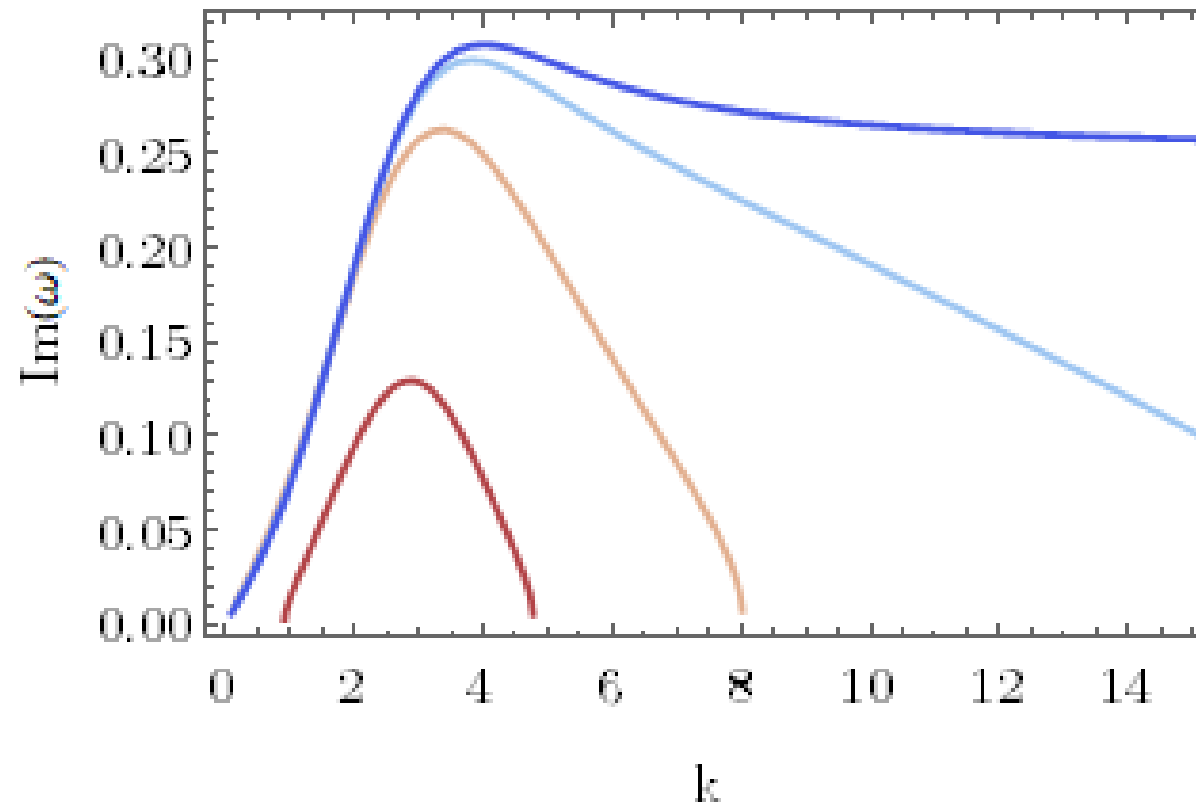




Costruzione curve



Dipendenza dalla temperatura



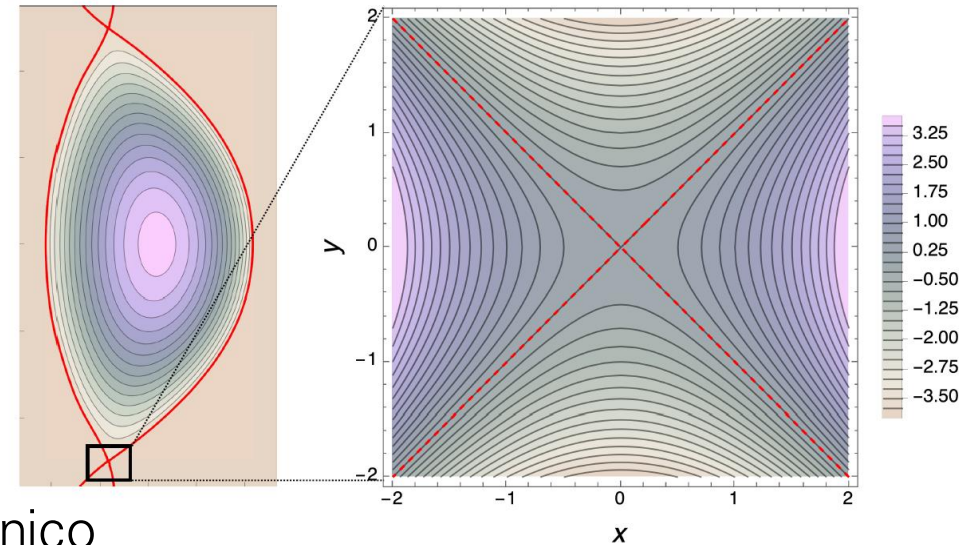
Parte II

Modello ridotto di turbolenza 3D

- Configurazione magnetica: punto a X

$$\mathbf{B}_0 = B\hat{\mathbf{b}} = B_p (y\hat{\mathbf{e}}_x + x\hat{\mathbf{e}}_y) + B_t\hat{\mathbf{e}}_z$$

- Trattazione a due fluidi
- Quasi-neutralità (scala turbolenza $\gg \lambda_D$)
- Corrente ortogonale causata dal drift di polarizzazione ionico
- No effetti diamagnetici
- Velocità parallela degli ioni trascurabile
- Quantità di background costanti



-
- Equazione continuità

$$\frac{d\delta\mathcal{N}}{dt} - \mathcal{D}\nabla_{\perp}^2\delta\mathcal{N} = \frac{1}{e}\nabla_{\parallel}\cdot\delta\mathbf{J}_{\parallel} \quad (\text{velocità avvettiva } \mathbf{v}_e = \frac{c}{B^2}\mathbf{E} \times \mathbf{B})$$

- Bilancio momento ionico

$$\frac{d\delta\mathbf{v}_{\perp}^i}{dt} = \frac{e}{m_i} \left(-\nabla_{\perp}\delta\phi + \delta\mathbf{v}_{\perp}^i \times \mathbf{B}/c \right) + \nu\nabla_{\perp}^2\delta\mathbf{v}_{\perp}^i$$

- Assumendo $\delta\mathbf{v}_{\perp}^i = \delta\mathbf{v}_{\perp}^e + \tilde{\mathbf{v}}_{\perp}^i$, si ricava al primo ordine

$$\tilde{\mathbf{v}}_{\perp}^i = \frac{cm_i}{Be} \left(\frac{d}{dt} - \nu\nabla_{\perp}^2 \right) (\hat{\mathbf{b}} \times \delta\mathbf{v}_{\perp}^e) = -\frac{c^2m_i}{Be} \left(\frac{d}{dt} - \nu\nabla_{\perp}^2 \right) \frac{1}{B} \nabla_{\perp}\delta\phi$$

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- Corrente ortogonale $\delta \mathbf{J}_\perp \equiv \mathcal{N}e(\delta \mathbf{v}_\perp^i - \delta \mathbf{v}_\perp^e) = \mathcal{N}e \tilde{\mathbf{v}}_\perp^i$. Conservazione carica $\nabla \cdot \delta \mathbf{J} = 0$ implica

$$\nabla_\perp \cdot \left(\frac{1}{B} \frac{d}{dt} \frac{1}{B} \nabla_\perp \delta \phi \right) - \nu \nabla_\perp \cdot \left(\frac{1}{B} \nabla_\perp^2 \frac{1}{B} \nabla_\perp \delta \phi \right) = \frac{1}{\mathcal{N}c^2 m_i} \nabla_\parallel \cdot \delta \mathbf{J}_\parallel$$

- Inoltre $\delta \mathbf{J}_\parallel = \frac{\sigma}{\mathcal{N}e} \nabla_\parallel \delta p - \sigma \nabla_\parallel \delta \phi = \frac{\sigma K_B T}{\mathcal{N}e} \nabla_\parallel \delta \mathcal{N} - \sigma \nabla_\parallel \delta \phi$

- Coordinate adimensionali

$$\tau \equiv \Omega_i t \quad u \equiv (2\pi/L_p)x \quad v \equiv (2\pi/L_p)y \quad w \equiv (2\pi/L_t)z \quad L_t \gg L_p$$

- Introducendo

$$\epsilon = \frac{B_p}{B_t} \left(\frac{L_p}{2\pi} \right) \quad \gamma(u, v) = \sqrt{1 + \epsilon^2(u^2 + v^2)} \quad B = \gamma B_t$$

- Modello

$$\frac{d}{d\tau} \bar{\mathcal{N}} - \bar{\mathcal{D}} D_{\perp}^2 \bar{\mathcal{N}} = \mathbf{D}_{\parallel} \cdot \mathbf{Y}_{\parallel}$$

$$\alpha_1 \mathbf{D}_{\perp} \cdot \left(\frac{1}{\gamma} \frac{d}{d\tau} \frac{1}{\gamma} \mathbf{D}_{\perp} \Phi \right) - \alpha_1 \alpha_2 \mathbf{D}_{\perp} \cdot \left(\frac{1}{\gamma} D_{\perp}^2 \frac{1}{\gamma} \mathbf{D}_{\perp} \Phi \right) = \mathbf{D}_{\parallel} \cdot \mathbf{Y}_{\parallel}$$

$$\alpha_3 (D_{\parallel}^2 \bar{\mathcal{N}} - D_{\parallel}^2 \Phi) = \mathbf{D}_{\parallel} \cdot \mathbf{Y}_{\parallel}$$

$$\Phi \equiv \frac{e\delta\phi}{K_B T} \quad \bar{\mathcal{N}} = \frac{\delta \mathcal{N}}{\mathcal{N}} \quad \mathbf{Y}_{\parallel} = \frac{2\pi}{L_p} \frac{\delta \mathbf{J}_{\parallel}}{\mathcal{N} e \Omega_i} \quad \alpha_1 = \rho_i^2 \left(\frac{2\pi}{L_p} \right)^2 \quad \alpha_2 = \frac{\nu}{\Omega_i} \left(\frac{2\pi}{L_p} \right)^2$$

$$\alpha_3 = \frac{v_A^2 \rho_i^2}{\eta_B \Omega_i} \left(\frac{2\pi}{L_p} \right)^2 \quad \bar{\mathcal{D}} = \frac{\mathcal{D}}{\Omega_i} \left(\frac{2\pi}{L_p} \right)^2$$

Simulazioni 3D con campo toroidale

- Assumendo $B_p = 0$ e $\mathcal{D} = \nu$: relazione costitutiva

$$\bar{\mathcal{N}} = \alpha_1 D_{\perp}^2 \Phi$$

- Equazione evolutiva per la vorticità

$$\partial_{\tau} D_{\perp}^2 \Phi + \alpha_1 (\partial_u \Phi \partial_v D_{\perp}^2 \Phi - \partial_v \Phi \partial_u D_{\perp}^2 \Phi) = \cancel{\alpha_2 D_{\perp}^4 \Phi} + \alpha_3 D_{\parallel}^2 D_{\perp}^2 \Phi - \frac{\alpha_3}{\alpha_1} D_{\parallel}^2 \Phi$$

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- Espansione in Fourier

$$\Phi(\tau, u, v, w) = \sum_{n,\ell,m} \varphi_{n,\ell,m}(\tau) e^{i(nw+\ell u+mv)}$$

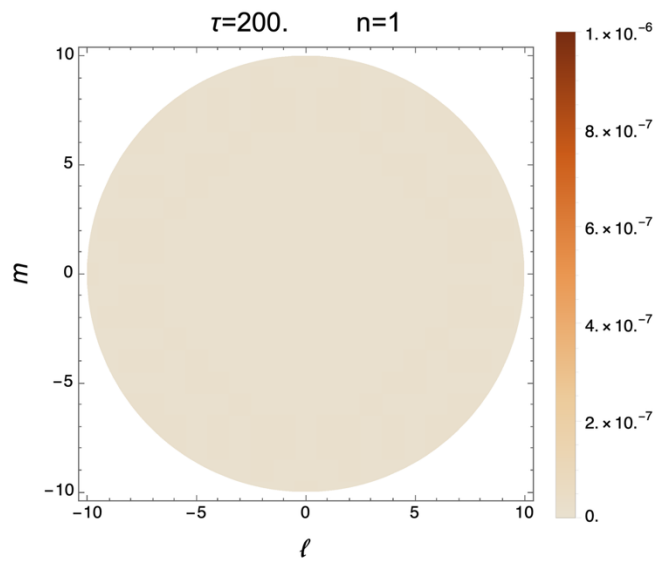
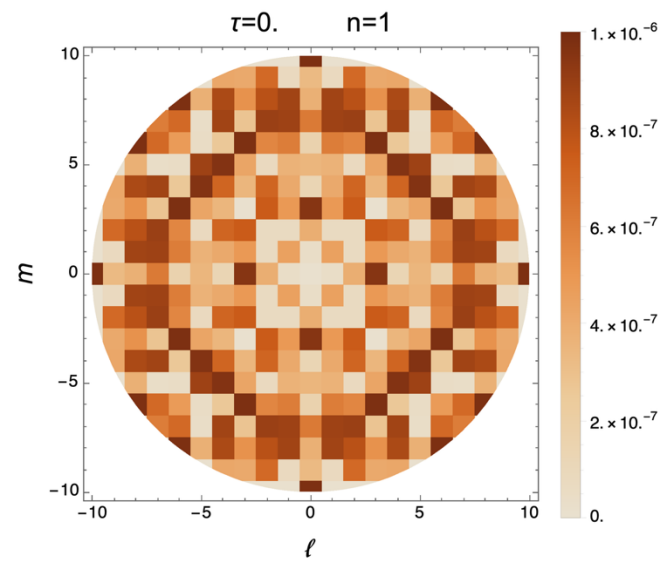
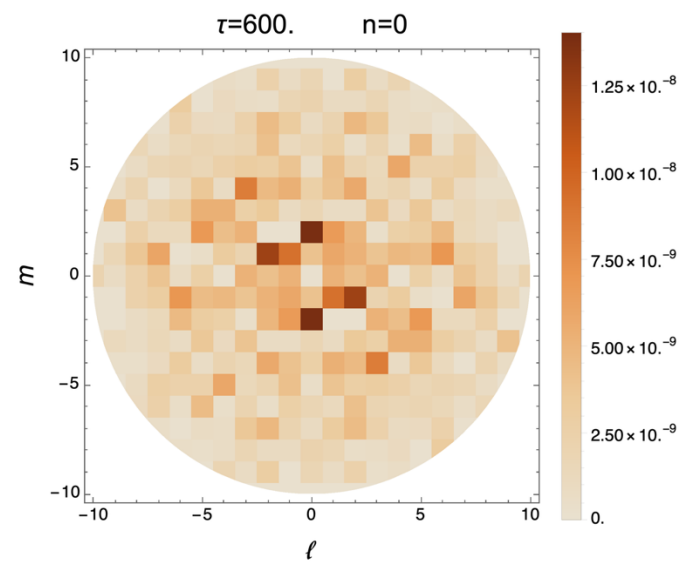
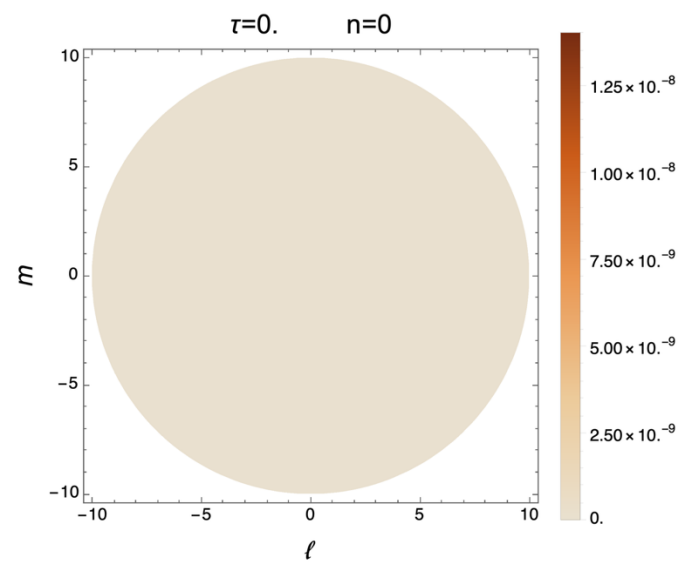
- Parametri di inizializzazione

$$T = 100 \text{ eV} \quad B_t = 3 \text{ T} \quad \mathcal{N} = 5 \times 10^{19} \text{ m}^{-3} \quad L_p = 1 \text{ cm} \quad L_t \simeq 1300 \text{ cm}$$

- Condizioni iniziali sui modi di Fourier

$$\varphi_{0,\ell,m}(0) = 0$$

$$\varphi_{n,\ell,m}(0) = \text{random noise} \\ (n \neq 0)$$



Ruolo del punto a X

- Assumendo per la dinamica 2D un profilo magnetico di background

$$\mathbf{B}_0 = B_0 \hat{\mathbf{b}} = B_p (y \hat{\mathbf{e}}_x + x \hat{\mathbf{e}}_y) + B_t \hat{\mathbf{e}}_z$$

- Direzione campo magnetico

$$\hat{\mathbf{b}} = \frac{1}{\sqrt{1 + \varepsilon^2 (u^2 + v^2)}} (\varepsilon v, \varepsilon u, 1) \simeq (\varepsilon v, \varepsilon u, 1) \quad \varepsilon = \frac{B_p L}{B_t} \ll 1$$

- Operatori differenziali

$$\mathbf{D}_{\parallel} = \varepsilon (v \partial_u + u \partial_v) \hat{\mathbf{b}}$$

$$\begin{aligned} D_{\parallel}^2 &\rightarrow \varepsilon^2 (v^2 \partial_u^2 + u^2 \partial_v^2 + 2uv \partial_u \partial_v + u \partial_u + v \partial_v) \\ D_{\perp}^2 &\rightarrow \partial_u^2 + \partial_v^2 - \varepsilon^2 (v^2 \partial_u^2 + u^2 \partial_v^2 + 2uv \partial_u \partial_v) \end{aligned}$$

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- Equazione di evoluzione linearizzata

$$\partial_\tau D_\perp^2 \Phi = \alpha_2 D_\perp^4 \Phi + \alpha_3 D_\parallel^2 D_\perp^2 \Phi - \frac{\alpha_3}{\alpha_1} D_\parallel^2 \Phi$$

- Approccio perturbativo

$$\Phi = \Phi^{(0)}(\tau, u, v) + \epsilon^2 \Phi^{(1)}(\tau, u, v)$$

- Equazioni ordine 0 e 1

$$\partial_\tau (\partial_u^2 + \partial_v^2) \Phi^{(0)} - \alpha_2 (\partial_u^2 + \partial_v^2)^2 \Phi^{(0)} = 0$$

$$\begin{aligned} \partial_\tau (\partial_u^2 + \partial_v^2) \Phi^{(1)} - \alpha_2 (\partial_u^2 + \partial_v^2)^2 \Phi^{(1)} = & \left[\partial_\tau (v^2 \partial_u^2 + u^2 \partial_v^2 + 2uv \partial_u \partial_v) - \right. \\ & - 2\alpha_2 \left(v^2 \partial_u^4 + u^2 \partial_v^4 + (u^2 + v^2) \partial_u^2 \partial_v^2 + 2uv \partial_u \partial_v (\partial_u^2 + \partial_v^2) + \right. \\ & \left. \left. + 4u \partial_u \partial_v^2 + 4v \partial_v \partial_u^2 + \partial_u^2 + \partial_v^2 \right) - \right. \\ & - \alpha_4 (v^2 \partial_u^2 + u^2 \partial_v^2 + 2uv \partial_u \partial_v + u \partial_u + v \partial_v) + \\ & \left. + \alpha_3 (v^2 \partial_u^2 + u^2 \partial_v^2 + 2uv \partial_u \partial_v + u \partial_u + v \partial_v) (\partial_u^2 + \partial_v^2) \right] \Phi^{(0)} \end{aligned}$$

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- Soluzione ordine 0: condizioni di Dirichlet nulle

$$\Phi^{(0)}(\tau, u, v) = e^{-\alpha_2(\ell_0^2 + m_0^2)\tau} \sin(\ell_0 u) \sin(m_0 v) \quad \ell_0, m_0 \in \mathbb{N}$$

- Equazione ordine 1

$$\partial_\tau(\partial_u^2 + \partial_v^2)\Phi^{(1)} - \alpha_2(\partial_u^2 + \partial_v^2)^2\Phi^{(1)} = e^{-\alpha_2(\ell_0^2 + m_0^2)\tau} F(u, v)$$

- Trasformata di Fourier sulle coordinate spaziali + trasformata di Laplace sul tempo

$$\Phi_{s,\ell,m}^{(1)} = \frac{\Phi_{\ell,m}^{(1)}(0)}{s + \alpha_2(\ell^2 + m^2)} - \frac{F_{\ell,m}}{(\ell^2 + m^2)(s + \alpha_2(\ell^2 + m^2))(s + \alpha_2(\ell_0^2 + m_0^2))}$$

- Antitrasfomata ($\omega = -is$)

$$\Phi^{(1)}(\tau, u, v) = \frac{1}{2\pi} \sum_{\ell,m} e^{i(\ell u + m v)} \int_{\mathbb{R}} d\omega \frac{e^{i\omega\tau}}{i(\omega - i\alpha_2(\ell^2 + m^2))} \left[\Phi_{\ell,m}^{(1)}(0) + \frac{iF_{\ell,m}}{(\ell^2 + m^2)(\omega - i\alpha_2(\ell_0^2 + m_0^2))} \right]$$

-
- Dipendenza temporale

$$\Phi^{(1)} \propto e^{-\alpha_2 k^2 \tau}$$

$$\ell^2 + m^2 \neq \ell_0^2 + m_0^2$$

$$\Phi^{(1)} \propto \tau e^{-\alpha_2 (\ell_0^2 + m_0^2) \tau}$$

$$\ell^2 + m^2 = \ell_0^2 + m_0^2$$

- Risonanza: tempi scala

$$\tau_{RES} = 1/\epsilon^2$$

$$\tau_{EXP} = \frac{1}{\alpha_2 (\ell_0^2 + m_0^2)}$$

- Condizione validità sviluppo perturbativo ($\epsilon^2 |\Phi^{(1)}| \ll |\Phi^{(0)}|$ ad ogni tempo)

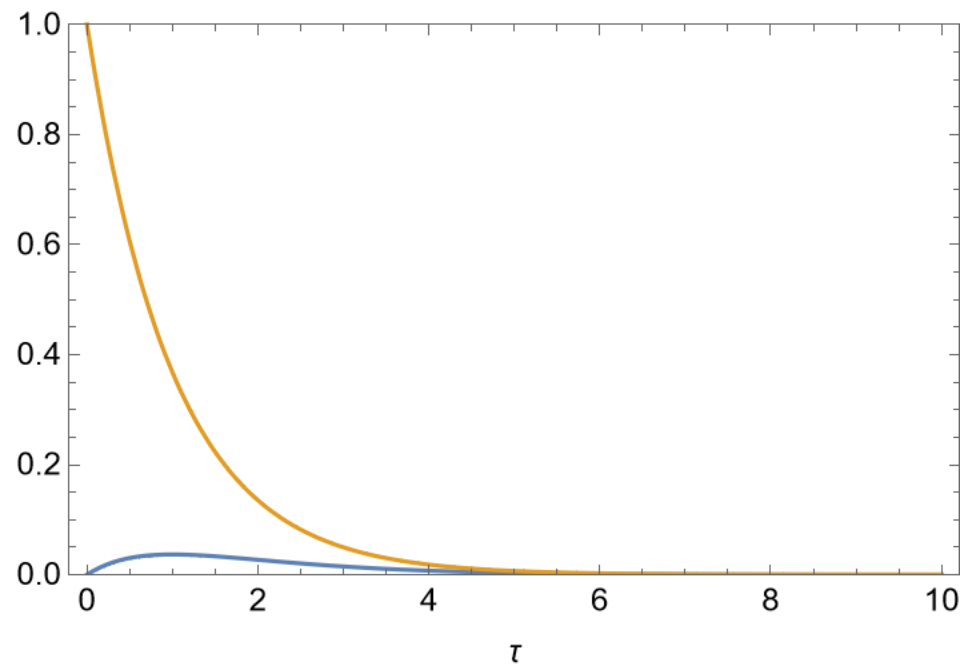
$$\tau_{RES} \gg \tau_{EXP} \quad \rightarrow \quad \ell_0^2 + m_0^2 \gg \frac{\epsilon^2}{\alpha_2} \quad \frac{\epsilon^2}{\alpha_2} \propto L_p^4$$

$$L_p = 1\text{cm}$$

$$\ell_0^2 + m_0^2 \gg 10^{-2}$$

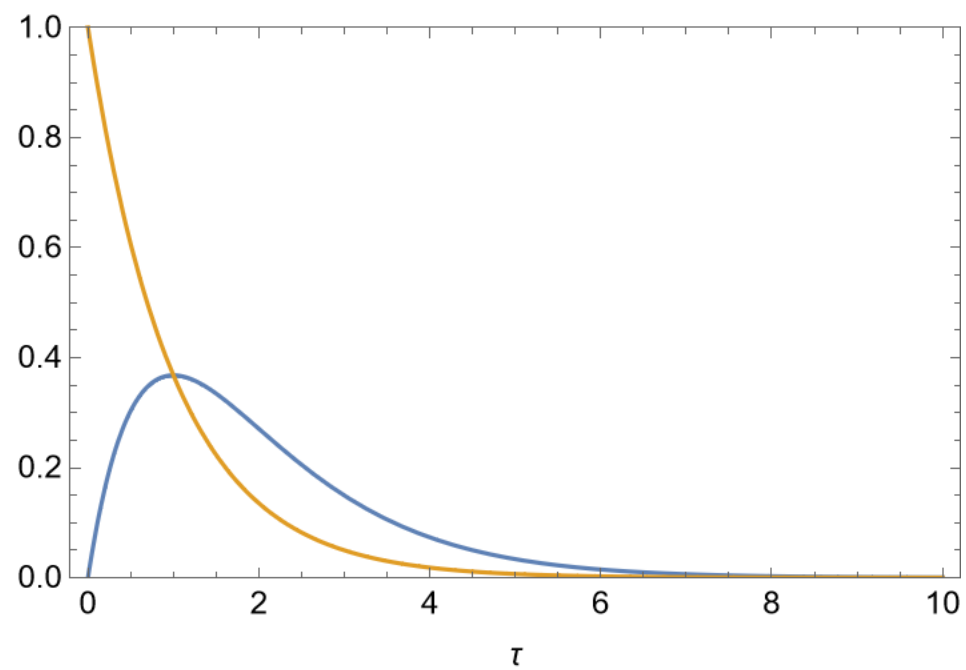
$$L_p = 5\text{cm}$$

$$\ell_0^2 + m_0^2 \gg 16$$



$$\tau_{EXP} = 1$$
$$\tau_{RES} = 10$$

$$\tau_{EXP} = 1$$
$$\tau_{RES} = 1$$





Tracers

- Equazione per il potenziale elettrico (equazione di Eulero 2D con termine viscoso)

$$\partial_\tau D^2\Phi + (\partial_u\Phi\partial_v D^2\Phi - \partial_v\Phi\partial_u D^2\Phi) = \eta D^4\Phi$$

- Integrazione numerica da $\tau = 0$ a $\tau = 3000$, step temporale $h = 0,001$
- Evoluzione tracers, sistema N corpi, metodo RK4

$$\begin{aligned}\frac{dx_i}{d\tau} &= v_i^x & \frac{dv_i^x}{d\tau} &= -\frac{\partial\Phi}{\partial x_i} \\ \frac{dy_i}{d\tau} &= v_i^y & \frac{dv_i^y}{d\tau} &= -\frac{\partial\Phi}{\partial y_i}\end{aligned}$$

Scarto quadratico medio

- Calcolo del MSD

$$\left\langle (\Delta s)^2 \right\rangle (\tau) = \frac{1}{N} \sum_i (x_i(\tau) - x_i(0))^2 + (y_i(\tau) - y_i(0))^2$$

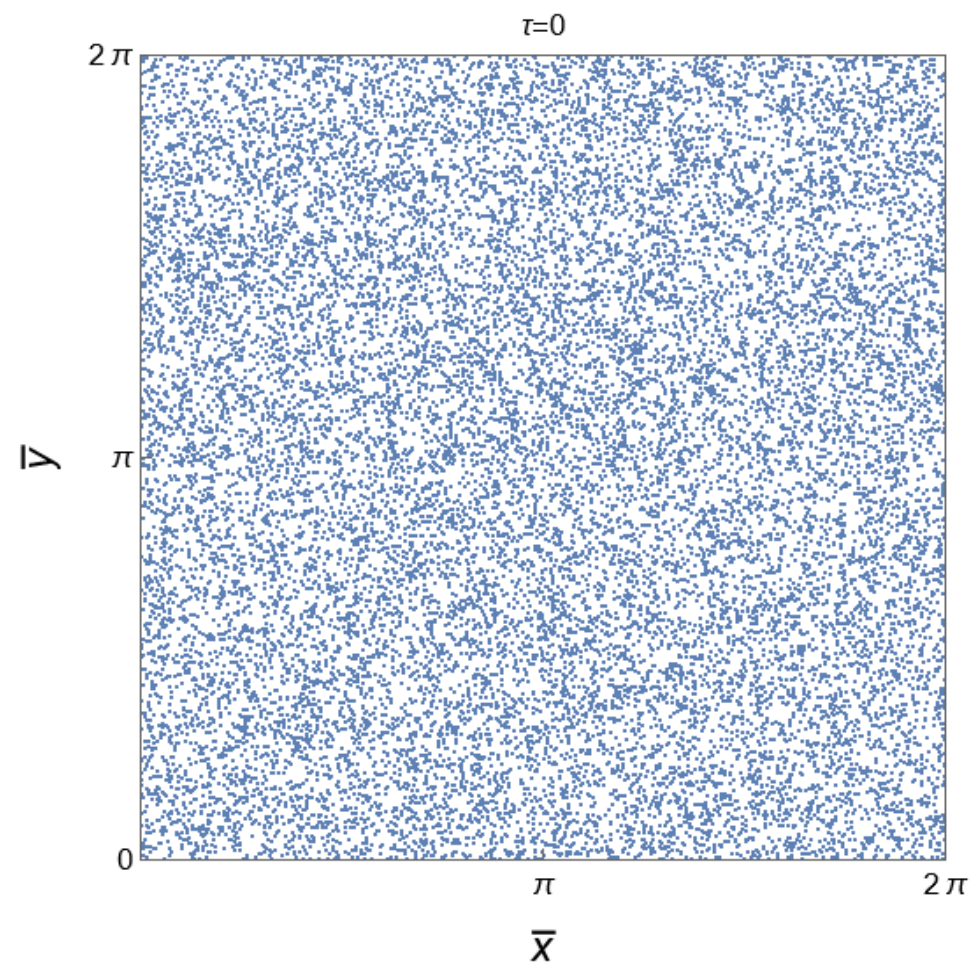
- Andamento temporale MSD

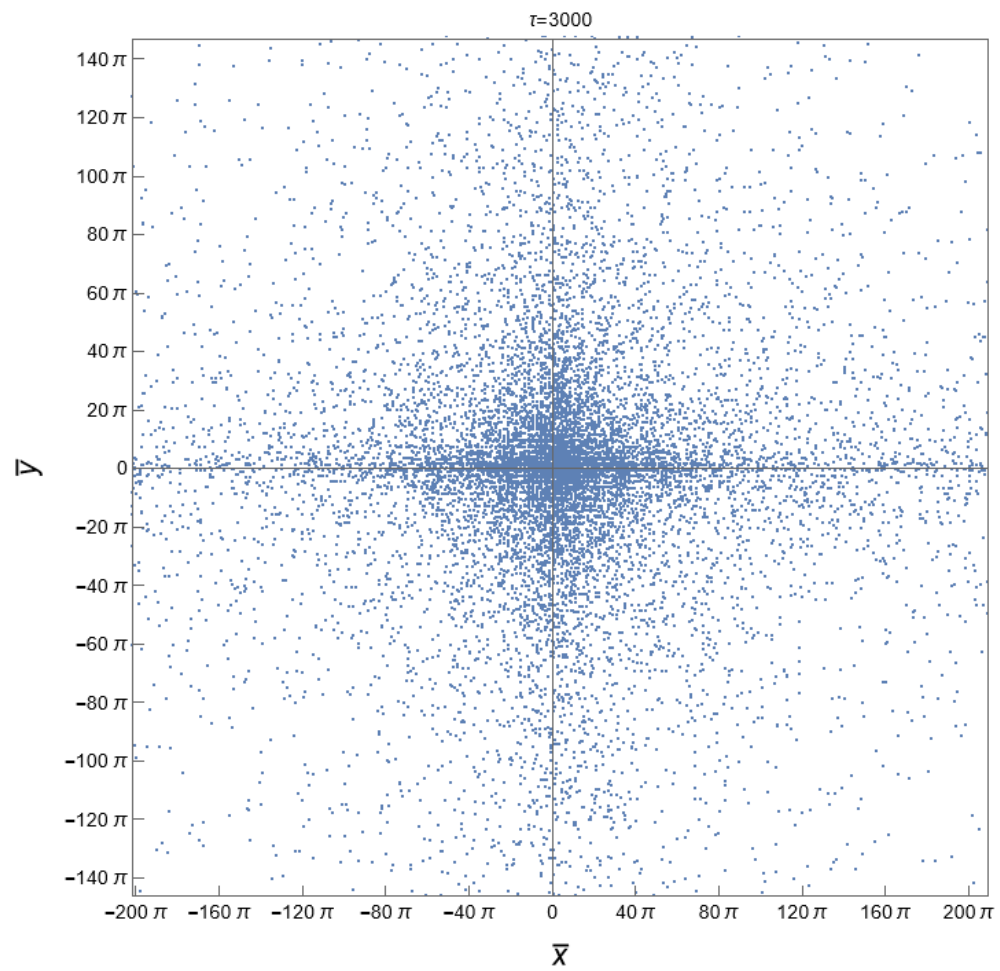
$$\left\langle (\Delta s)^2 \right\rangle (\tau) \propto \tau^\gamma$$

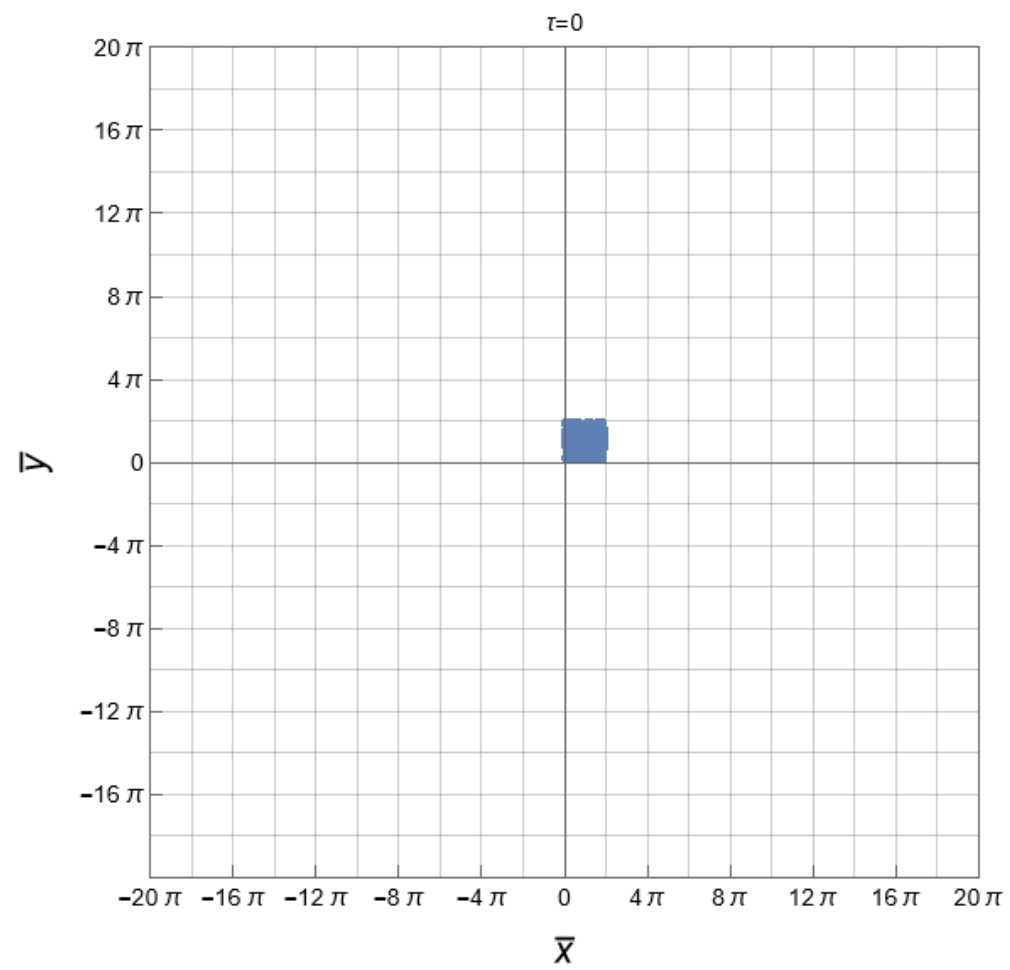
$\gamma = 2$	Avvettivo
$\gamma > 1$	Supradiffusivo
$\gamma = 1$	Diffusivo
$\gamma < 1$	Subdiffusivo

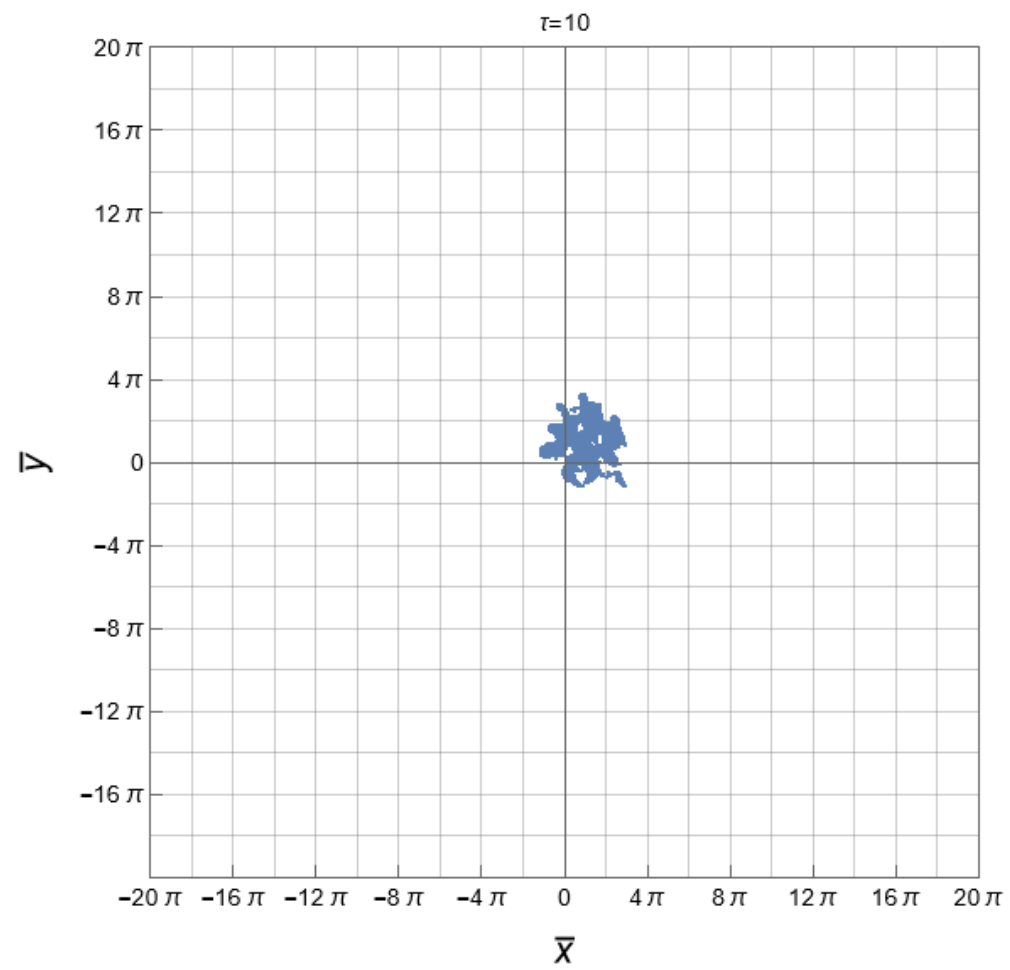
Coefficiente di diffusione

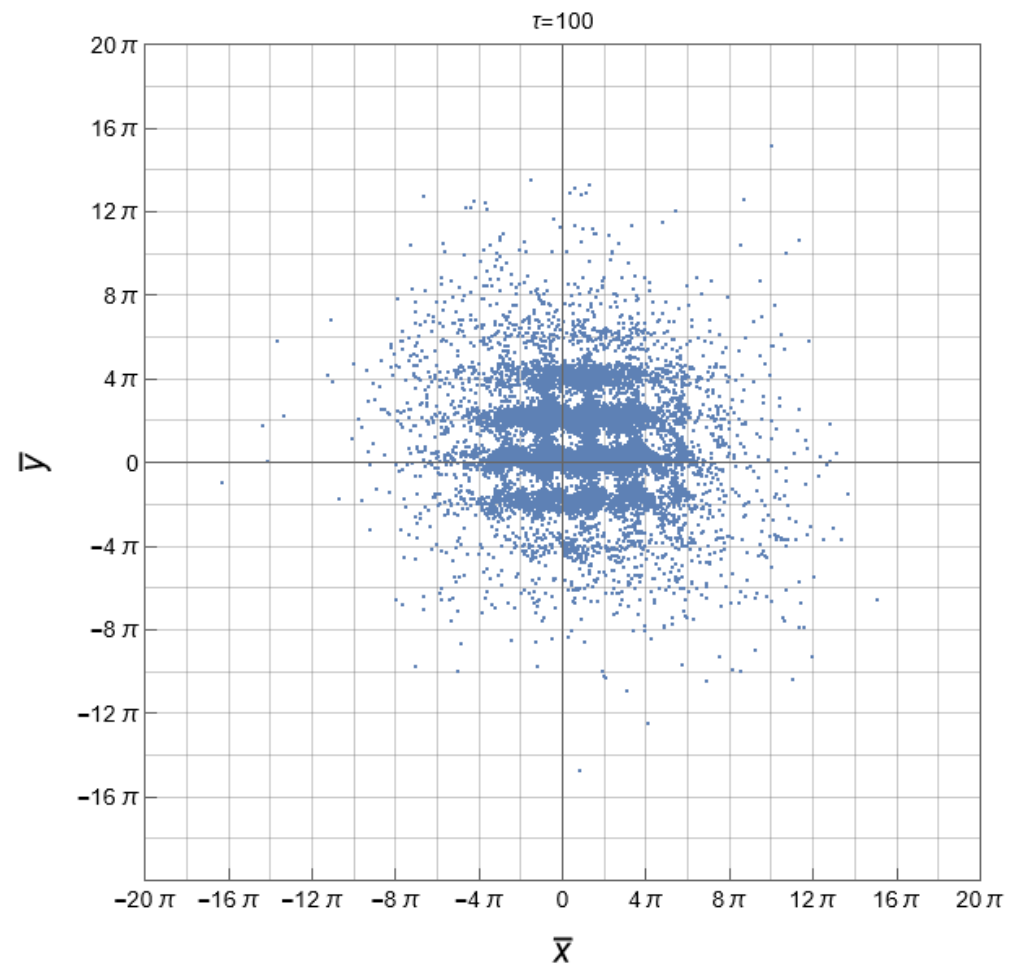
$$\left\langle (\Delta s)^2 \right\rangle (\tau) = 4D\tau$$

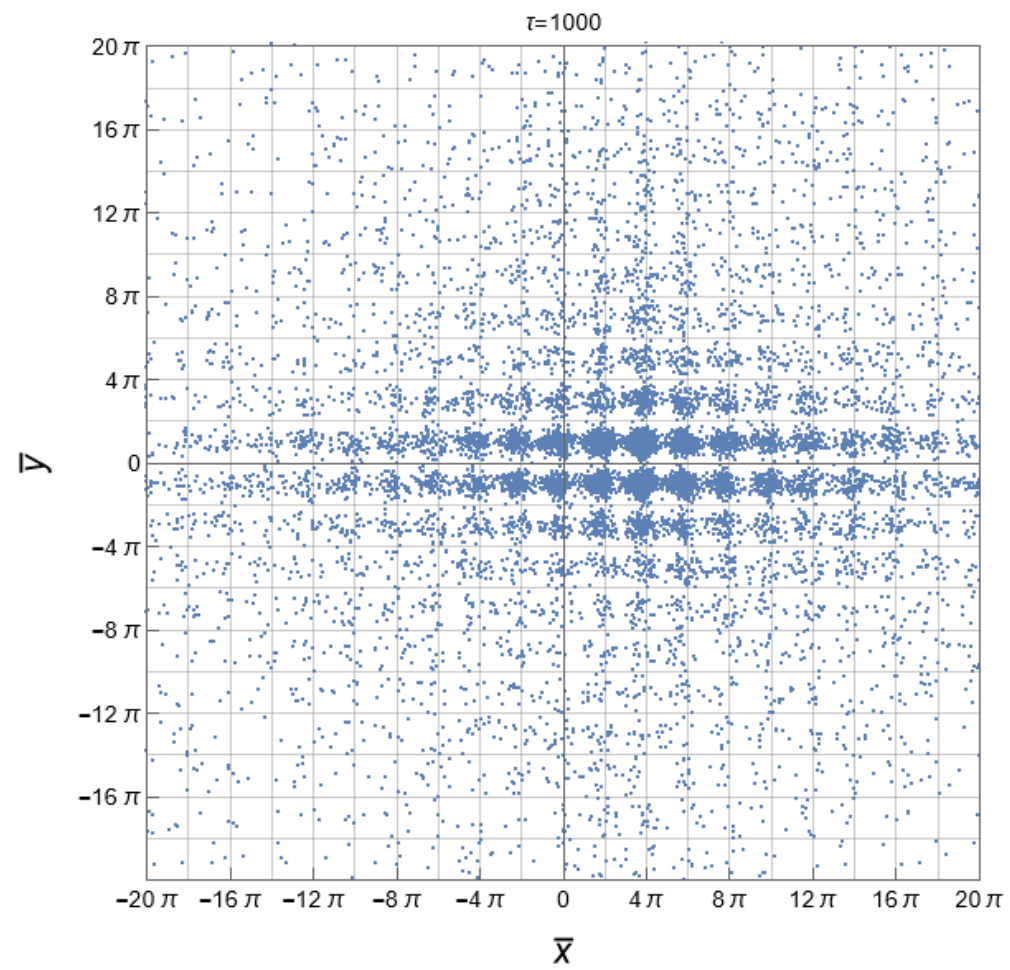


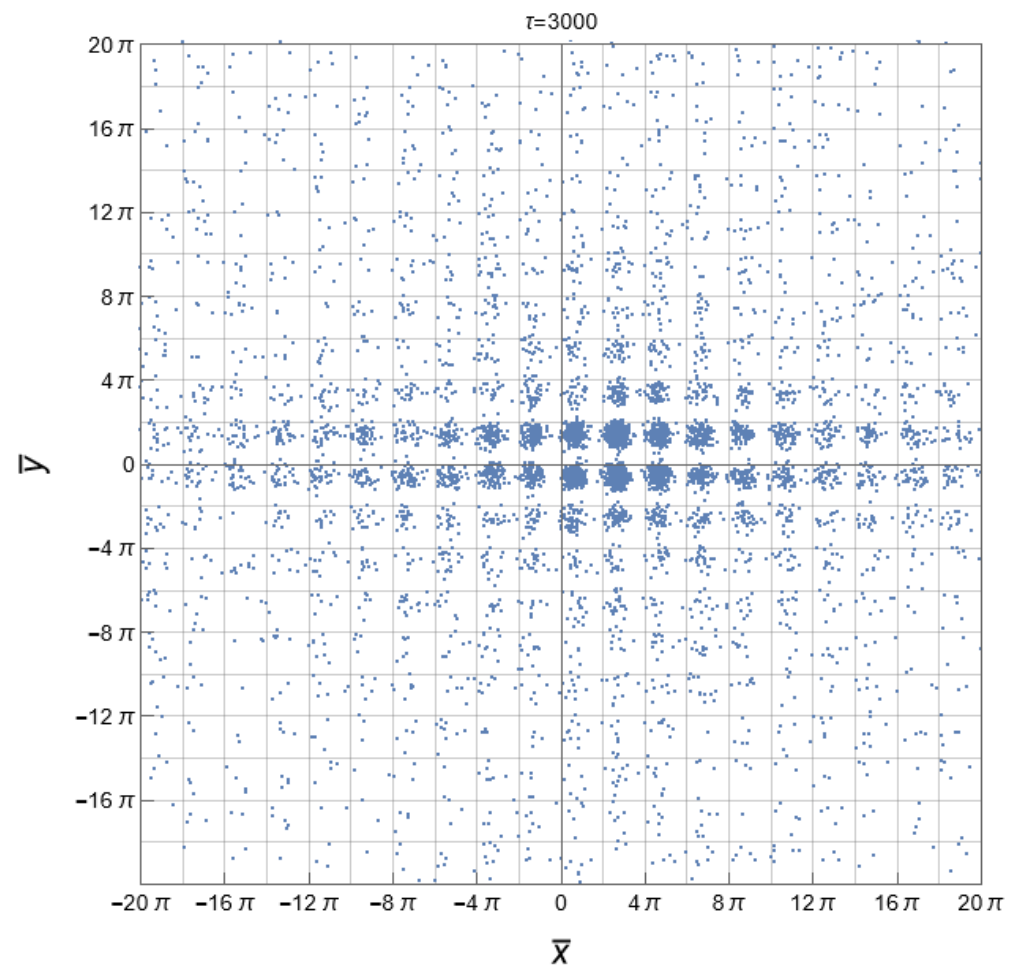


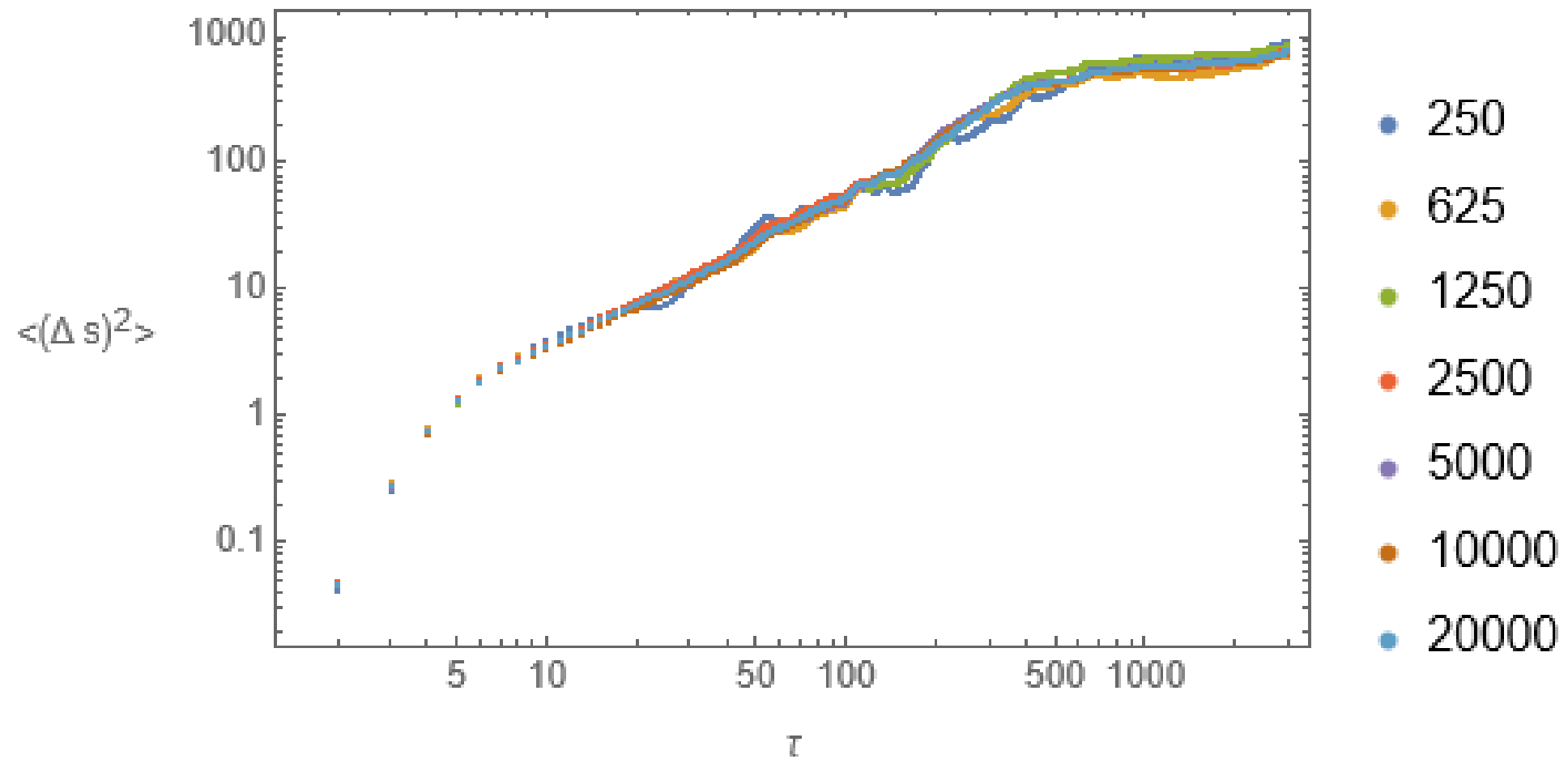


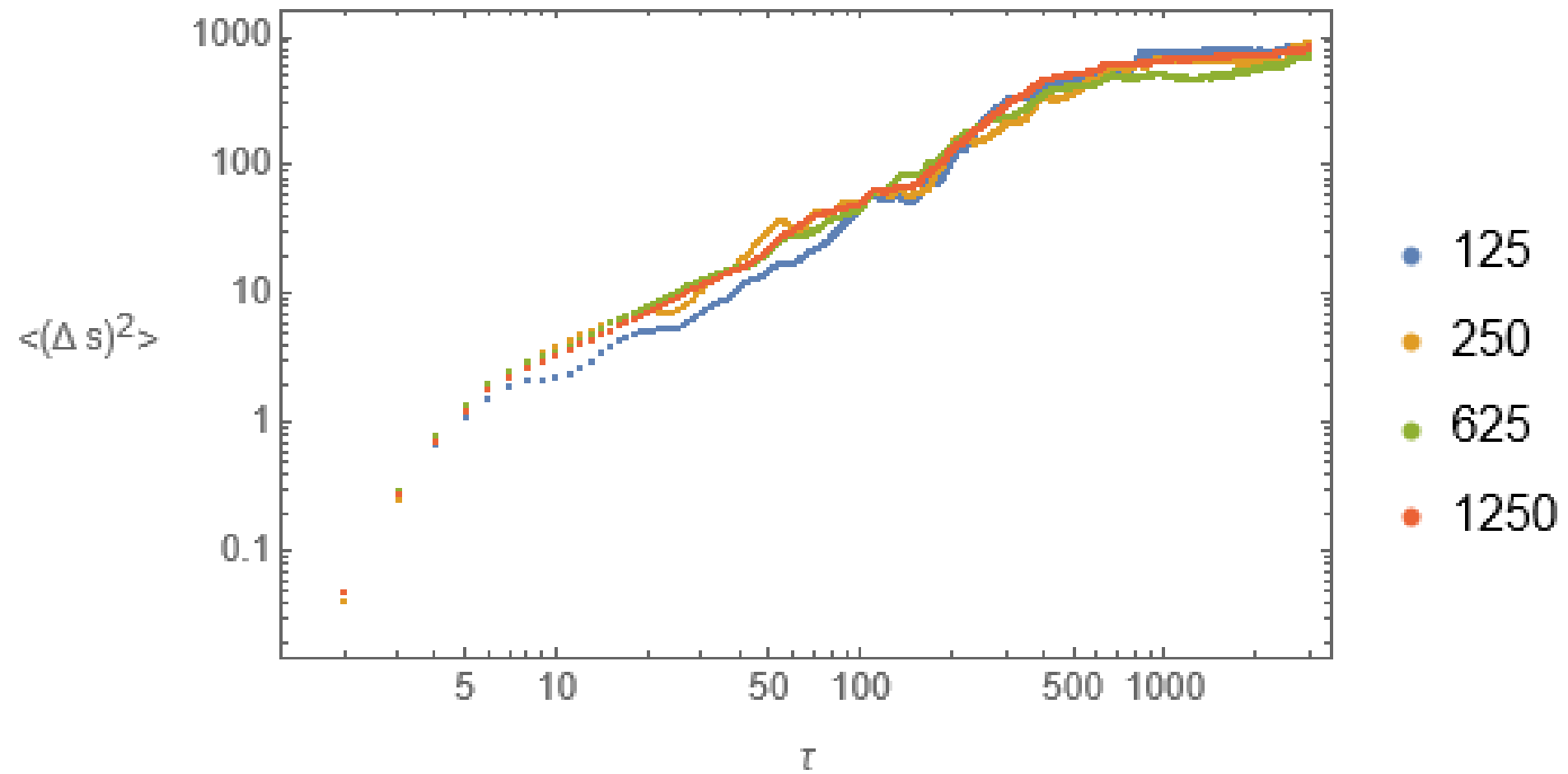


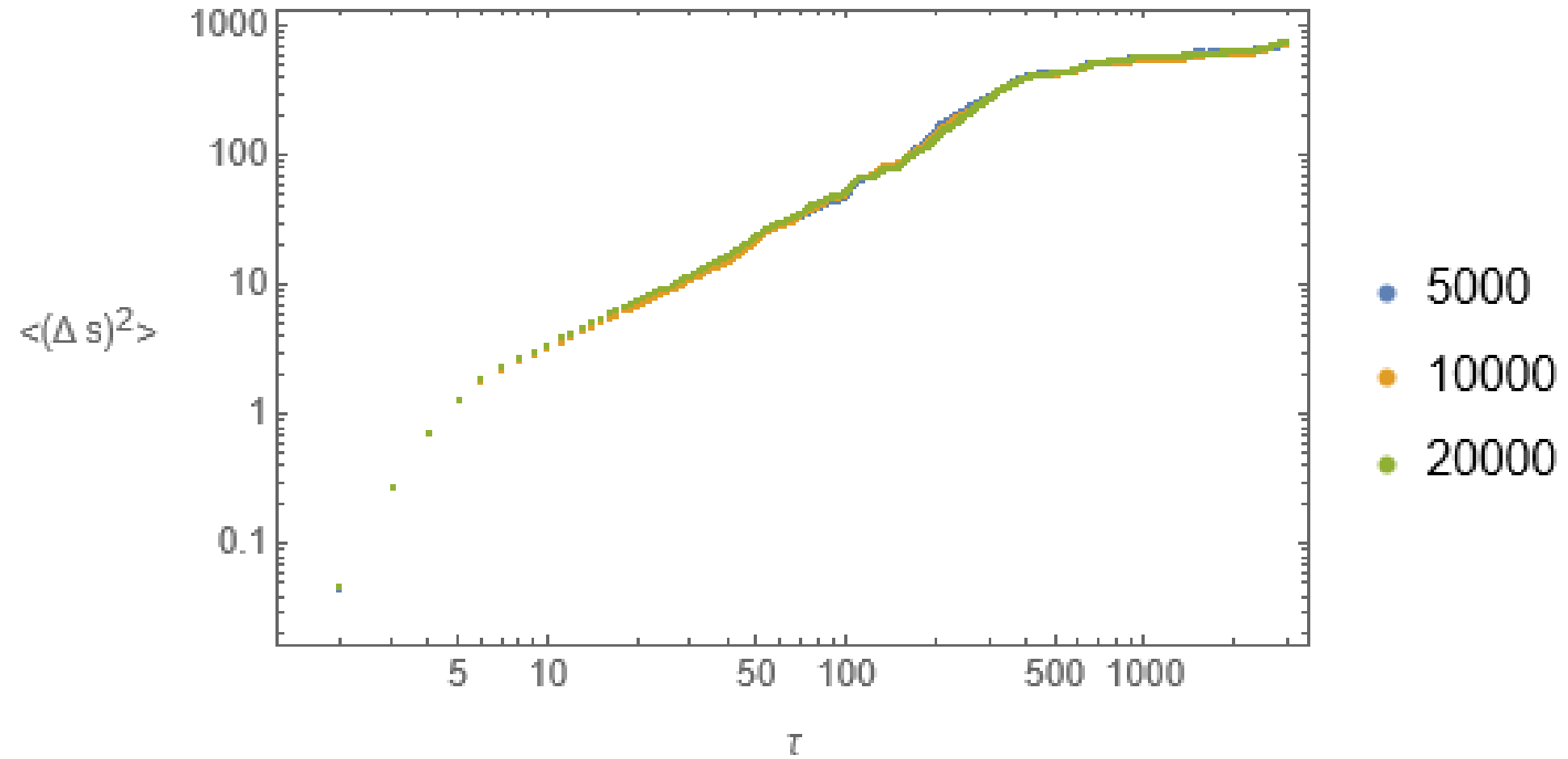


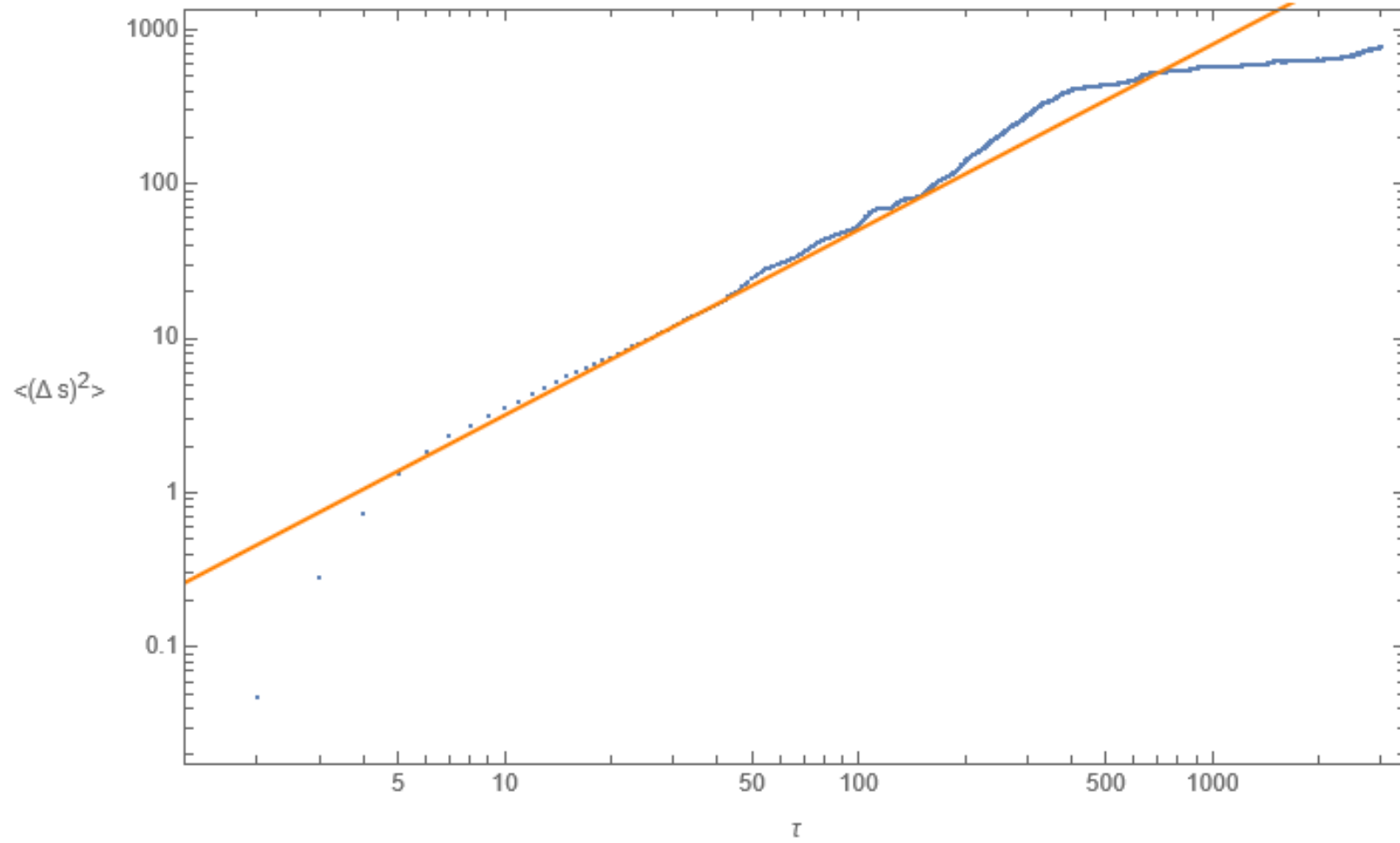












$$\langle (\Delta s)^2 \rangle (\tau) = 0.2 \tau^{1.2}$$

Grazie per l'attenzione