

## ABSTRACT

- Recent progress on the theory, numerical simulations, and experimental observations of Vertical Displacement Oscillatory Modes (VDOM) in tokamak experiments is reported.
- VDOM ( $n=0$ ) are driven unstable by energetic particles and can have an impact on plasma disruptions, plasma edge stability and confinement.
- These modes are a candidate to explain Alfvén-frequency  $n=0$  modes recently observed on JET (see Ref. [1]).
- The specific types of fast ion distribution functions that can provide an instability drive for VDOM are discussed.
- The difference between VDOM and Global Alfvén Eigenmodes (GAE) is discussed.
- In this poster we shall concentrate on analytic results. For numerical work on VDOM using the NIMROD code, see poster by D. Banerjee et al, this conference.

## BACKGROUND

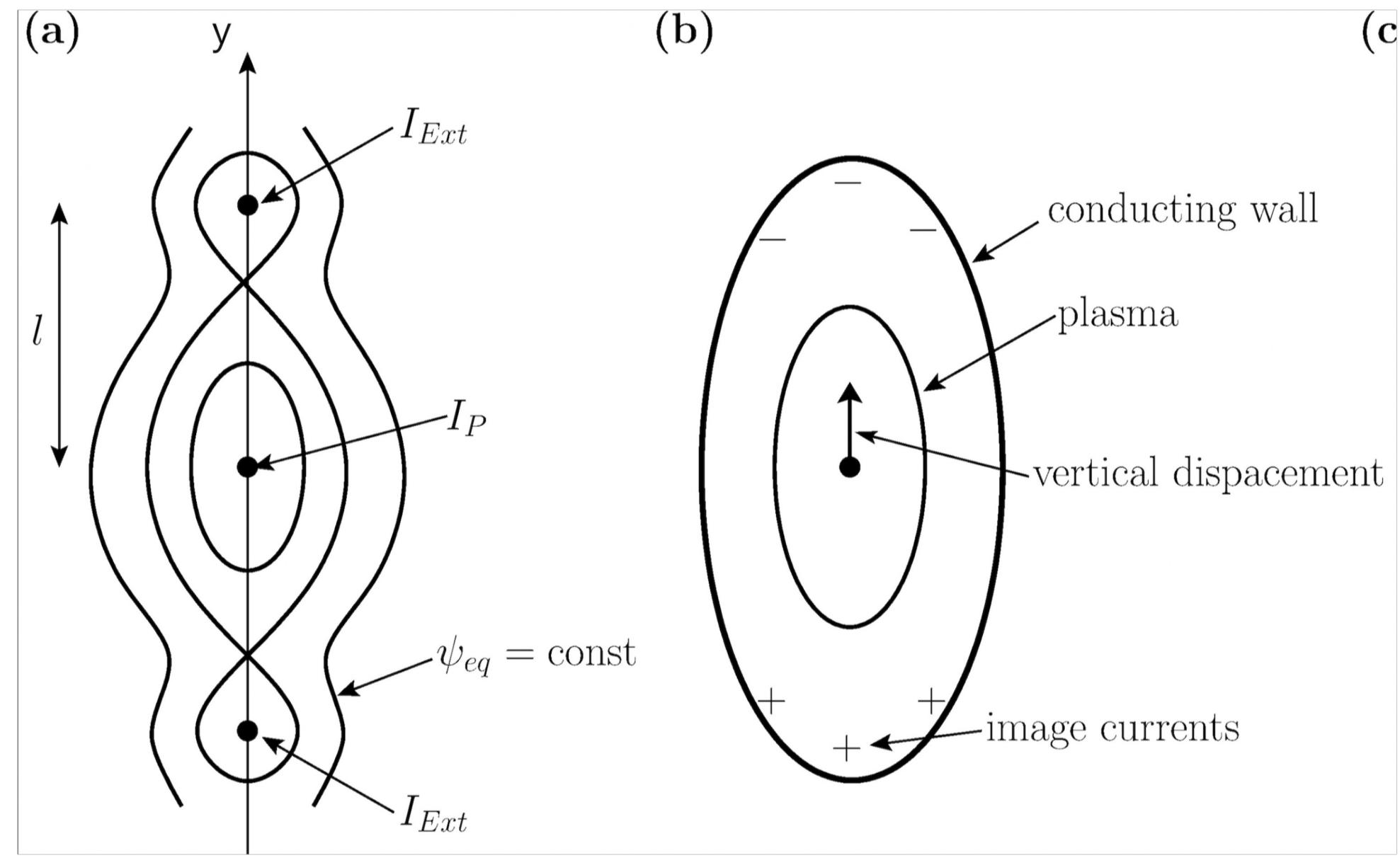
- VDOM: New type of fast-ion-driven instability [2-5].
- VDOM oscillate with a frequency  $\omega \sim$  poloidal Alfvén frequency. They can resonate with fast ions with energies  $\sim 100$  keV to a few MeV.
- They may explain observations on JET [1, 5], TCV, MAST-U.
- VDOM are intimately connected with the Vertical Instability.

## Heuristic model and analytic theory for VDOM

Different types of axisymmetric,  $n=0$  modes:

- Geodesic acoustic modes (GAM)  $\rightarrow$  low frequency (sound)
- Global Alfvén eigenmodes (GAE)  $\rightarrow$  high (Alfvén) frequency
- Vertical displacements (VDE)  $\rightarrow$  purely growing, zero frequency
- Vertical Displacement Oscillatory Modes (VDOM)  $\rightarrow$  high (Alfvén) frequency

## VDOM MECHANISM: Heuristic model



## ANALYTIC DISPERSION RELATION (Reduced Ideal MHD + Resistive Wall)

$$\gamma^3 + \frac{\gamma^2}{(1-\hat{e}_0)\tau_\eta} + \gamma\omega_0^2 + \frac{\omega_0^2}{(1-D)\tau_\eta} = 0$$

where

$$\omega_0 = \left(\frac{D-1}{1-\hat{e}_0}\right)^{1/2} \gamma_\infty, \quad \gamma_\infty = \frac{2\kappa}{1+\kappa^2} \left(\frac{\kappa-1}{\kappa}\right)^{1/2} \tau_A^{-1} \quad \text{and} \quad \tau_A = \frac{B_p'}{(4\pi\varrho_m)^{1/2}}$$

When  $D > 1$  and  $\omega_0\tau_\eta \gg 1$ , we find three roots:

$$\omega = \pm\omega_0 - i\frac{(1-\hat{e}_0)D}{2(D-1)(1-\hat{e}_0)D\tau_\eta}; \quad \gamma = \frac{1}{(D-1)\tau_\eta}$$

VDOM correspond to the first two roots  $\rightarrow$  stable in the absence of fast ions.

The third root is the Vertical Displacement growing on the resistive wall time  $\rightarrow$  suppressed by active feedback stabilization.

 The Geometric Wall Parameter,  $D(\kappa, b/b_w)$ 

Passive wall stabilization of vertical displacement requires  $D \geq 1$ . Assume elliptical confocal plasma boundary and elongation  $\rightarrow D$  depends on  $\kappa$  and  $b/b_w$

$$D(\kappa, b/b_w) = \frac{\kappa^2+1}{(\kappa-1)^2} \left\{ 1 - \left[ 1 - \frac{\kappa^2-1}{\kappa^2} \left( \frac{b}{b_w} \right)^2 \right]^{1/2} \right\}$$

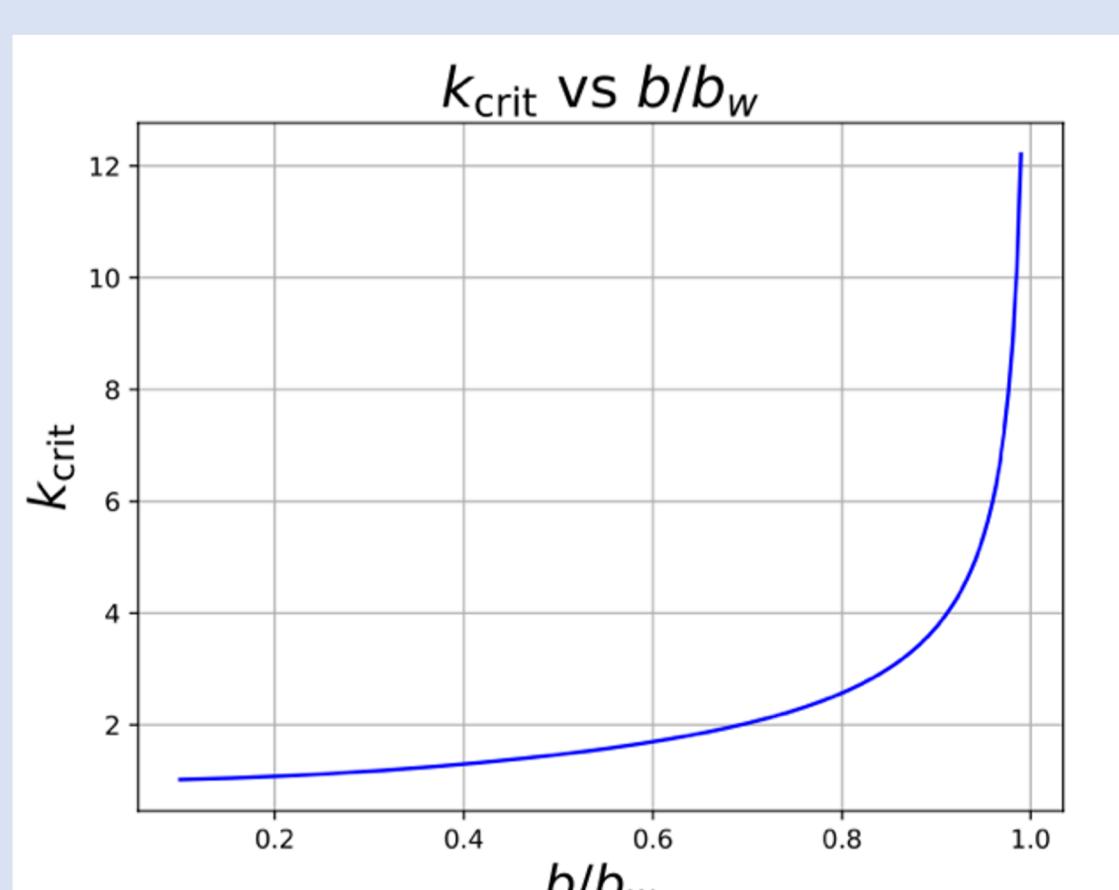


Fig. 1: Critical elongation,  $\kappa_{crit}$ , vs normalized plasma-wall distance,  $b/b_w$ . When the plasma touches the wall,  $b/b_w = 1$ .

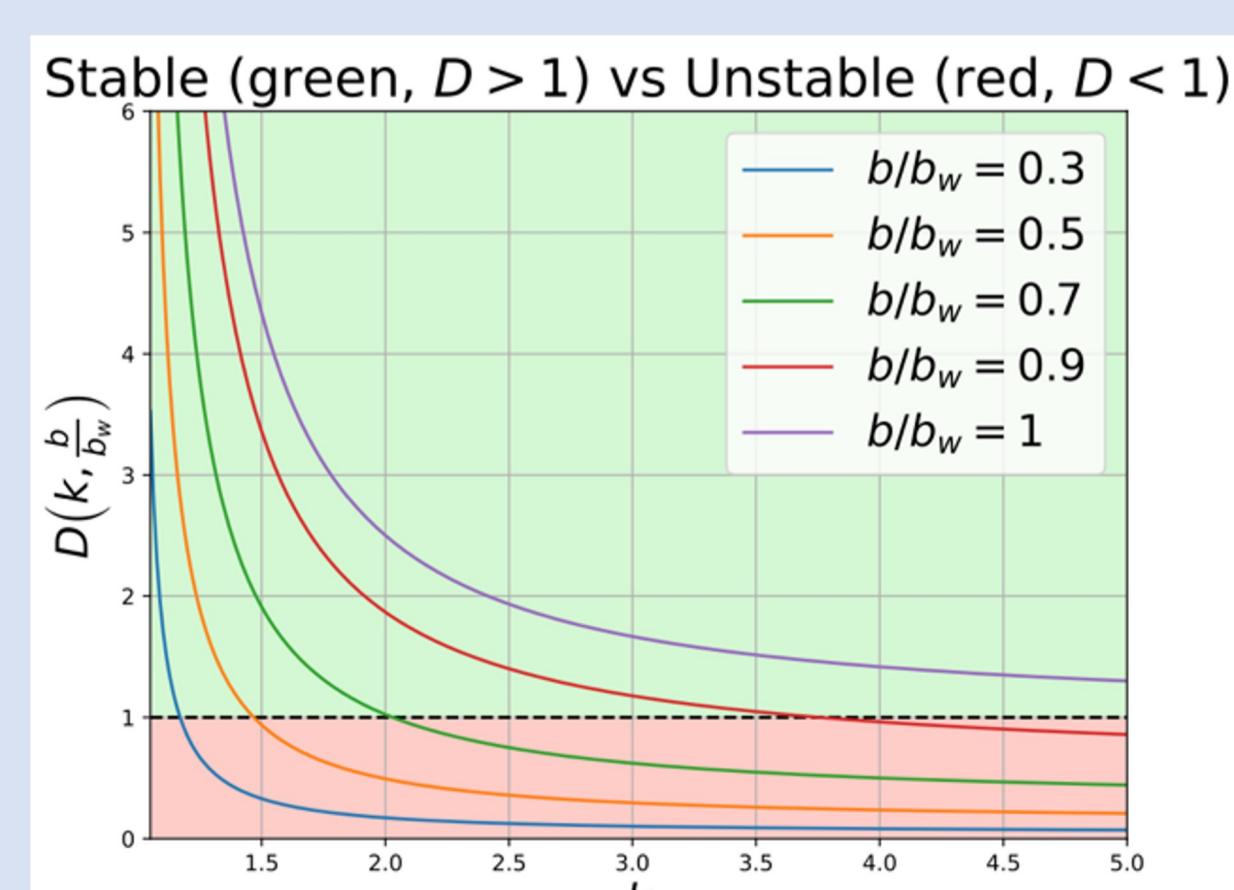


Fig. 2: The geometric wall parameter  $D(\kappa, b/b_w)$  vs elongation  $\kappa$  for different values of the normalized plasma-wall distance,  $b/b_w$ .

## FAST ION DRIVEN VDOM

Resonance condition:  $\omega = p\omega_{t/b}$ . A necessary condition for a positive fast ion drive is

$$\left. \frac{\partial F_h}{\partial \varepsilon} \right|_\mu = \left. \frac{\partial F_h}{\partial \varepsilon} \right|_\Lambda - \left. \frac{\Lambda \partial F_h}{\varepsilon \partial \Lambda} \right|_\varepsilon > 0$$

The required bump-on-tail-like distribution functions may form with NBI or ICRF, or after a sawtooth crash.

A positive drive is obtained for  $0.4 \leq \Lambda_0 \leq 0.95$ ; It favors high-field side ICRF [6], or fusion alpha particles with non-standard, potato like orbits [1].

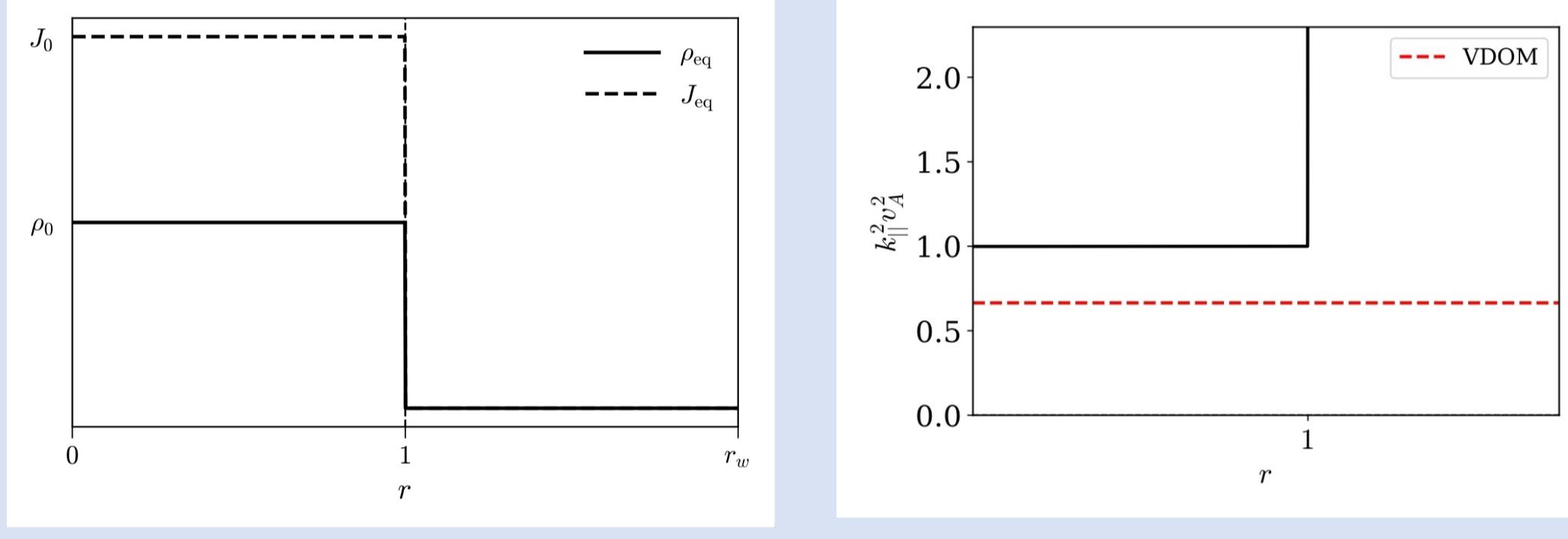
## THE DIFFERENCE BETWEEN VDOM AND GAE

GAE (first discussed in cylindrical geometry in Ref. [7]) are shear-Alfvén waves. The restoring force is field-line bending. They exist because of finite magnetic shear. Their discrete frequency lies just below the minimum of the Alfvén continuum spectrum,  $k_{\parallel}v_A$ . Internal modes: frequency largely independent on plasma-wall distance.

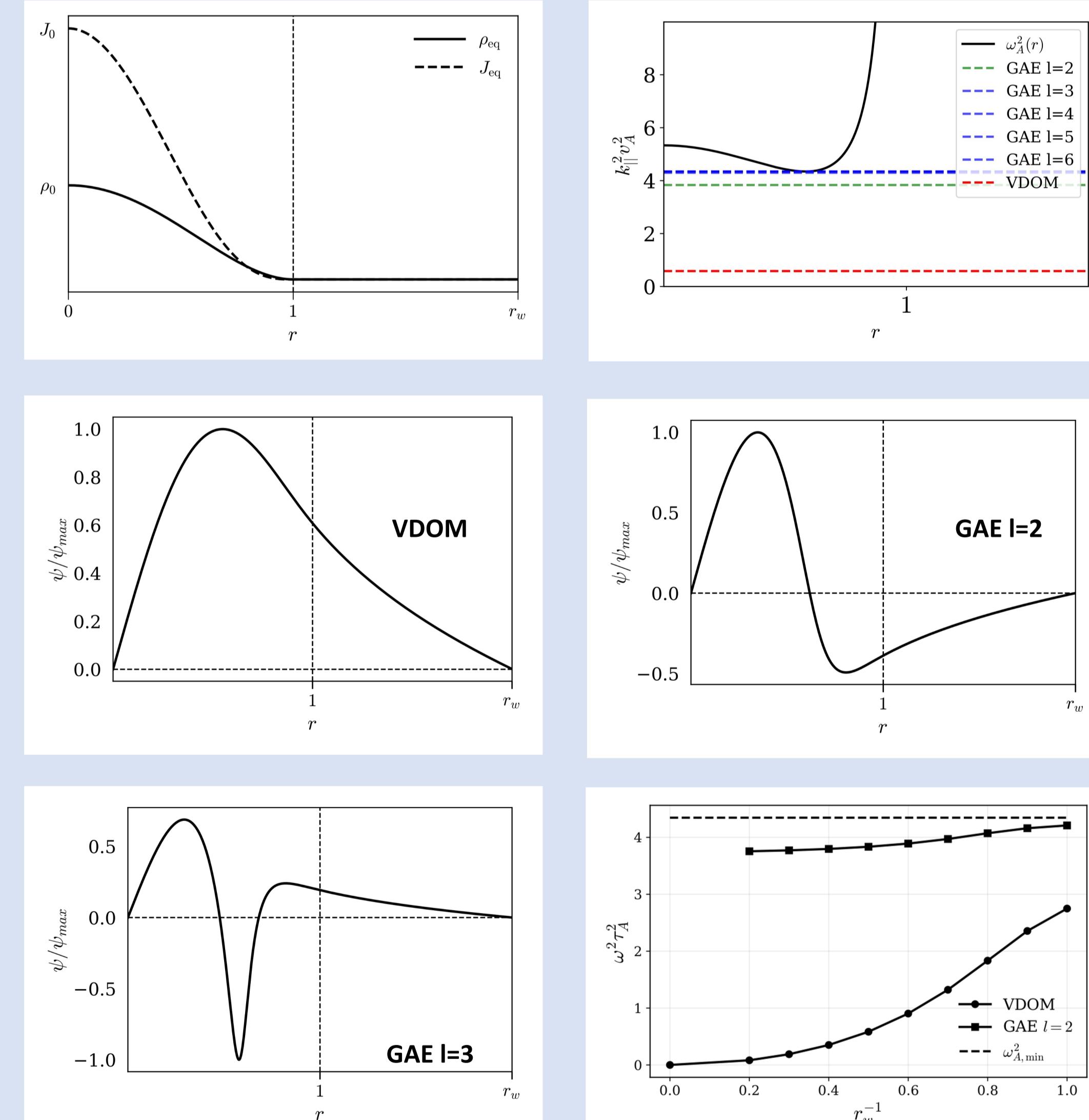
By contrast, VDOM are external modes. The restoring force is the force of interaction between the plasma current and currents induced on the wall when the plasma shifts vertically. For this reason, the VDOM frequency depends importantly on the plasma-wall distance. VDOM and vertical instability are closely related; however, VDOM exist also in the circular cross section limit. Normally, VDOM are immune of continuum damping, except for particular equilibrium profiles and the wall is very close to the plasma boundary.

The difference between VDOM and GAE is illustrated by the following solutions of the normal mode equation for the perturbed magnetic flux  $\psi$  and two eq. profiles.

Case 1: Flat  $J$  and  $\varrho_m$ , shearless (no GAE);  
Perturbed magnetic flux for  $r_w = 2$ , and  
normalized frequency square as function of  
 $r_w$ ; continuum damping  
may occur for  $r_w < \sqrt{3}$ .



Case 2:  
 $\varrho_m = \varrho_0(1-r^2)^2$   
 $J = J_0(1-r^2)^3$   
The Alfvén continuum exhibits a minimum.  
VDOM and GAE coexist.  
For  $r_w = 2$ , the VDOM frequency is far below the minimum of  $k_{\parallel}v_A$ , while several GAE with different values of the radial mode number  $l$  are found with a frequency very close to the minimum of  $k_{\parallel}v_A$ .  
 $\omega^2 \tau_A^2$  vs  $r_w$  for  
VDOM shows that in this case the VDOM frequency stays below the minimum of  $k_{\parallel}v_A$  (no continuum damping) even when  $r_w \rightarrow 1$ .



## CONCLUSIONS

An overview of the state-of-the-art on  $n=0$  VDOM has been presented. This theory may help with the interpretation of experimental observations of  $n=0$  modes in recent tokamaks. VDOM are driven unstable by fast ions but are also closely associated with vertical displacements that can evolve into VDE and disruptions. The solution of the normal mode equation in the cylindrical limit with circular cross-section highlights the qualitative and quantitative differences between GAE and VDOM.

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## REFERENCES

- [1] J. Oliver et al, Phys. Rev. Lett. 2026 in press; [2] T. Barberis et al, NF2022; [3] T. Barberis et al, PPCF2022; [4] F. Porcelli et al, FPP2023; [5] T. Barberis et al, NF2024; [6] L-G. Eriksson and F. Porcelli, NF2025; [7] L. Villard and J. Vaclavik, NF1997.