Phase space transport in burning plasmas

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ENEA — AGENZIA NAZIONALE PER LE NUOVE TECNOLOGIE, L'ENERGIA E LO SVILUPPO ECONOMICO SOSTENIBILE

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Transport processes in burning plasmas

- Predicting the dynamics of a burning plasma on long time scales, comparable with the energy confinement time or longer, is essential in order to understand next generation fusion experiments, e.g., ITER;
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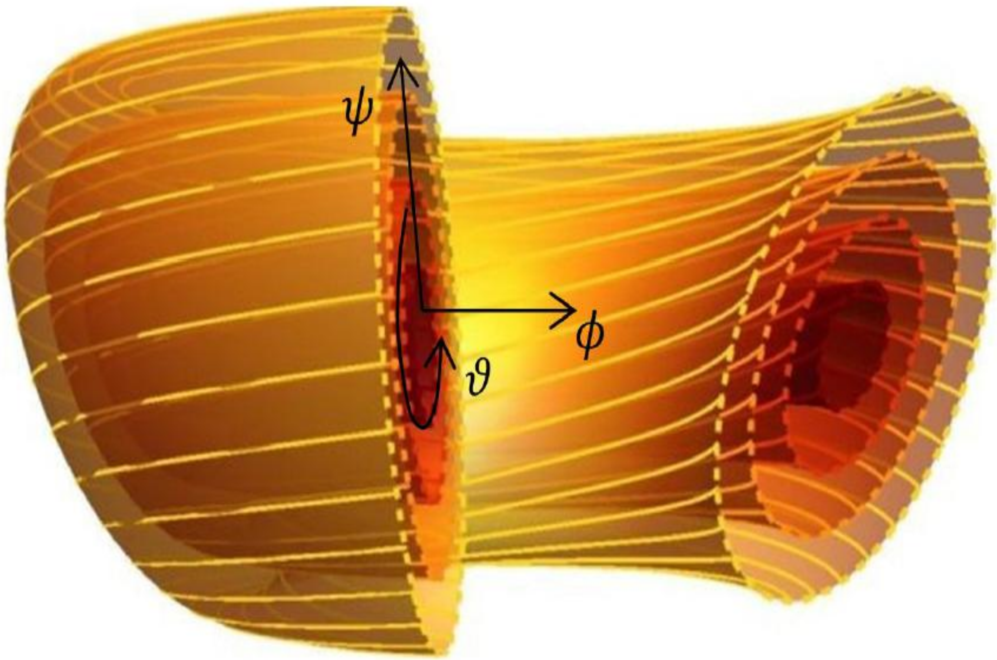
- the crucial role of energetic particles Zonca et al. 2015; Chen and Zonca 2016, must be properly described;
- a first-principle-based, self-consistent approach is crucial;
- extending gyrokinetic simulations to these time scales is a challenging task from the computational resource point of view, i.e., $\sim 10^{24}$grid points;
- simplifying assumptions based on physics understanding and first principles must be introduced;

Transport processes in burning plasmas



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- Transport equations define the evolution of radial profiles

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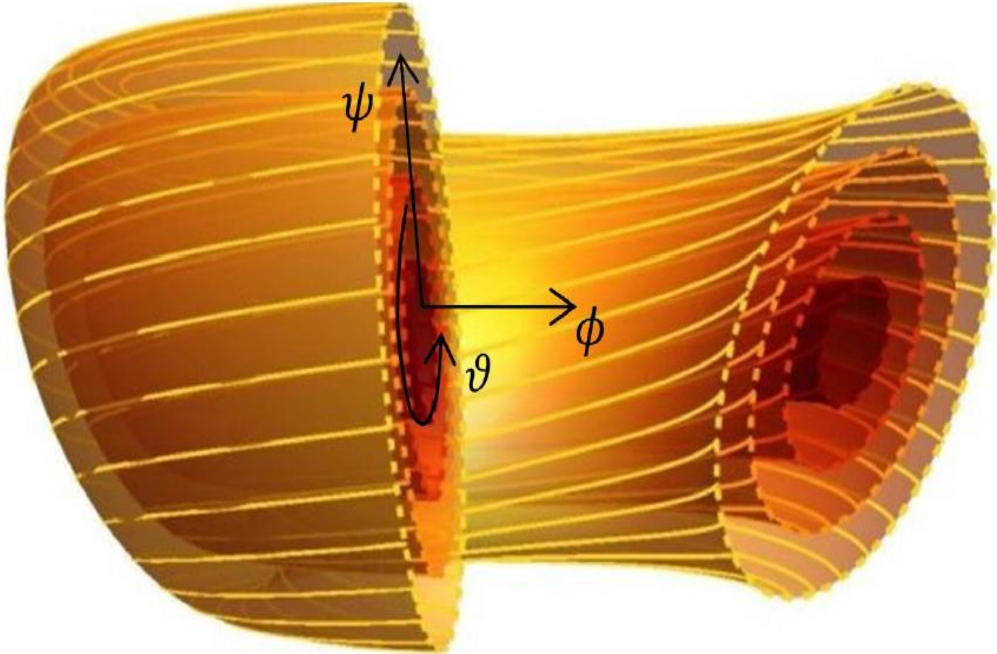

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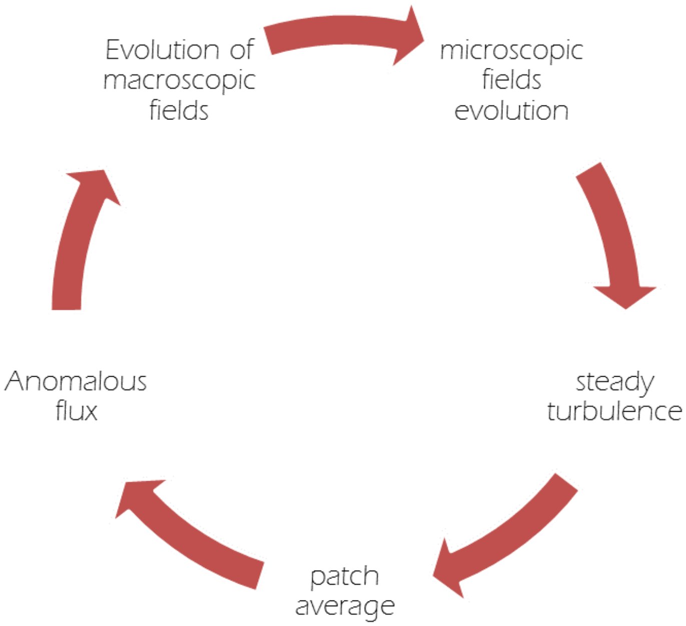

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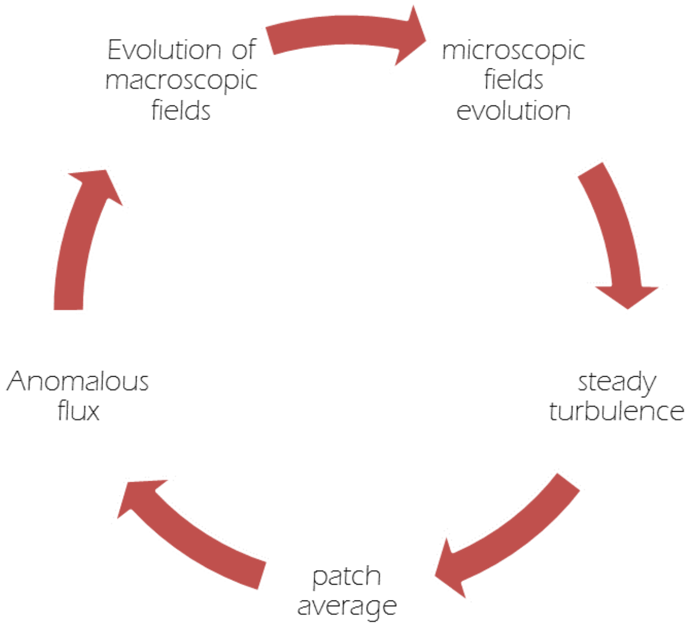
- Implicit separation of scales between equilibrium and fluctuations;
- Local Maxwellian is assumed;



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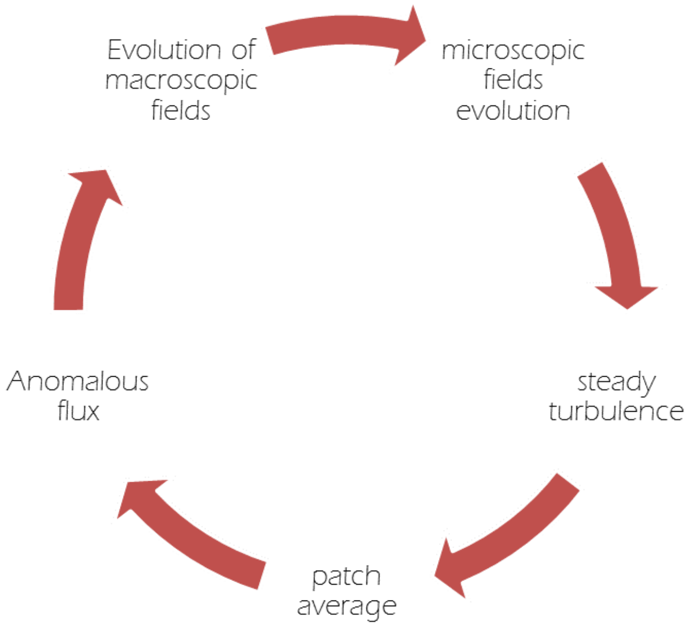

Need for generalization!!!

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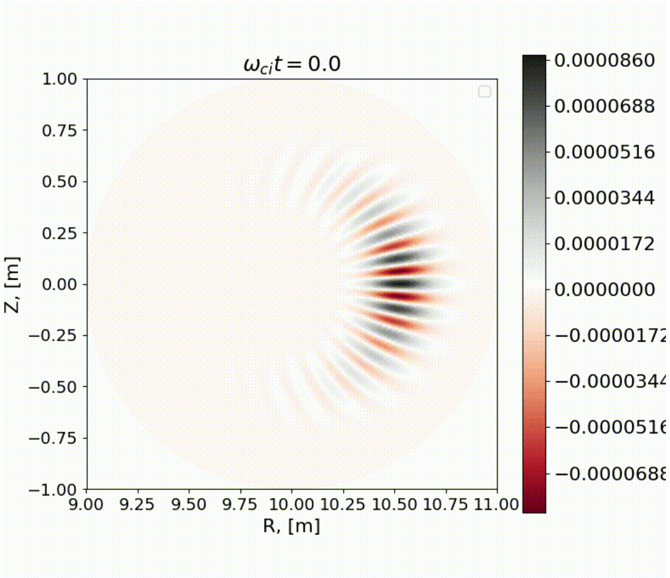
- Small but finite distortions w.r.t. local thermodynamic equilibrium may evolve very differently due to collisionless behaviors and resonances;
- interaction with thermal plasma over long timescales can modify bulk transport processes;

Neighboring nonlinear equilibria



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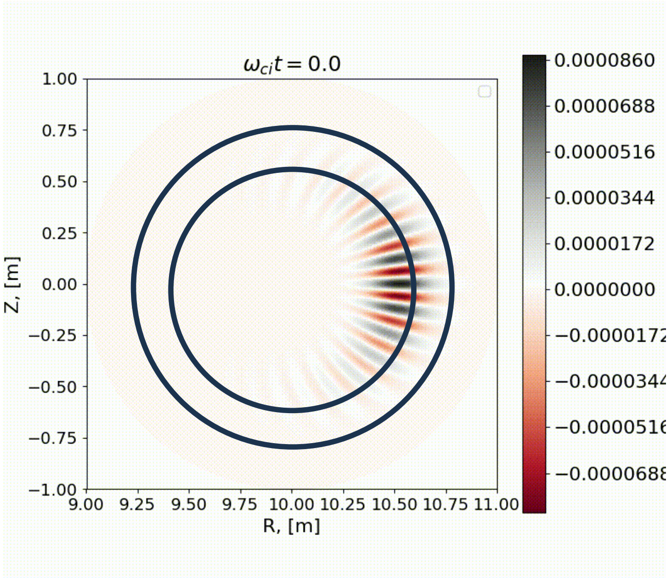
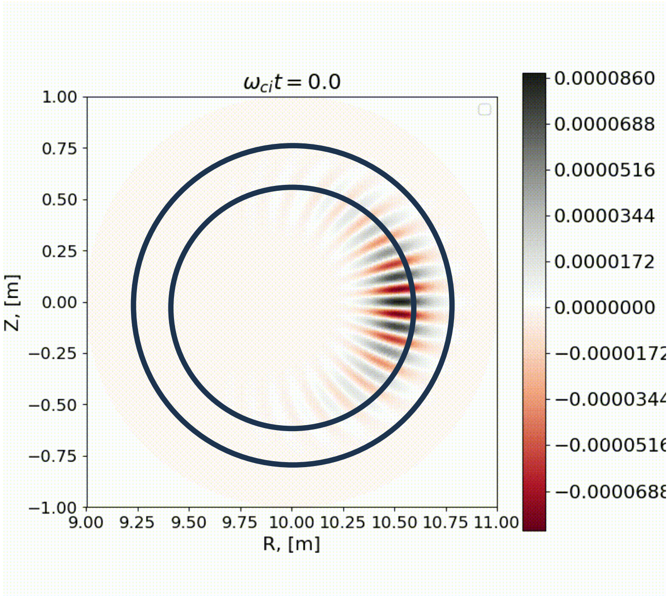
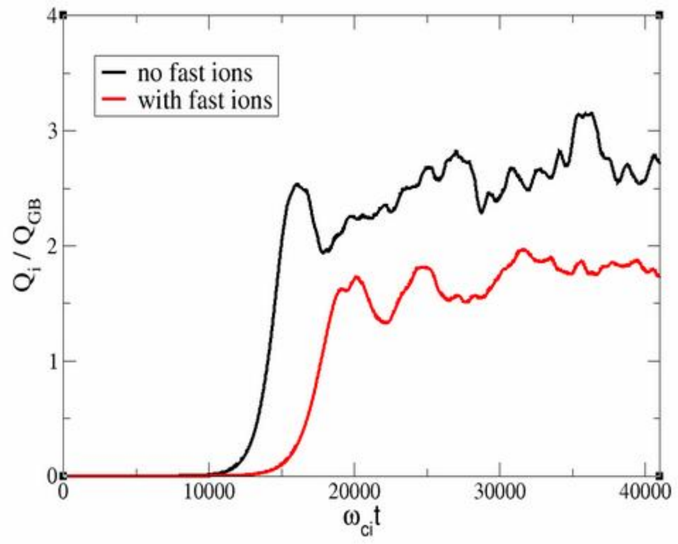


Neighboring nonlinear equilibria

- Nonlinear gyrokinetic simulations demonstrate clear reduction in the heat flux in the presence of EPs.





Purpose statement

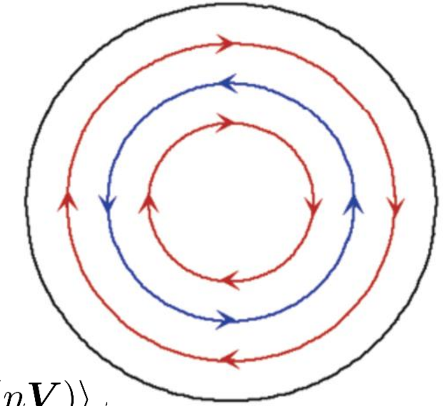
We aim at 1. providing the general expressions describing EPs (plasma) dynamics on long time scales (transport) and 2. introducing a framework to solve these equations within different levels of reduced dynamics.

By means of this approach it will be possible to:

- define the concept of nonlinear equilibrium;
- describe the physics of burning plasmas where alpha particles will play a key role in transport studies by interacting with thermal components;
- the derivation, see Falessi 2017; Falessi and Zonca 2019, Falessi et al 2023 is based on the Phase space zonal structures theory, see Chen and Zonca 2016.

Phase Space Zonal Structures

- Coupling between fluctuating fields can generate zonal flows and zonal fields;
- Crucial elements for regulating turbulent fluxes e.g., by scattering instability turbulence to shorter radial wavelength stable domain…



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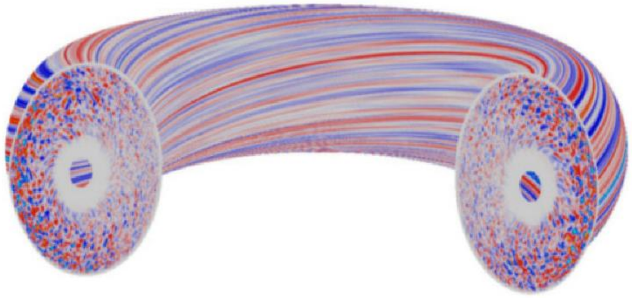

Figure: Courtesy of Y. Xiao et al., PoP 2015.

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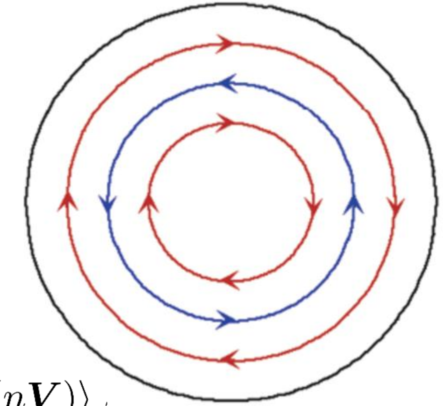

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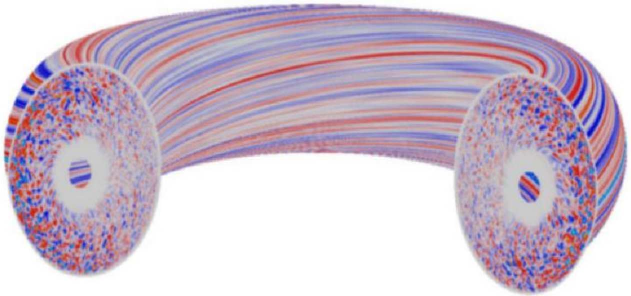

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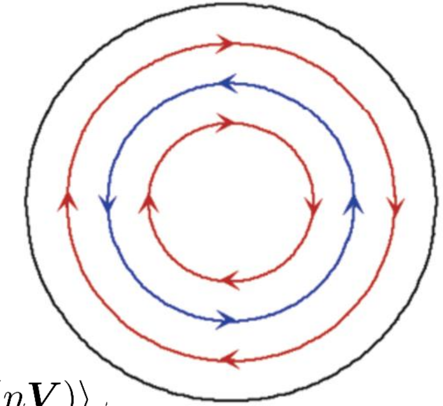
- Crucial elements for regulating turbulent fluxes e.g., by scattering instability turbulence to shorter radial wavelength stable domain…
- zonal structures in the density and temperature profiles are unaffected by rapid collision-less dissipation;
- collision-less undamped fluctuations in the phase space are called phase space zonal structures Chen and Zonca 2016, Falessi and Zonca 2019;



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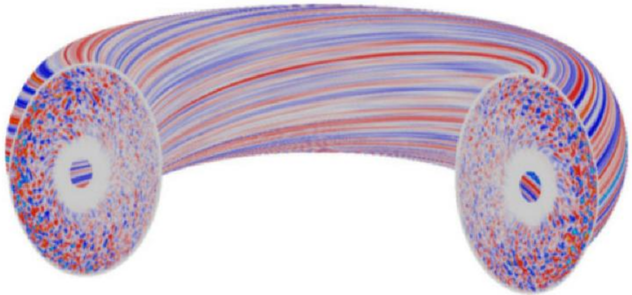

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Orbit averaging

- Particle motion in the reference magnetic field is characterized by three integrals of motion, i.e. $P_\phi, \mu, \mathcal{E}$;
- Phase Space Zonal Structures equation is connected with the macro-/meso- scopic component, i.e. $[\dots]_S$, unperturbed orbit averaged distribution function (Falessi and Zonca 2019);

$$\partial_t \overline{F_{z0}} + \frac{1}{\tau_b}\left[\frac{\partial}{\partial P_\phi}\overline{\left(\tau_b \delta \dot{P}_\phi \delta F\right)}_z + \frac{\partial}{\partial \mathcal{E}}\overline{\left(\tau_b \delta \dot{\mathcal{E}} \delta F\right)}_z\right]_S = \overline{\left(\sum_b C_b^g\left[F, F_b\right] + \mathcal{S}\right)}_{z\,S}$$

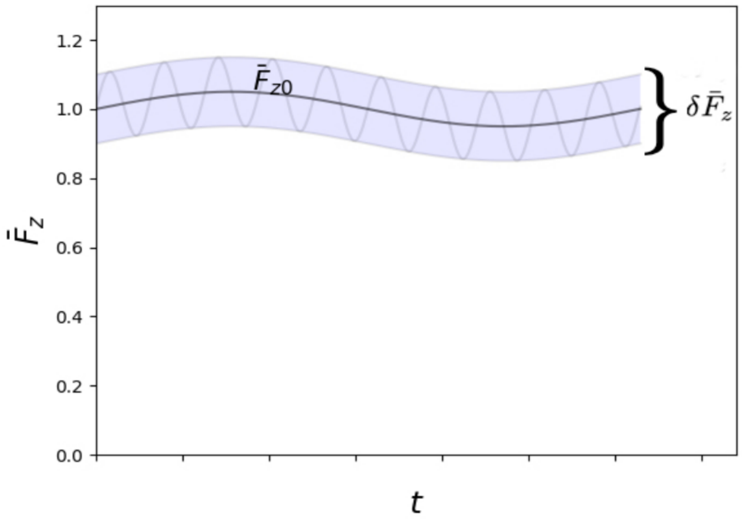
where $\overline{(\dots)} = \tau_b^{-1}\oint d\theta/\dot{\theta}\,(\dots) = \oint d\theta/\dot{\theta}\, e^{iQ_z}(\dots)(\bar{\psi},\theta)$, with $\tau_b = \oint d\theta/\dot{\theta}$ and $\psi = \overline{\psi} + \delta\tilde{\psi}(\theta)$;

- This expression describe transport processes in the phase space due to fluctuations, collisions and sources.

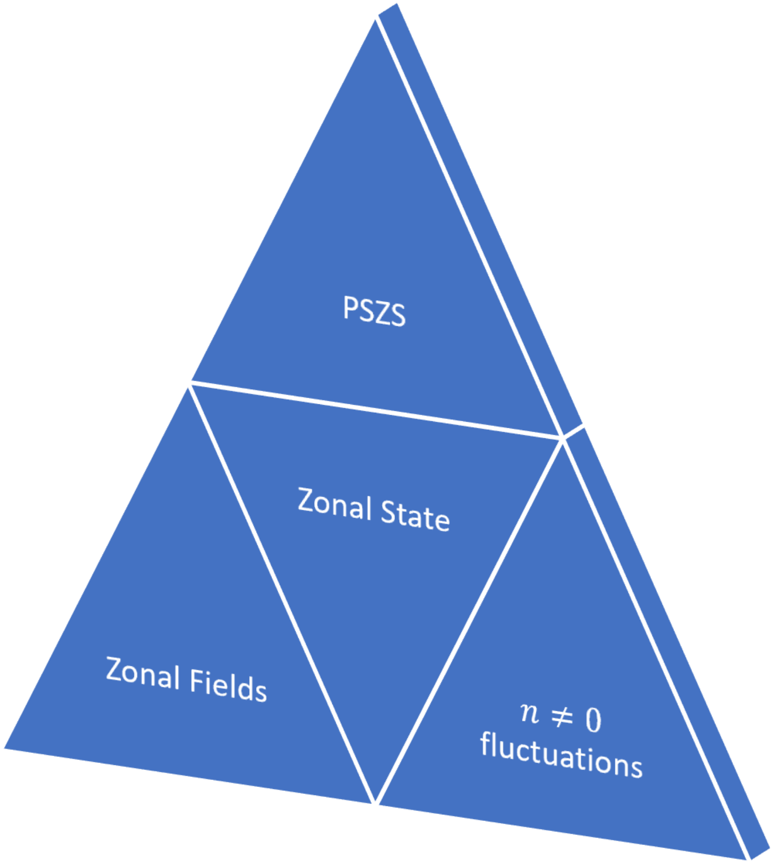
Neighboring nonlinear equilibria

- Having defined Phase Space Zonal Structures, we can decompose the toroidally symmetric distribution function;
- $\overline{F_{z0}}$ describe macro- & meso-scales;
- micro-scales are accounted by $\delta\overline{F}_z$;
- they describe system transitions between neighboring nonlinear equilibria, see Chen and Zonca 2007; Falessi and Zonca 2019;
- nonlinear equilibria, together with zonal fields, form a zonal state.

$$F_z = \overline{F_{z0}} + \delta\bar{F}_z + \delta\tilde{F}_z$$



Zonal state



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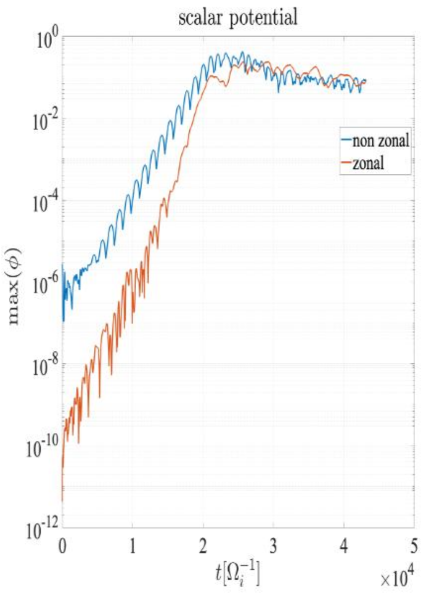

Figure 10: BAE time evolution

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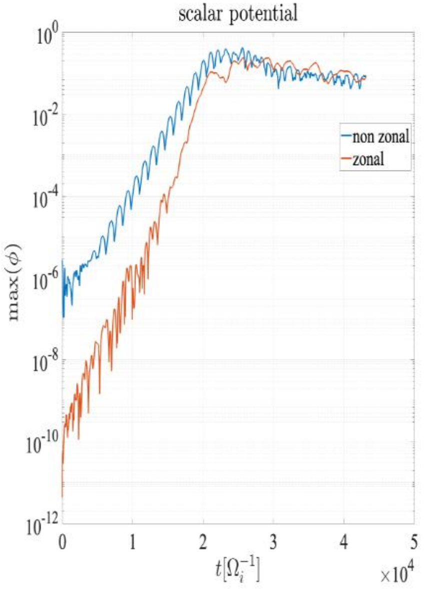

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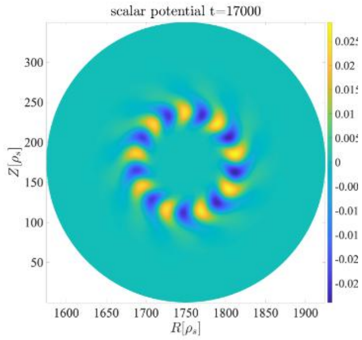

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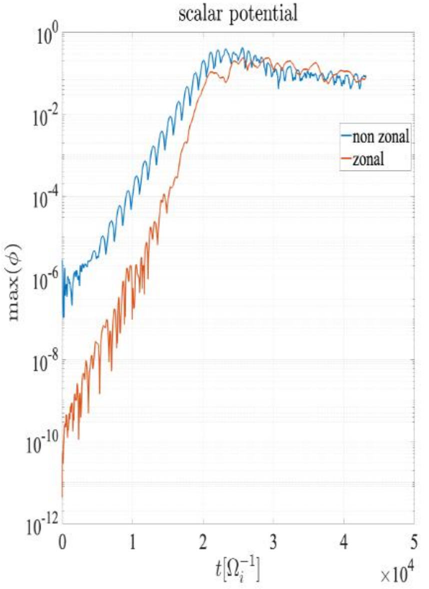

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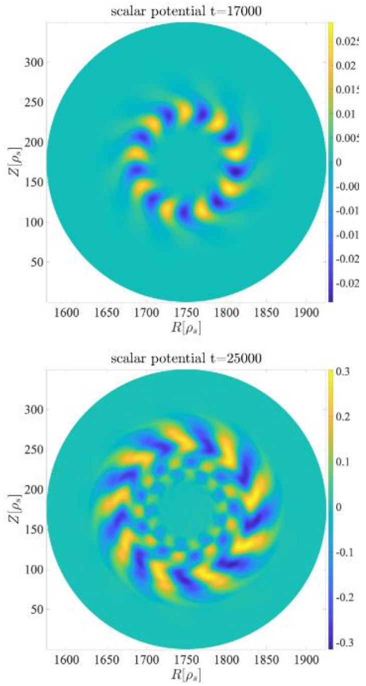

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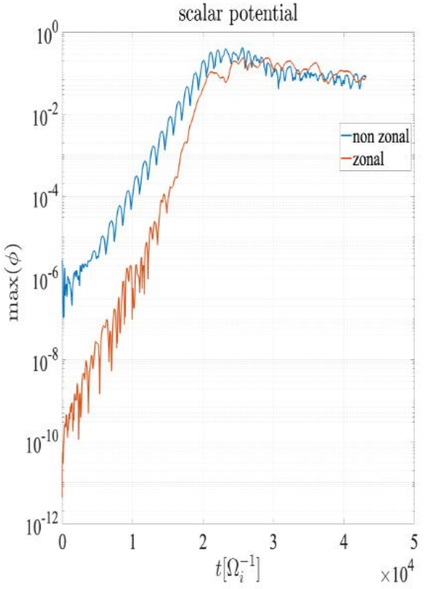

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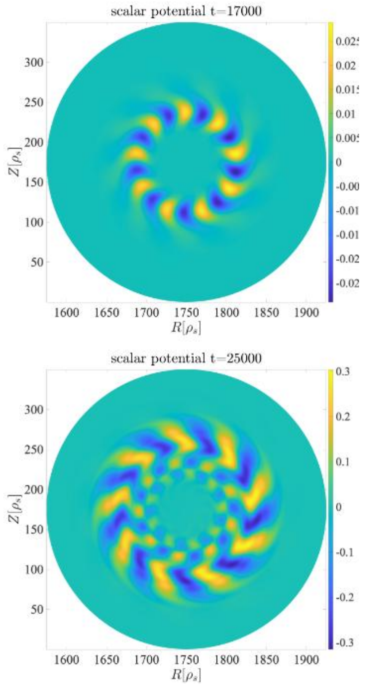

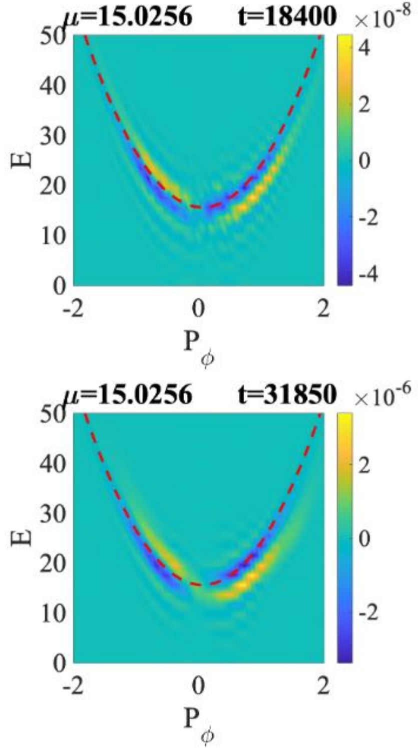

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Reduced transport models

- PSZS transport is studied by means of a hierarchy of verified and validated reduced models within an EUROfusion Enabling Research Project with P. Lauber as P.I.;

- explicit expression of EP fluxes in PSZS equations have been calculated within the following hierarchy of simplifying assumptions:

- the zeroth level of simplification consist in the gyrokinetics description of plasma dynamics;

- the first level of simplification consist in assuming $|\omega| \gg \tau_{NL}^{-1} \sim \gamma_L$, Zonca et al 2021;

- the second and final level of simplification is the quasilinear model

Reduced transport models

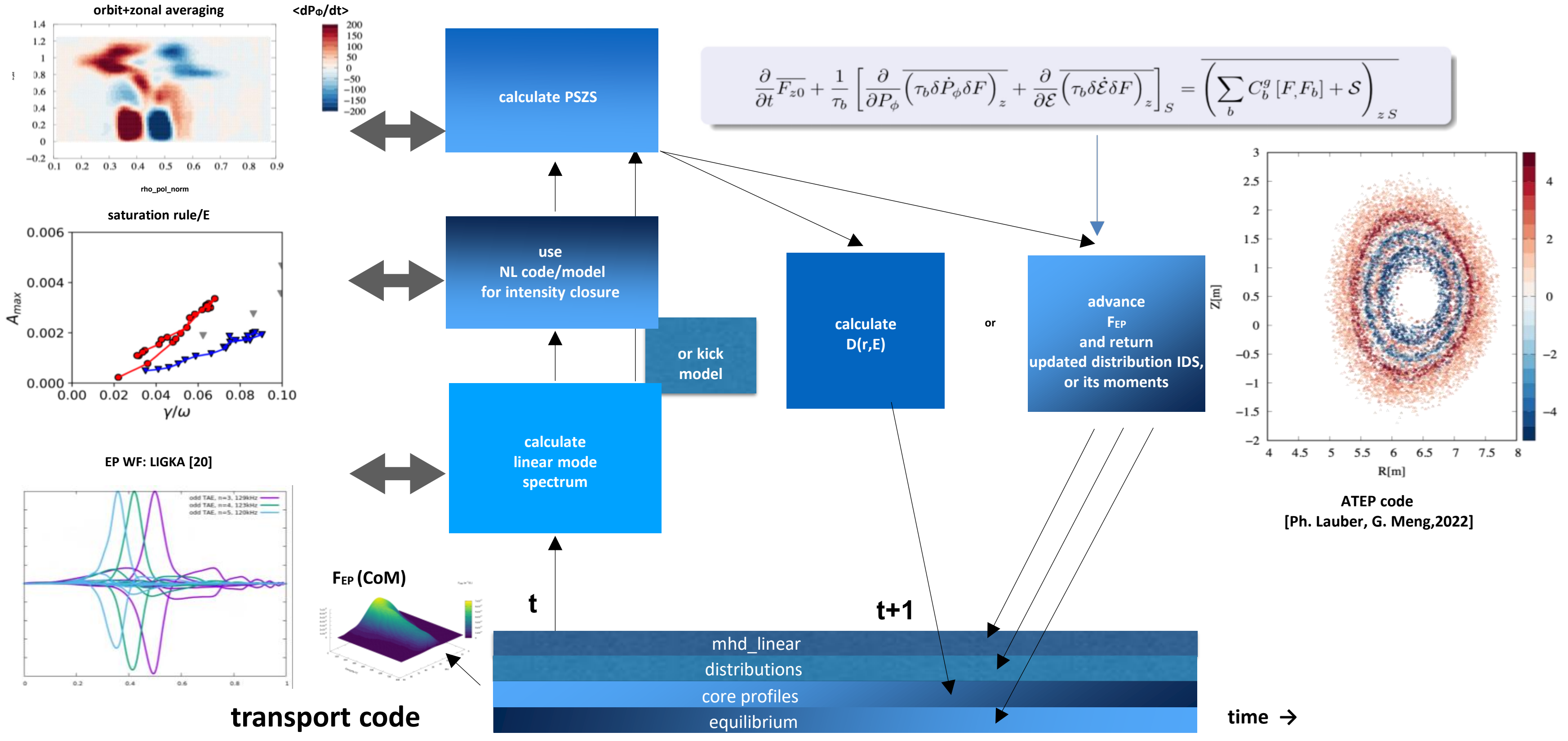
DAEPS (Y. Li et al 2020, G. Wei et al 2025)

- Ballooning decomposition for fluctuations;
- Based on fish-bone like dispersion relation;
- Mode structure decomposition, separation of radial envelope and parallel mode structure;
- Calculate nonlinear fluxes by the DSM model or a saturation rule.

LIGKA-HAGIS (Lauber et al 2007)

- Fourier decomposition for fluctuations;
- Solve linear gyrokinetic equation;
- Assume fluctuations amplitude, e.g., kick-model or quasilinear;
- Use IMAS-coupled EP stability WF (HAGIS/LIGKA) to calculate Phase Space Zonal Structures fluxes.

ATEP: kick model limit



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time

Phase Space fluxes

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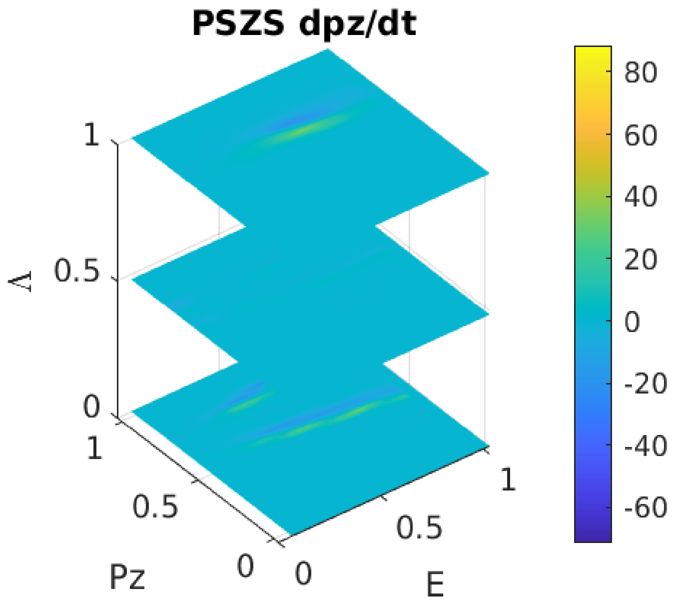

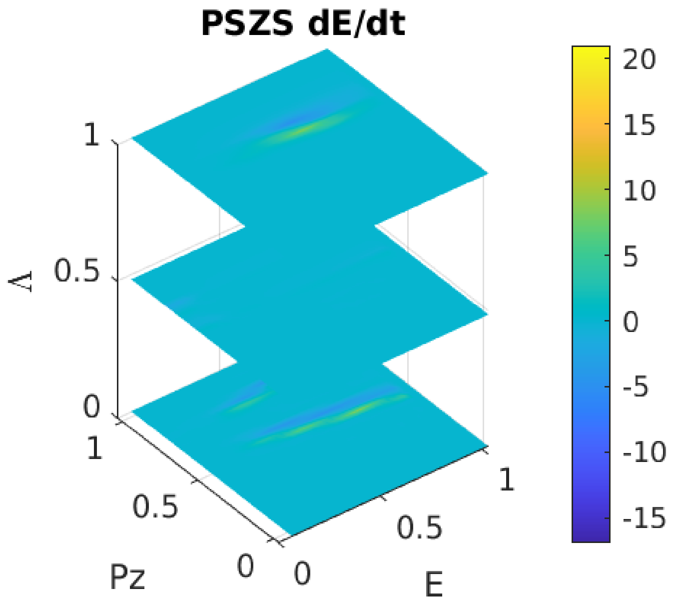

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PSZS evolution

ATEP code: solve transport equation for PSZS with sources and collisions, Lauber 2022

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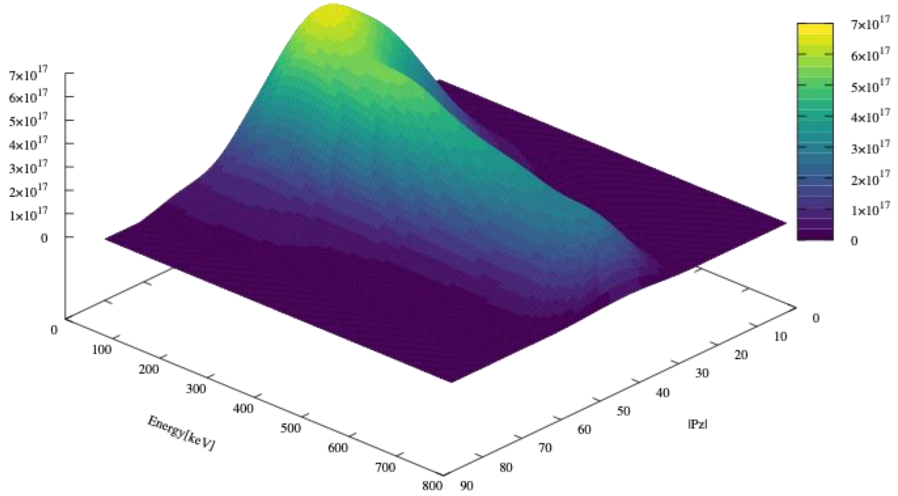
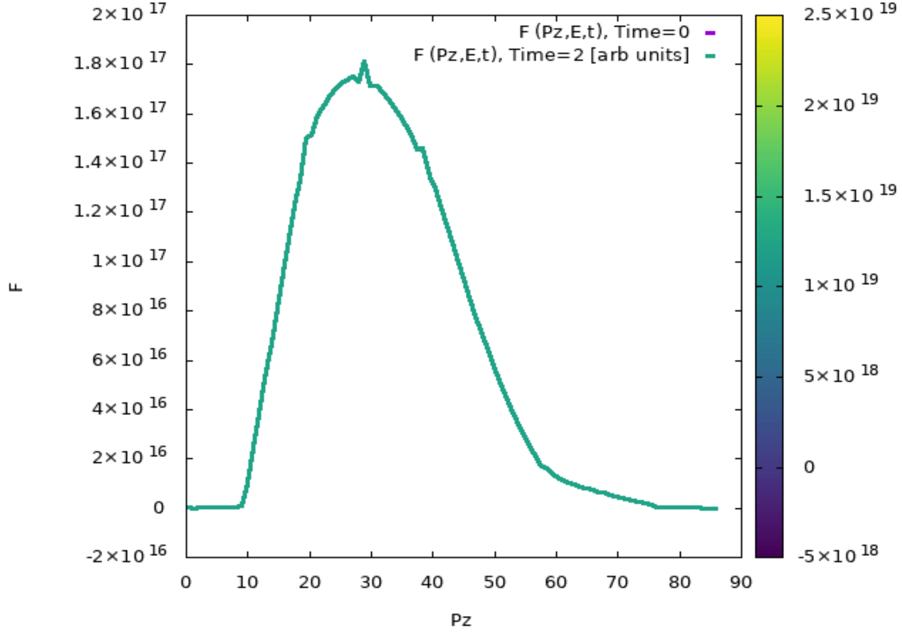
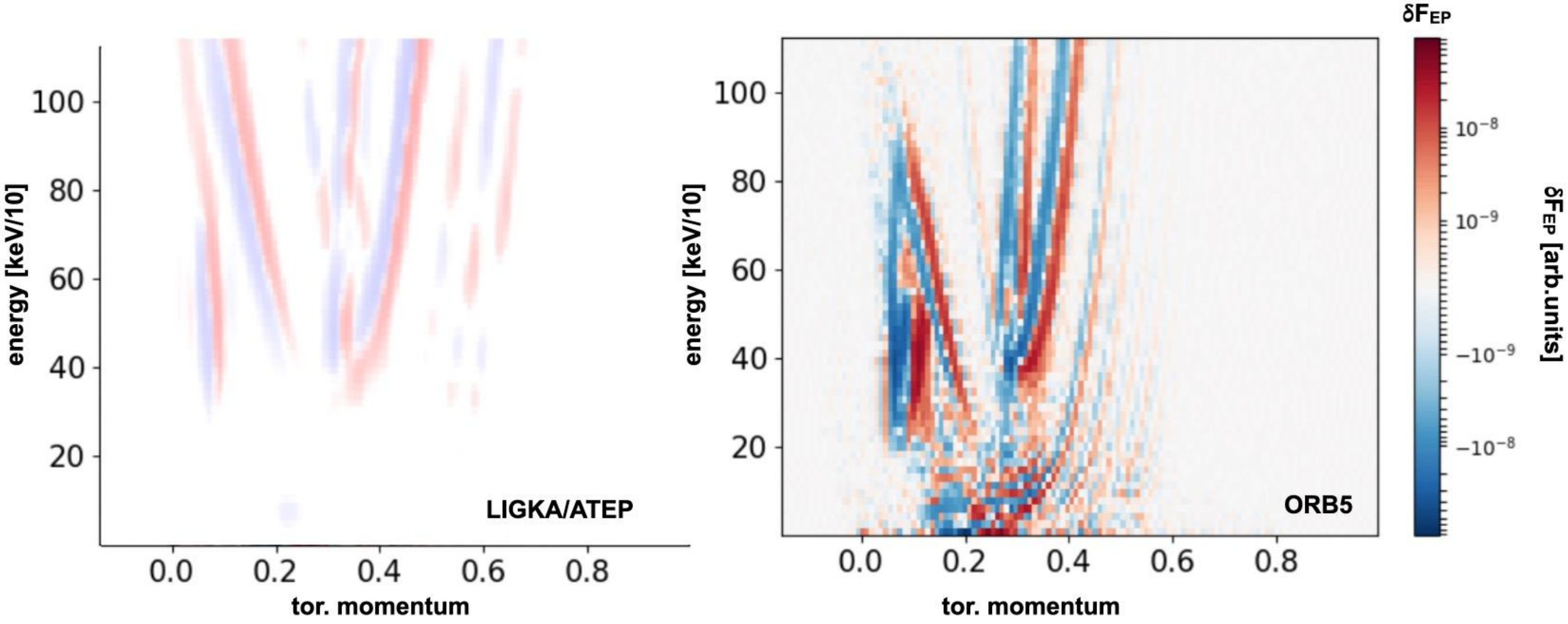




ITER plasma from H&CD WF by M. Schneider

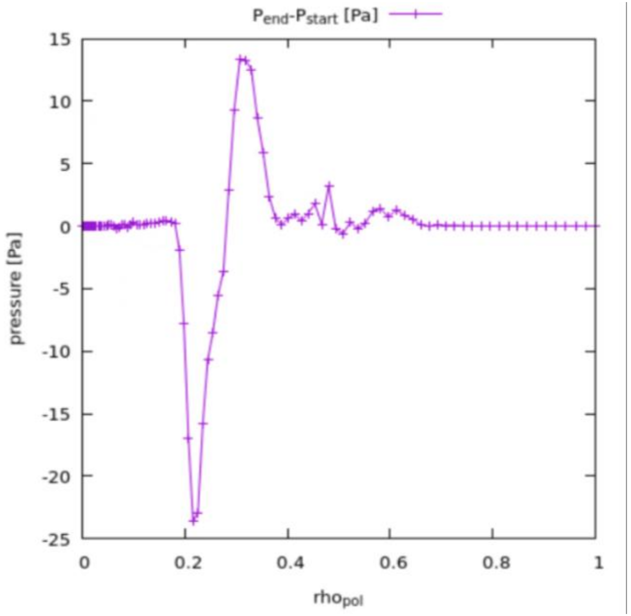
PSZS moments

- Phase-space fluxes computed with ATEP 3D and ORB5 demonstrate a good level of consistency



PSZS moments

- Mapping to 1d profile of PSZS;
- The CGL equilibrium can be readily constructed Falessi et al NJP 2023;



PSZS moments

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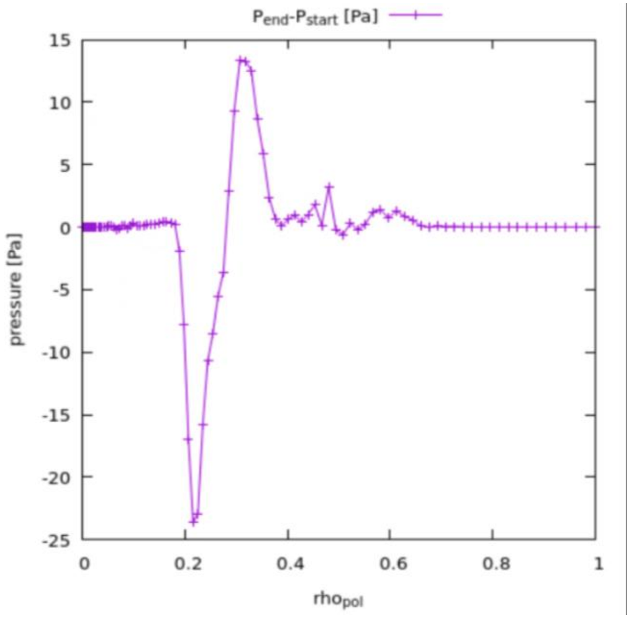
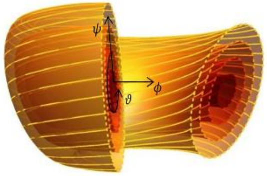
- Next step is the calculation of the corresponding magnetic equilibrium;

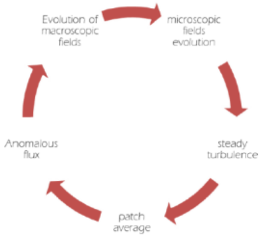
$$\sigma \frac{\mathbf{J}_z \times \mathbf{B}}{c} = \nabla P_{\parallel} + (\sigma - 1)\nabla\left(\frac{B^2}{8\pi}\right) + \frac{B^2}{4\pi}\nabla_{\perp}\sigma$$



Summary & Conclusions

- Need of predictive transport models on timescales comparable with magnetic fusion experiments, i.e., ITER, properly describing EPs;





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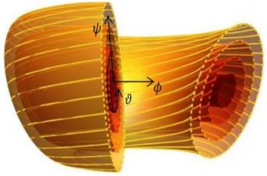

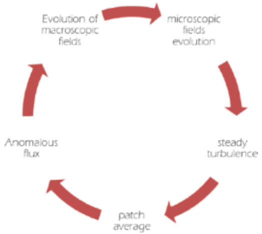

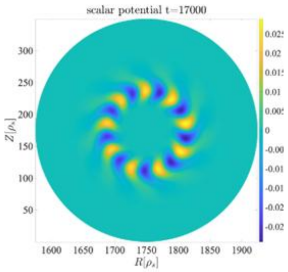

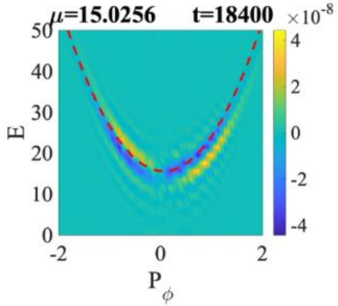

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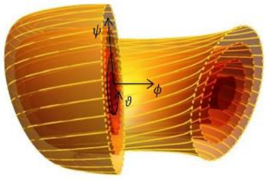
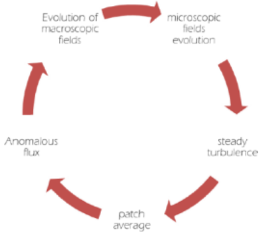
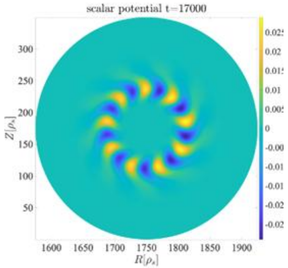
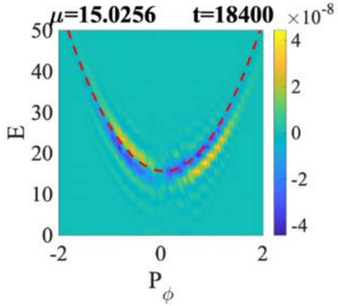
- we have introduced the concept of zonal state to describe the evolution of the plasma between neighboring nonlinear equilibria;









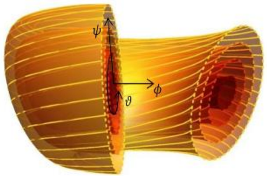
Summary & Conclusions

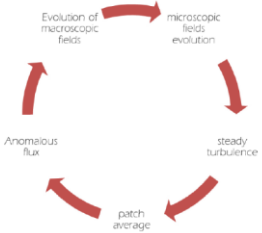
- Need of predictive transport models on timescales comparable with magnetic fusion experiments, i.e., ITER, properly describing EPs;

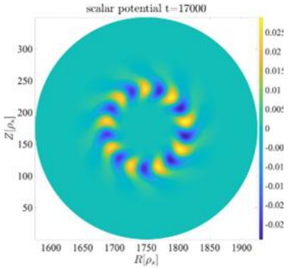
- we have shown how to describe meso-scales and non-Maxwellian distribution functions using appropriate phase transport equations;

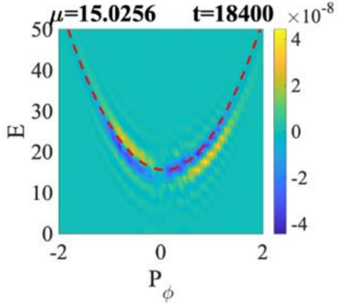
- we have introduced the concept of zonal state to describe the evolution of the plasma between neighboring nonlinear equilibria;

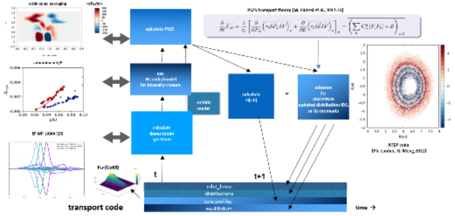
- we have introduced a framework which allow to study EP phase space transport over long time scales.

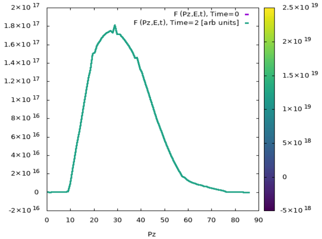




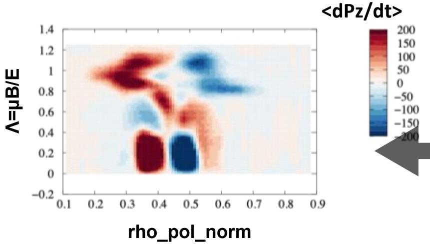




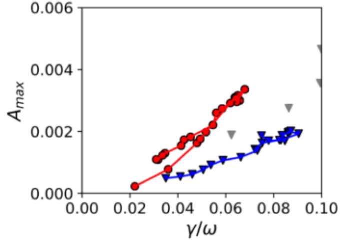




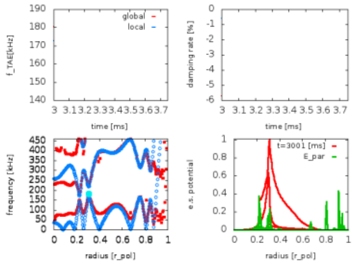
ATEP: Quasilinear model



saturation rule: HAGIS or ORB5



EP WF (LIGKA)



calculate PSZS

use NL code/model for intensity closure

calculate linear mode spectrum

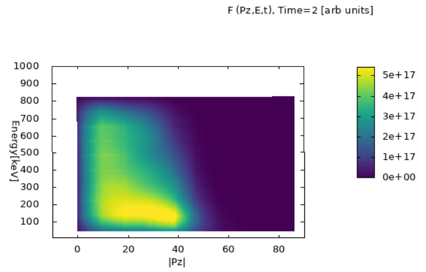
ATEP code: solve transport equation for PSZS with sources and collisions Lauber 2022

$$\frac{\partial}{\partial t}\overline{F_{z0}} + \frac{1}{\tau_b}\left[\frac{\partial}{\partial P_\phi}\overline{\left(\tau_b \delta\dot{P}_\phi \delta F\right)}_z + \frac{\partial}{\partial \mathcal{E}}\overline{\left(\tau_b\delta\dot{\mathcal{E}}\delta F\right)}_z\right]_S = \overline{\left(\sum_b C_b^g\left[F,F_b\right] + \mathcal{S}\right)}_{z\,S}$$

calculate D(r,E)

or

advance F_{EP} and return updated distribution IDS, or its moments



t

t+1

core_transport

equilibrium

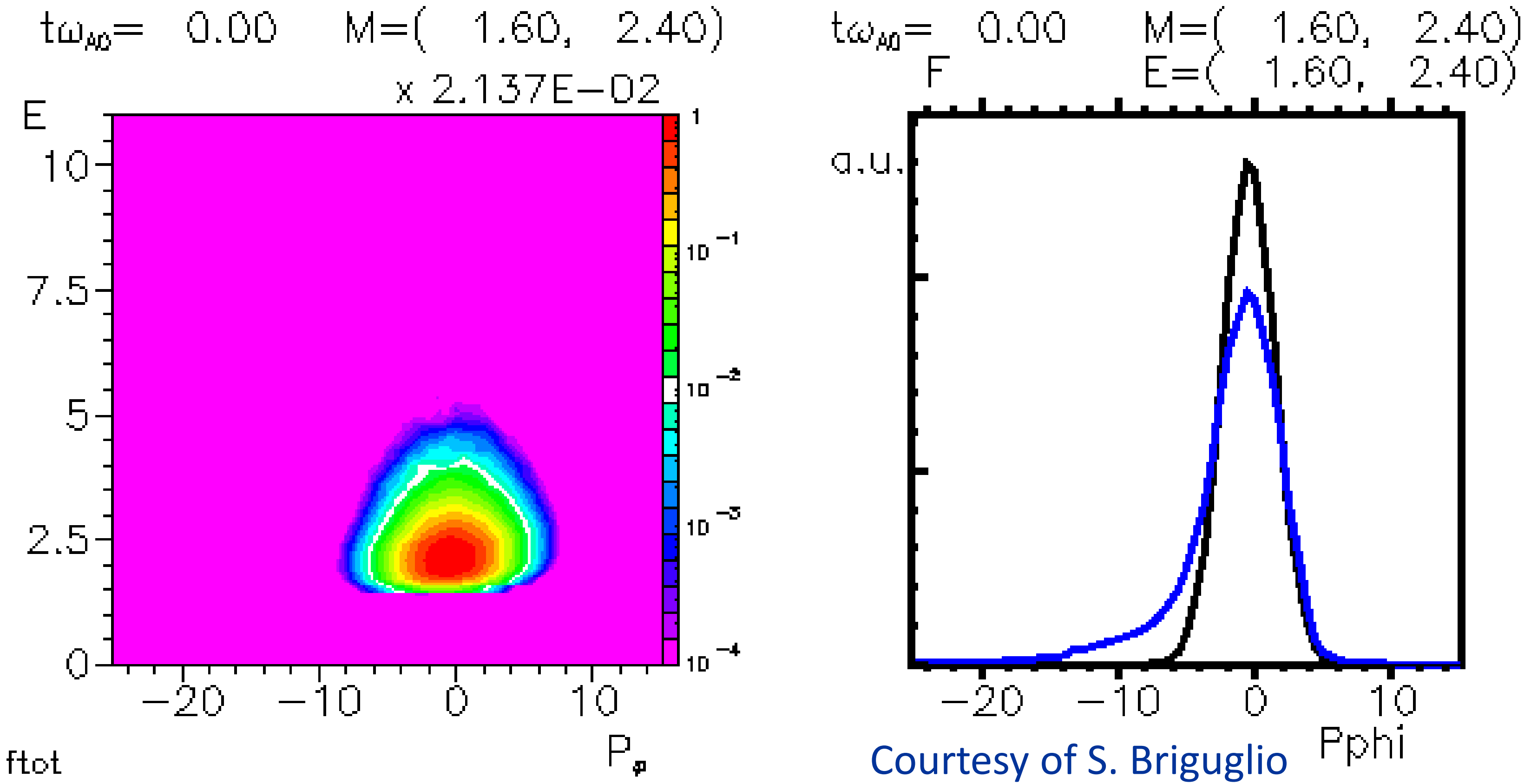
mhd_linear

distributions

transport code

time

PSZS diagnostic in HMGC



Courtesy of S. Briguglio

Zonal fields: macro- /meso- scopic component

- the current $\mathbf{J}_z$ and the pressure tensor $\mathbf{P}$ satisfy the following force balance equation:

$$\sigma\frac{\mathbf{J}_z\times\mathbf{B}}{c}=\boldsymbol{\nabla}P_{\parallel}+(\sigma-1)\boldsymbol{\nabla}\left(\frac{B^2}{8\pi}\right)+\frac{B^2}{4\pi}\boldsymbol{\nabla}_{\perp}\sigma$$

where $\sigma=1+\frac{4\pi}{B^2}(P_{\perp}-P_{\parallel})$;

- the self-consistent modification of the equilibrium magnetic field due to PSZS can be calculated solving this equation, e.g.:

$$\Delta^{*}\psi+\boldsymbol{\nabla}\ln\sigma\cdot\boldsymbol{\nabla}\psi=-\frac{4\pi R^2}{\sigma}-\frac{1}{\sigma^2}\frac{\partial G}{\partial\psi}$$

- where $G\equiv\sigma F^2/2$ is a flux function;

Reduced transport models

- PSZS transport is studied by means of a hierarchy of verified and validated reduced models within an EUROfusion Enabling Research Project with P. Lauber as P.I.;

- explicit expression of EP fluxes in PSZS equations have been calculated within the following hierarchy of simplifying assumptions:

- the zeroth level of simplification consist in the gyrokinetics description of plasma dynamics;

- the first level of simplification consist in assuming $|\omega| \gg \tau_{NL}^{-1} \sim \gamma_L$, Zonca et al 2021;

- the second and final level of simplification is the quasilinear model

Transport processes in burning plasmas

- EPs are generally non Maxwellian, and transport processes take place in the phase space!
- Separation of scales is questionable…
- Particularly important for ITER physics;
- interplay of meso-scale structures with micro-scales generated by EPs, i.e., $\rho_{LE} \sim (\rho_L L)^{1/2}$, Zonca et al 2015;

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- Particularly important for ITER physics;
- interplay of meso-scale structures with micro-scales generated by EPs, i.e., $\rho_{LE} \sim (\rho_L L)^{1/2}$, Zonca et al 2015;
- interaction with thermal plasma over long timescales can modify bulk transport processes;
- the derivation, see Falessi 2017; Falessi and Zonca 2019, is based on the theory of Phase space zonal structures (PSZS), see Chen and Zonca 2016.

Zonal fields

- We have shown how to calculate an evolving renormalized distribution function consistent with the finite level of fluctuations; this is connected to the evolution of a macro-/meso- scopic corresponding CGL equilibrium …

- Following Cary & Brizard 2009 , we write every moment of the zonal distribution function:

$$\mathbf{J}_z = e\int d\mathcal{E}d\mu d\alpha d^3\mathbf{X}D(T^{-1}\mathbf{v})\delta(\mathbf{X}+\boldsymbol{\rho}-\mathbf{r})\left[F_0+\delta F_z-\frac{e}{m}\langle\delta L_g\rangle_z\frac{\partial}{\partial\mathcal{E}}F_0-\frac{e}{m}\frac{\partial}{\partial\mu}F_0\langle\delta L_g\rangle_z\right] + \frac{e^2}{m}\int d\mathcal{E}d\mu d\alpha d^3\mathbf{X}D\left[\frac{\partial}{\partial\mathcal{E}}F_0\delta\phi_z+\frac{1}{B_0}\frac{\partial}{\partial\mu}F_0\delta L_z\right]$$

- we can substitute F_z for the Phase Space Zonal Structures $\bar{F}_{z0}=\left[F_0+e^{-iQ_z}\delta\bar{F}_{Bz}\right]_S$;

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$$+\frac{e^2}{m}\int d\mathcal{E}d\mu d\alpha d^3\mathbf{X}D\left[\frac{\partial}{\partial\mathcal{E}}F_0\delta\phi_z+\frac{1}{B_0}\frac{\partial}{\partial\mu}F_0\delta L_z\right]$$

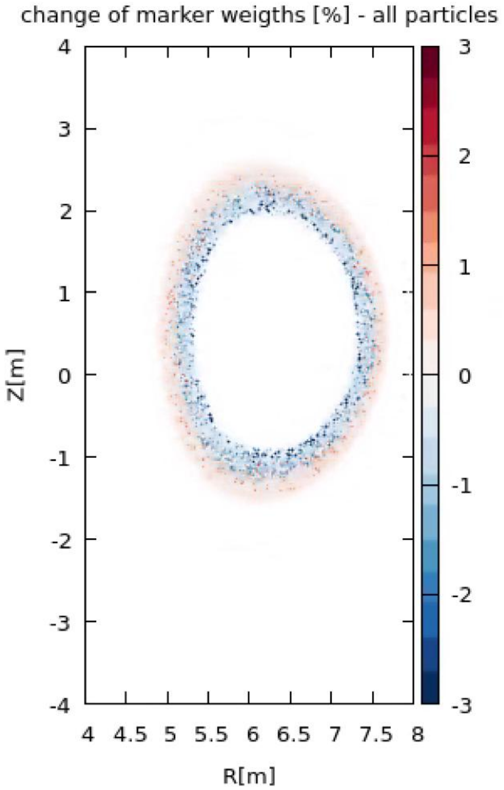
- we can substitute F_z for the Phase Space Zonal Structures $\bar{F}_{z0}=\left[F_0+e^{-iQ_z}\delta\bar{F}_{Bz}\right]_S$;

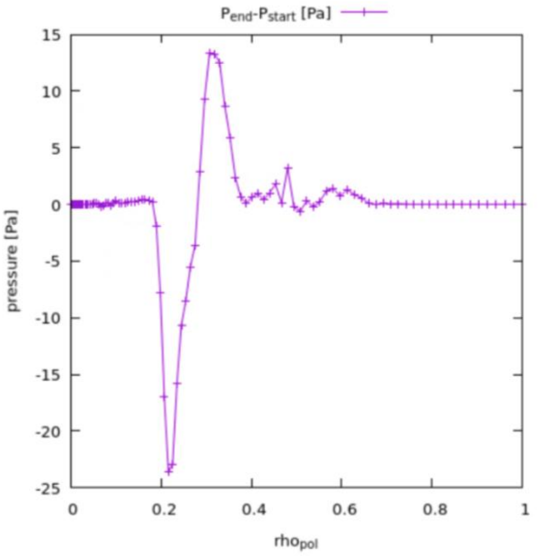
- we obtain, from a multipole expansion, an equilibrium consistent with a CGL pressure tensor:

$$\sigma\frac{\mathbf{J}_z\times\mathbf{B}}{c}=\nabla P_\parallel+(\sigma-1)\nabla\left(\frac{B^2}{8\pi}\right)+\frac{B^2}{4\pi}\nabla_\perp\sigma \qquad \Delta^*\psi+\nabla\ln\sigma\cdot\nabla\psi=-\frac{4\pi R^2}{\sigma}-\frac{1}{\sigma^2}\frac{\partial G}{\partial\psi}$$

PSZS moments

- Mapping to 1d profile of PSZS;
- The CGL equilibrium can be readily constructed Falessi et al 2023 sub;
- transport is zonal by construction;
- Next step is the calculation of the corresponding magnetic equilibrium;





ATEP code: solve transport equation for PSZS with sources and collisions, Lauber 2022

Purpose statement

We aim at 1. providing the general expressions describing EPs (plasma) dynamics on long time scales (transport) and 2. introducing a framework to solve these equations within different levels of reduced dynamics.

By means of this approach it will be possible to:

- define the concept of nonlinear equilibrium;
- describe the physics of burning plasmas where alpha particles will play a key role in transport studies by interacting with thermal components;
- the derivation, see Falessi 2017; Falessi and Zonca 2019, is based on the Phase space zonal structures theory, see Chen and Zonca 2016.

Neighboring nonlinear equilibria

- Nonlinear gyrokinetic simulations with ORB5 code demonstrate clear reduction in the heat flux for both the bulk ions and the electrons