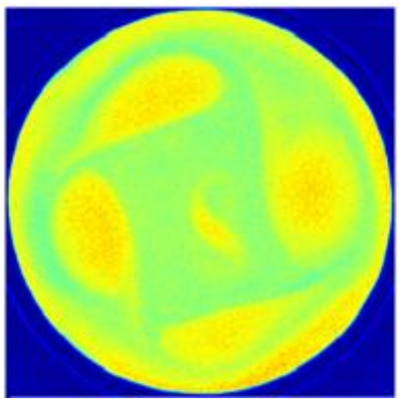
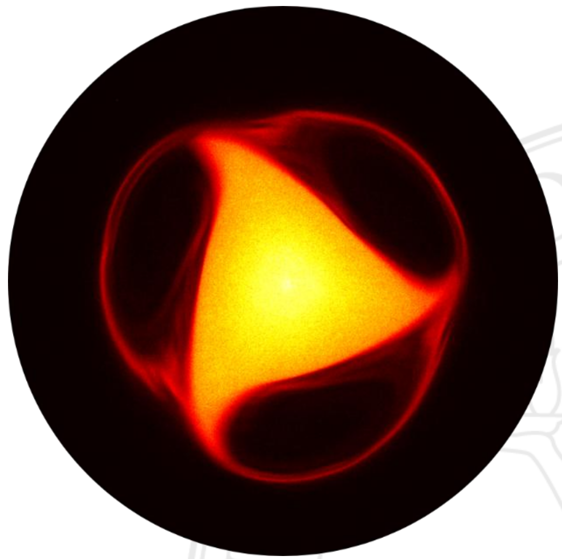
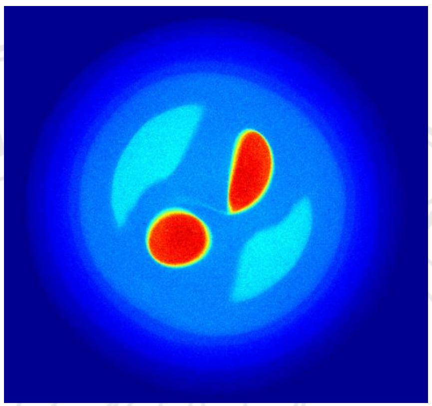
Dynamics of a two-dimensional fluid vortex explored via magnetized electron plasmas

Giancarlo Maero

Università degli Studi di Milano
Dipartimento di Fisica 'Aldo Pontremoli'

Starting from the end…

Several helping hands (and various body parts) were involved at different times and in different capacities:

• Nicola Panzeri (PhD), Samuele Altilia, Martino Bonisolli, Giordano Brighenti, Ludovico Casaccia, Guglielmo Canziani, Francesca Ferrero, Tommaso Fogliani, Davide Invernizzi, Lorenzo Lanzanova, Lorenzo Patricelli, Martino Zanetti, Matteo Zuretti (BSc and MSc thesis students)

• prof. Roberto Pozzoli (former group head), prof. Massimiliano Romé, dr. Marco Cavenago (INFN-LNL)

Funding: MIUR/MUR, UNIMI , INFN

Synopsis

Introduction

- 2D fluids and vorticity – non-axisymmetric vortices
- Trapped nonneutral plasmas as 2D fluids

Non-axisymmetric vortex dynamics

- V-state preparation through Kelvin-Helmholtz wave excitation
- Forced (resonant and autoresonant) dynamics and free evolution

Weirdos – RF-generated plasmas and self-organizing structures

- Coherent structures wih sources/sinks during ionizing RF excitation

Conclusions and outlook

G. Maero et al., 'Fundamental physics and applications using nonneutral plasma', Adv. Phys.: X, 2024

Introduction

2D fluid vortices

Trapped nonneutral plasmas

Fluid – NNP analogy

2D ideal incompressible fluids and vortices

Many examples of 2D fluids in nature – often geophysical systems

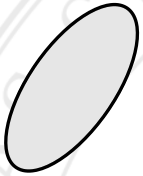
$$\begin{cases} \boldsymbol{u}(\boldsymbol{x},t) = \nabla\psi(\boldsymbol{x},t) \times \hat{\boldsymbol{e}}_z & [\,\psi \textit{ stream function }] \\ \frac{\partial}{\partial t}\zeta(\boldsymbol{x},t) + \boldsymbol{u}(\boldsymbol{x},t)\cdot\nabla\zeta(\boldsymbol{x},t) = 0 & [\,\zeta = (\nabla\times\mathbf{u})_z \textit{ vorticity }] \\ \nabla^2\psi(\boldsymbol{x},t) = -\zeta(\boldsymbol{x},t) & \end{cases}$$


Kelvin-Helmholtz clouds

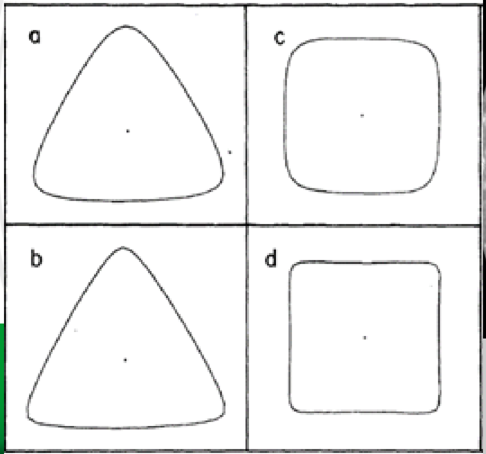

Satellite view of a hurricane[NASA]

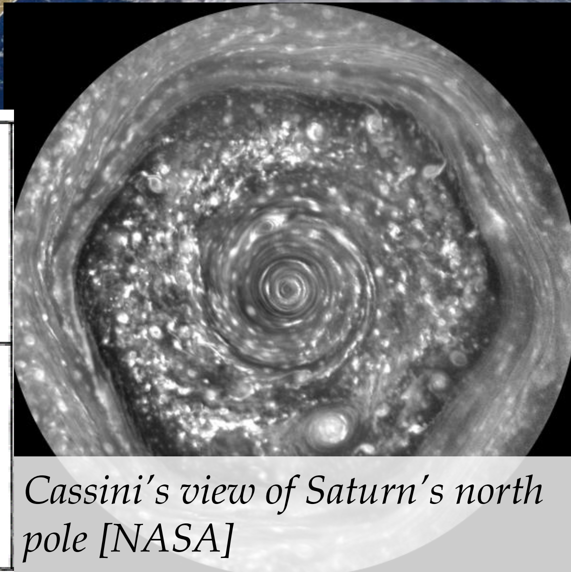
Nonlinear long-lived structures arise in strain fields, e.g. Kelvin-Helmholtz wave perturbation (velocity shear)

Kirchhoff vortex: elliptical patch of uniform vorticity can be indefinitely stable (solution to Euler eq.)



V-state: generalization to any deformation order (*l*-fold symmetric vortex) also stable (*Deem & Zabusky, PRL 1978 and further*)




Cassini's view of Saturn's north pole [NASA]

V-states: what we know, and what we don’t

Qualitatively, parameters of interest are:
vorticity/strain ratio on growth/evolution details
vorticity profile ("generalized" V-state) on growth and stability
wavenumber (*l*-mode) on stability range -> higher *l* are more fragile *[Zabusky]*

Quantitatively
for *l*=2 some theoretical modelling exists *[Kirchhoff, Kida, Saffman, Dritschel]*
for *l*>2 … not so much
mostly simulation and heuristic arguments, limited case studies

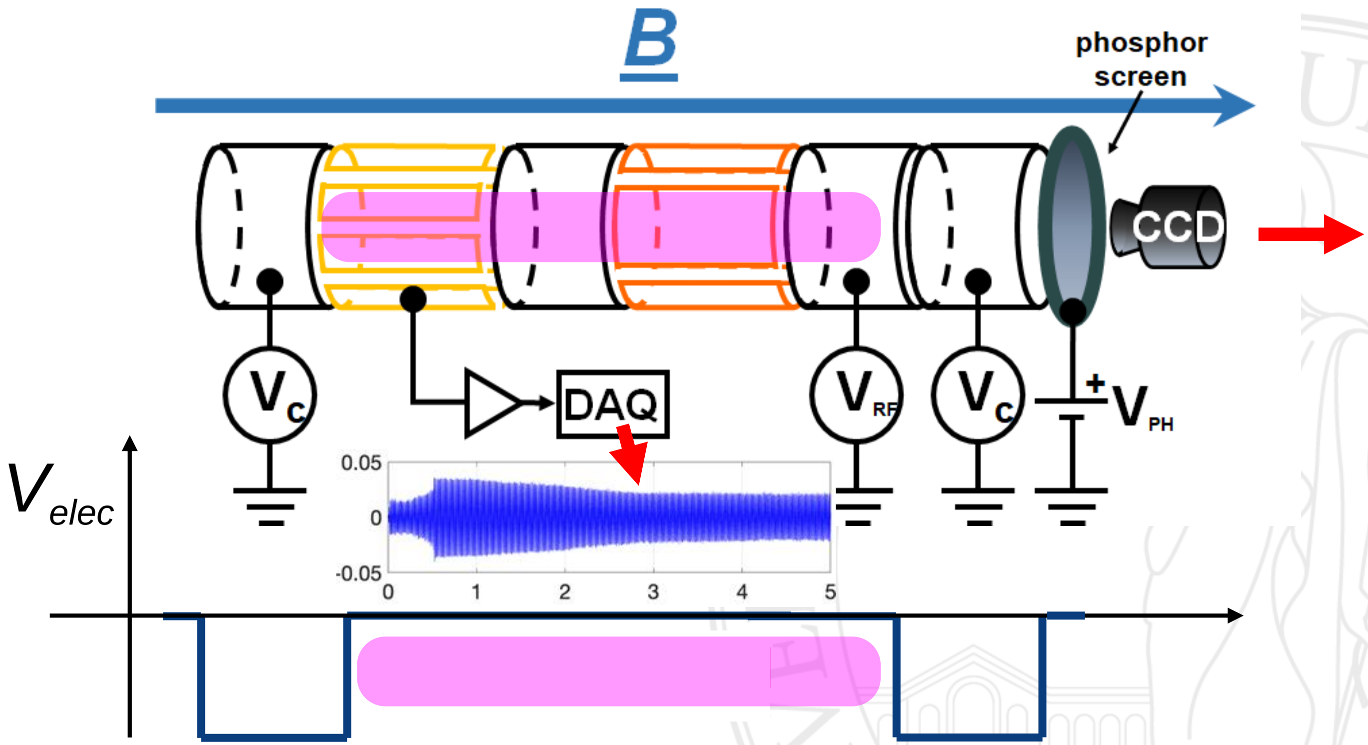
Experimentally: scarce laboratory reproducibility (3D effects, friction, weak parameter control…) → largely simulation work

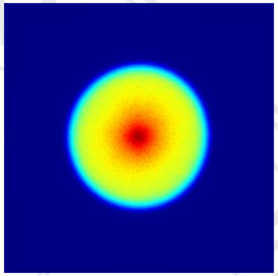
<u>Nonneutral plasmas are a better experimental environment</u>; coupled to modelling, they have been answering some questions
[UC San Diego, UC Berkeley, University of Kyoto… and Milano]

Nonneutral plasma - Penning-Malmberg trap

Nonneutral plasma (NNP): A plasma out of global charge balance – possibly single species (e.g., e^-) ***[→ also F. Romano's talk about T-REX]***

NNP confinement principle: Electrostatic potential well (axial trapping) + uniform magnetic field from solenoid (radial trapping) + HV/UHV (low collisionality and diffusion)

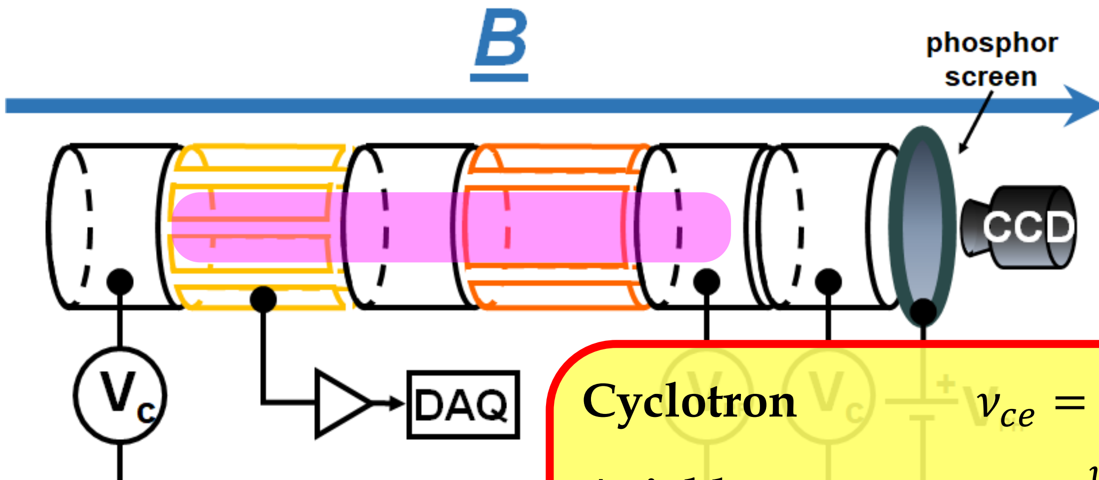


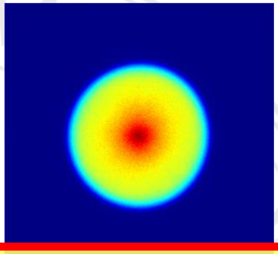


Nonneutral plasma - Penning-Malmberg trap

Nonneutral plasma (NNP): A plasma out of global charge balance – possibly single species (e.g., e$^-$) ***[→ also F. Romano's talk about T-REX]***

NNP confinement principle: Electrostatic potential well (axial trapping) + uniform magnetic field from solenoid (radial trapping) + HV/UHV (low collisionality and diffusion)





central clump in
rotational equilibrium

$$P_\theta \sim \frac{eB}{2c}\sum_{j=1}^{N} r_j^2 \sim < r^2 >$$

Cyclotron $\nu_{ce} = \frac{eB}{m_e 2\pi} \sim 1 - 10 \text{ GHz}$

Axial bounce $\nu_b = \frac{v_z}{2L} \sim 1 - 10 \text{ MHz}$

Slow ExB drift $\nu_{rot} = \frac{e n_e}{4\pi\varepsilon_0 B} \sim 1 - 100 \text{ kHz}$

$$\boldsymbol{\nu_{rot} \ll \nu_b \ll \nu_{ce}}$$

NNPs as 2D ideal incompressible fluids

Highly-magnetized plasma: average over axial and cyclotron motion, neglect inertial effects

plasma

$$\left.\begin{aligned} \frac{\partial}{\partial t} n(\boldsymbol{x},t) + \boldsymbol{V}(\boldsymbol{x},t)\cdot\nabla n(\boldsymbol{x},t) = 0 \\ \boldsymbol{V}(\boldsymbol{x},t) = -\frac{1}{B}\nabla\phi(\boldsymbol{x},t)\times\hat{\boldsymbol{e}}_z \\ \nabla^2\phi(\boldsymbol{x},t) = \frac{en(\boldsymbol{x},t)}{\varepsilon_0} \end{aligned}\right\}$$

$\longleftrightarrow$

2D ideal incompressible fluid

$$\left\{\begin{aligned} \frac{\partial}{\partial t}\zeta(\boldsymbol{x},t) + \boldsymbol{u}(\boldsymbol{x},t)\cdot\nabla\zeta(\boldsymbol{x},t) = 0 \\ \boldsymbol{u}(\boldsymbol{x},t) = \nabla\psi(\boldsymbol{x},t)\times\hat{\boldsymbol{e}}_z \\ \nabla^2\psi(\boldsymbol{x},t) = -\zeta(\boldsymbol{x},t) \end{aligned}\right.$$

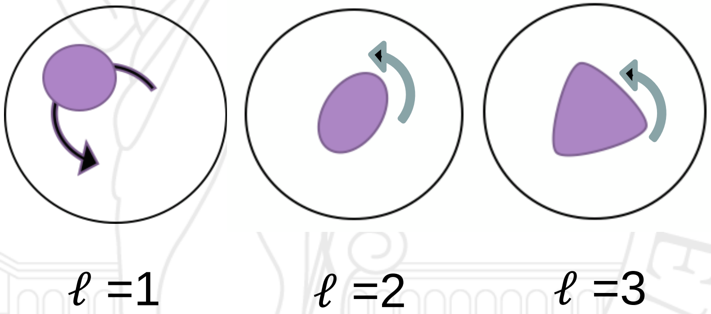
$\boldsymbol{V} \leftrightarrow \boldsymbol{u}$ $\qquad \phi/B \leftrightarrow \psi$ $\qquad en/\varepsilon_0 B \leftrightarrow \zeta$

Diocotron (Kelvin-Helmholtz) modes: Normal mode decomposition of non-axisymmetric density perturbations – **V-state is a nonlinear (high-amplitude) KH mode**

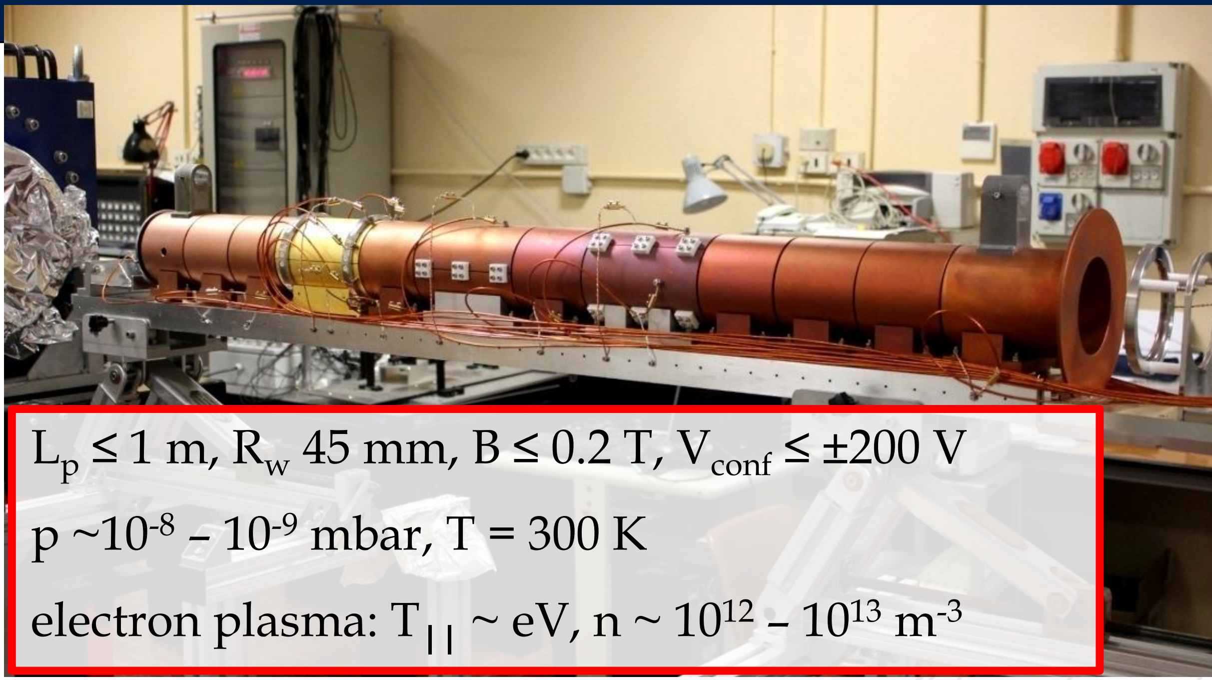
$$n(r,\theta,t) = n^0(r) + \sum_l \delta n^l(r)\exp(il\theta - i2\pi\boldsymbol{\Omega}_l t)$$

$$\boldsymbol{\omega}_l/2\pi = \nu_{rot}\left[l - 1 + \left(\frac{R_p}{R_W}\right)^{2l}\right]$$

(uniform density + linear regime)



Traps in Milano: ELTRAP/Eltrappino



$L_p \leq 1$ m, R_w 45 mm, $B \leq 0.2$ T, $V_{conf} \leq \pm 200$ V

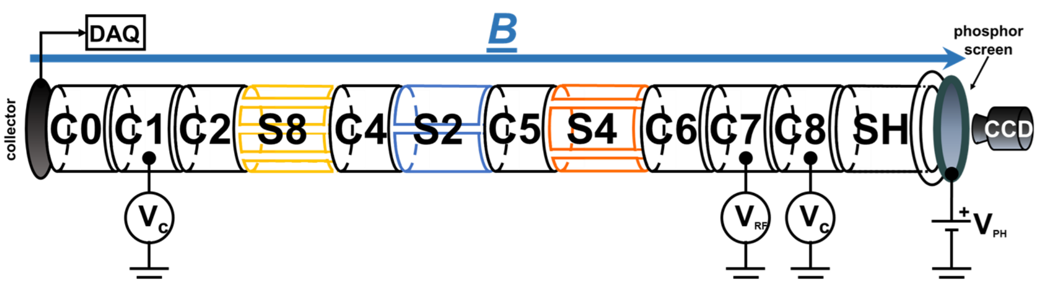
p ~10^{-8} – 10^{-9} mbar, T = 300 K

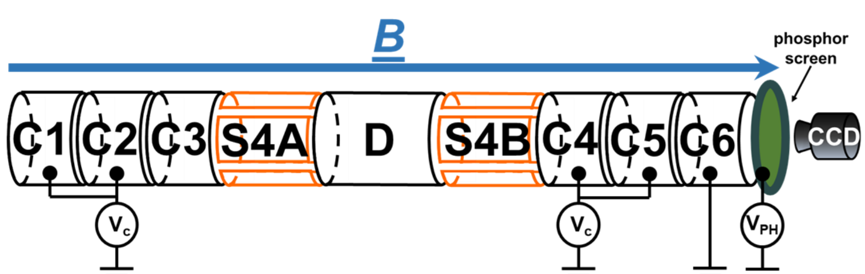
electron plasma: $T_{||}$ ~ eV, n ~ 10^{12} – 10^{13} m^{-3}

$L \leq 250$ mm, R_w 22.5 mm

$B \leq 0.88$ T, $|V_{conf}| \leq \pm 200$ V

n ~ 10^{13} – 10^{14} m^{-3}





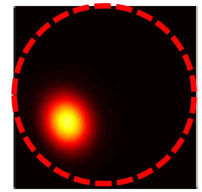
Non-axisymmetric vortex dynamics

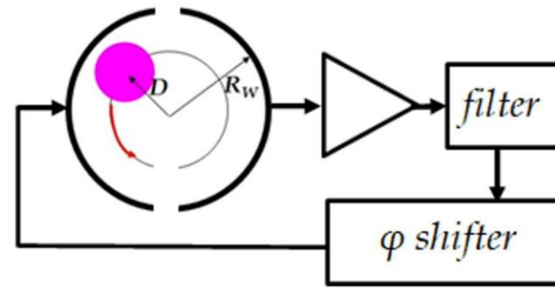
Non-axisymmetric vortices in a NNP

V-state forced and free dynamics

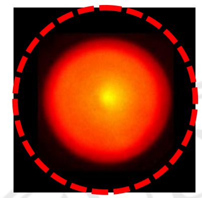
Vortex preparation: Density profile

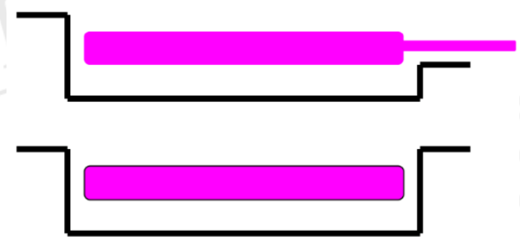
Electron plasma generation: off-axis vortex





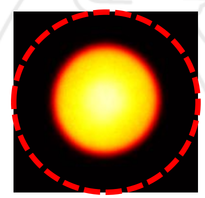
Circular vortex centering and relaxation

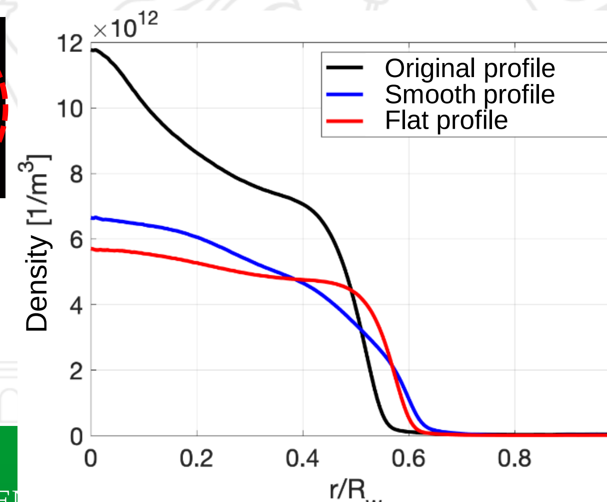




Profile manipulation: evaporation

(and again relaxing to equilibrium)

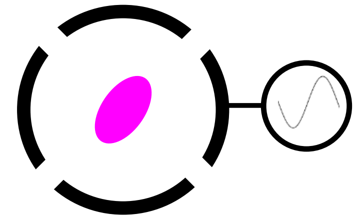




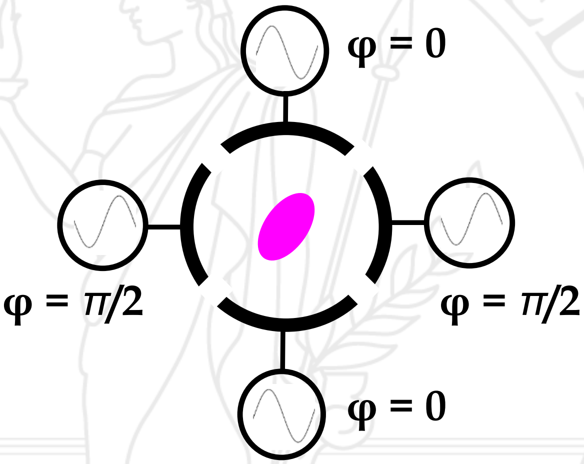
Result: tailored profile (from <u>central clump</u>, to <u>smooth</u>, to <u>flat</u>)

How to excite a single diocotron mode

- ‘Straightest’ way: One sector at resonant ω_l
- Limit: Not very effective for high l (limited deformation amplitude)

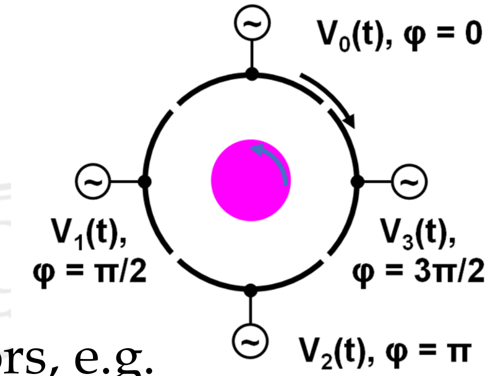


- ‘Ideal’ way: For l-th mode, apply corresponding multipole field on 2l-split electrode at the resonant ω_l
- Limit: $l \leq \pi/\Delta\theta$ ($\Delta\theta$ = sector span) or $l \leq N/2$ (N = # sectors)



Rotating-field mode excitation

- Linearized perturbation theory for flat profile: Given N sectors and a multipolar rotating/oscillating field, resonant modes are found
- Example: N = 4, with proper φ
  → either co-rotating/counter-rotating dipole or oscillating quadrupole at frequency ω_{dr}
  resonant at $\omega_{dr} = \omega_l$ with l = 4k+1 // 4k+2 // 4k+3



- The result is generalized for any number N of sectors, e.g.

***N* = 4**, $\Delta\varphi = \pi/2$

$$\begin{cases} l = 4k+1, \ \sigma = -1 \\ l = 4k+3, \ \sigma = +1 \\ l = 4k+2 \ (\sigma = \pm 1) \end{cases}$$

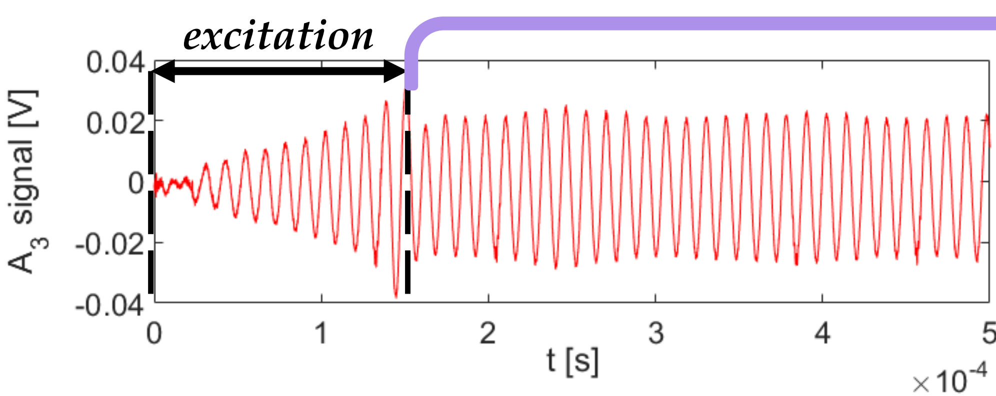
***N* = 8**

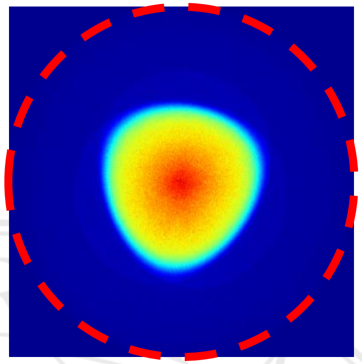
	σ=-1 (co-rotating)		σ=+1 (counter-rotating)	
[D]	$\Delta\varphi = -\pi/4$,	l = 8k+1	$\Delta\varphi = \pi/4$,	l = 8k+7
[Q]	$\Delta\varphi = -\pi/2$,	l = 8k+2	$\Delta\varphi = \pi/2$,	l = 8k+6
[S]	$\Delta\varphi = -3\pi/4$,	l = 8k+3	$\Delta\varphi = 3\pi/4$,	l = 8k+5
[O]	$\Delta\varphi = -\pi$,	l = 8k+4	$\Delta\varphi = \pi$,	l = 8k+4

Mode growth general features

Case study: *l*=3 mode

At resonance, with small perturbation $V_{dr} / \Phi_p < 5 \cdot 10^{-2}$
mode growth to nonlinear stage ~ few ten $\boldsymbol{\tau_{rot}}$





Optical detection: Fourier analysis over θ; total mode amplitude $\boldsymbol{A_l}$

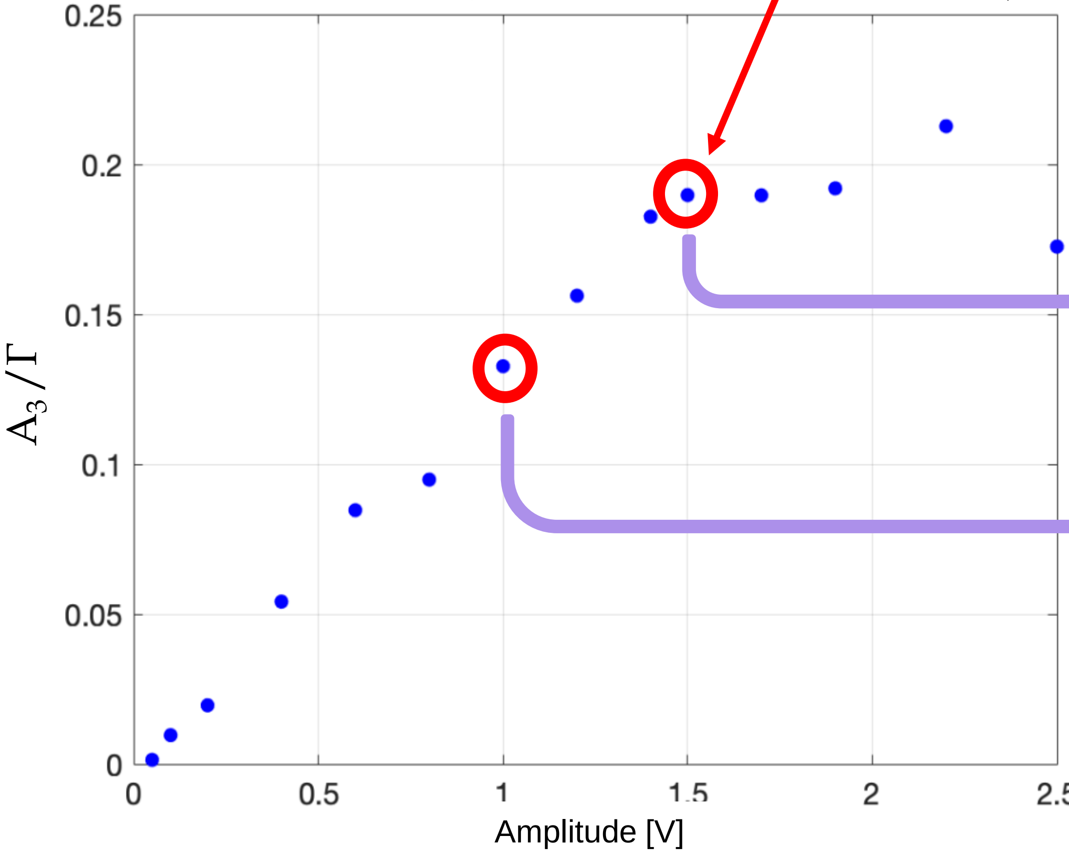
$$A_l = 2\pi \sum_{i=0}^{R_W} |\delta n^l(r)| r_i$$ δn^l perturbation at radius r

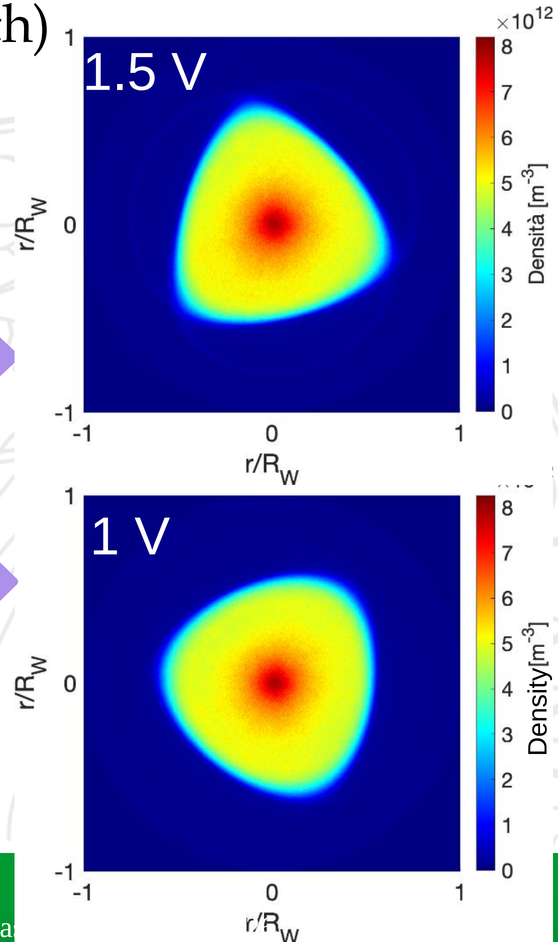
Initially, mode amplitude is linear with excitation time and amplitude

Mode growth general features

Even with small perturbation, **critical amplitude** can be reached
→ **filamentation**, mode **collapse and cascade** to lower modes

exc. time ~ 20 τ_{rot} Γ total circulation (charge/length)

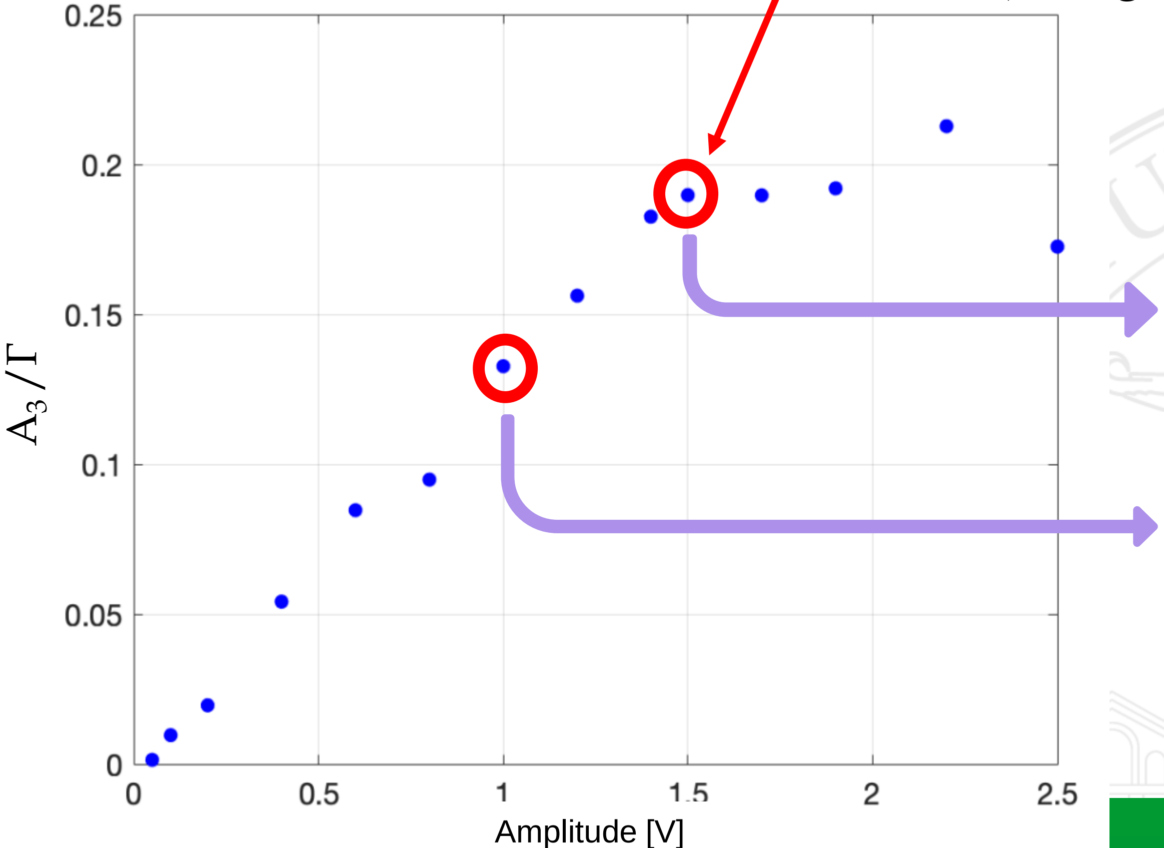


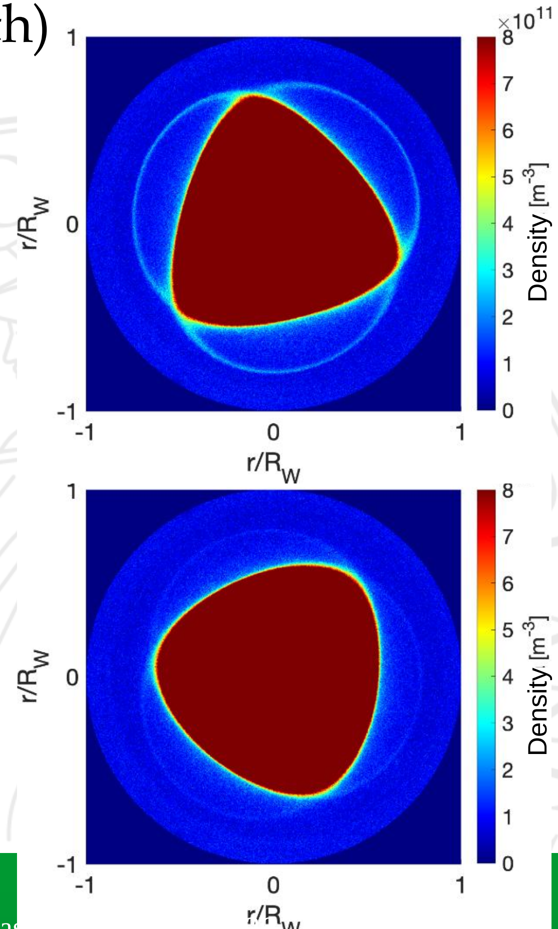


Mode growth general features

Even with small perturbation, **critical amplitude** can be reached
saturation → filamentation, mode **collapse and cascade** to lower modes

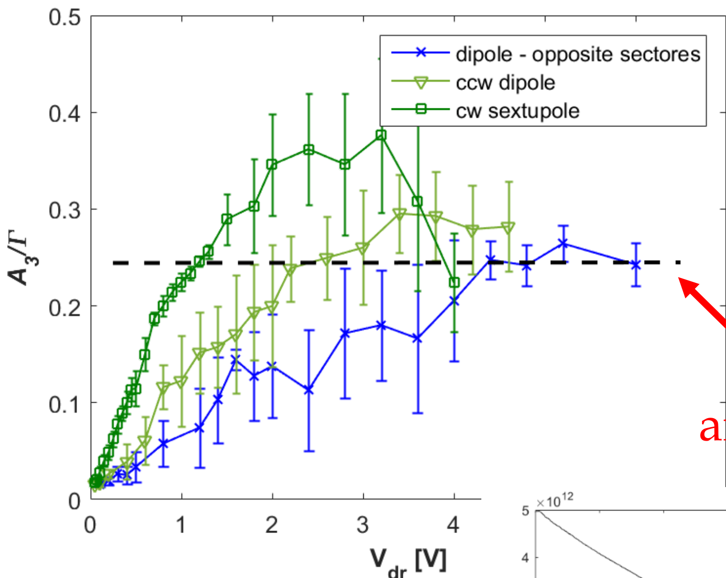
exc. time ~ 20 τ_{rot} Γ total circulation (charge/length)

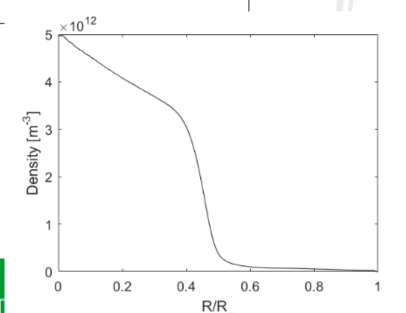




‘Optimal’ technique – strain field structure

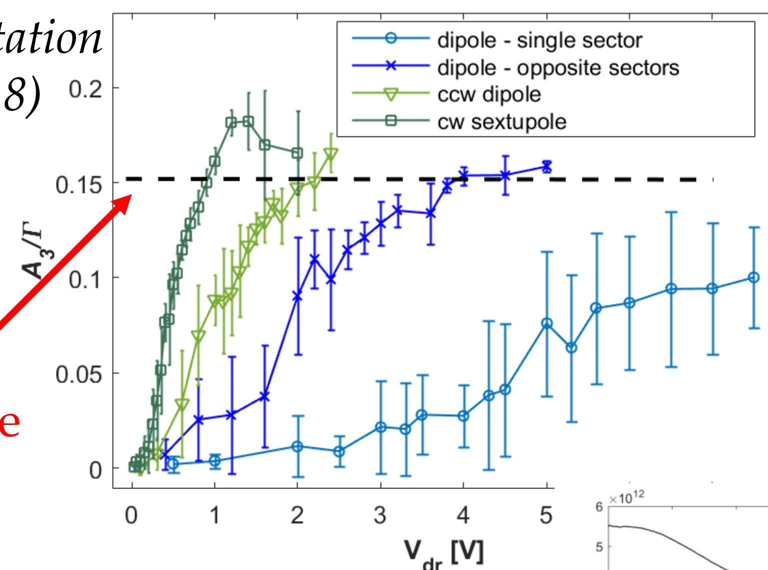
- Full-electrode rotating field technique (green hues)
  → vortex ‘embedded’ in strain field, ‘optimal’ (fully developed V-state)
- Simpler schemes (blue hues)
  → higher intensity, longer t_{exc}, not to critical amplitude

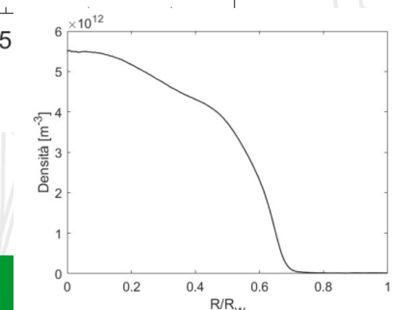




l=3 excitation
(N = 8)

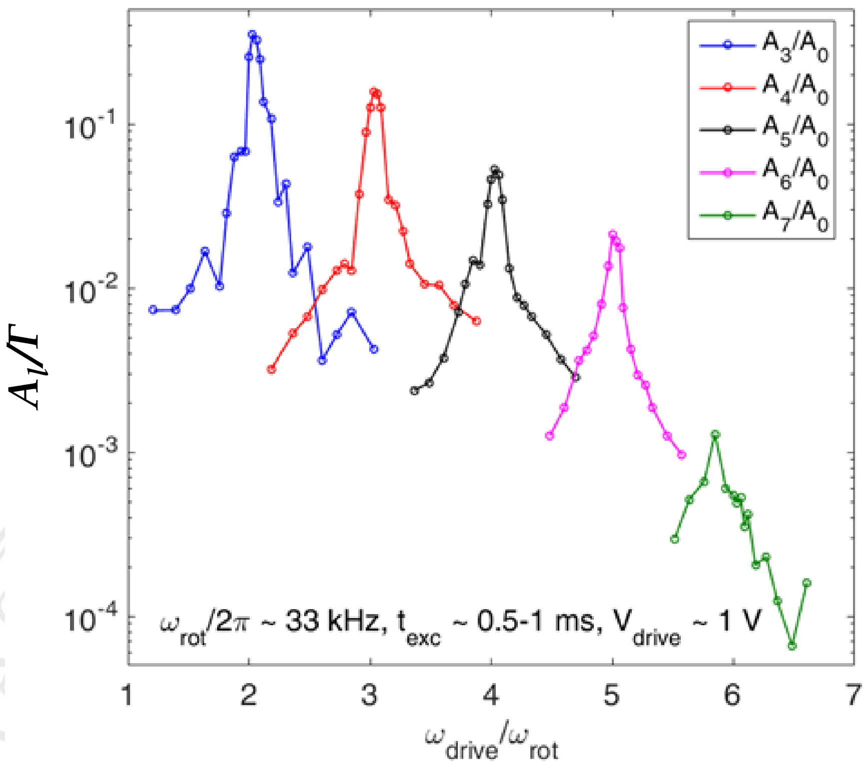
critical amplitude

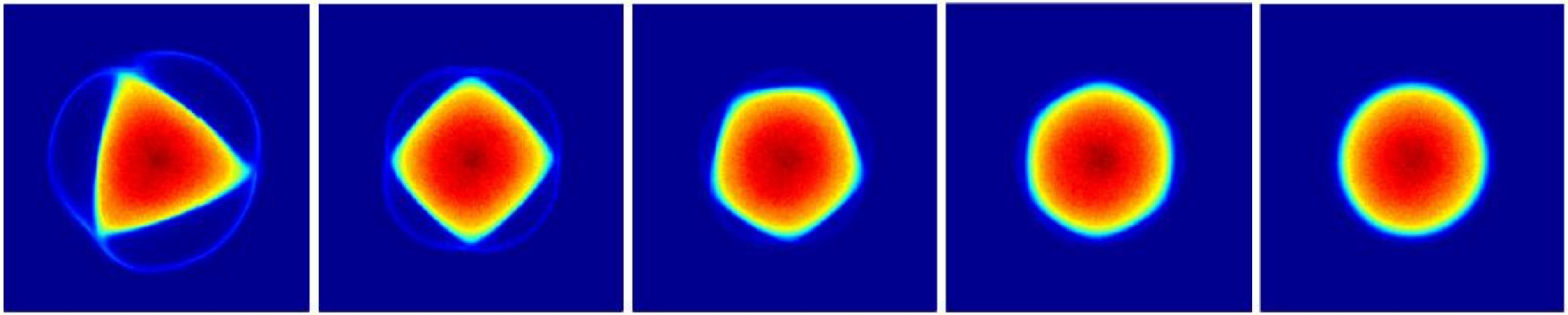




Early stage: resonance for modes l = 3 → 7

<u>Set frequency excitation, early stage ($t < 10^2 \cdot \tau_{rot}$):</u> Fast mode growth can reach critical amplitude (polygonal shape). V-state saturates creating filamentation, hence mode damping and creation of a low-vorticity background halo. Resonant excitation of KH modes up to l = 7 is clearly measured optically. *[G. Maero et al., JPP 2023]*



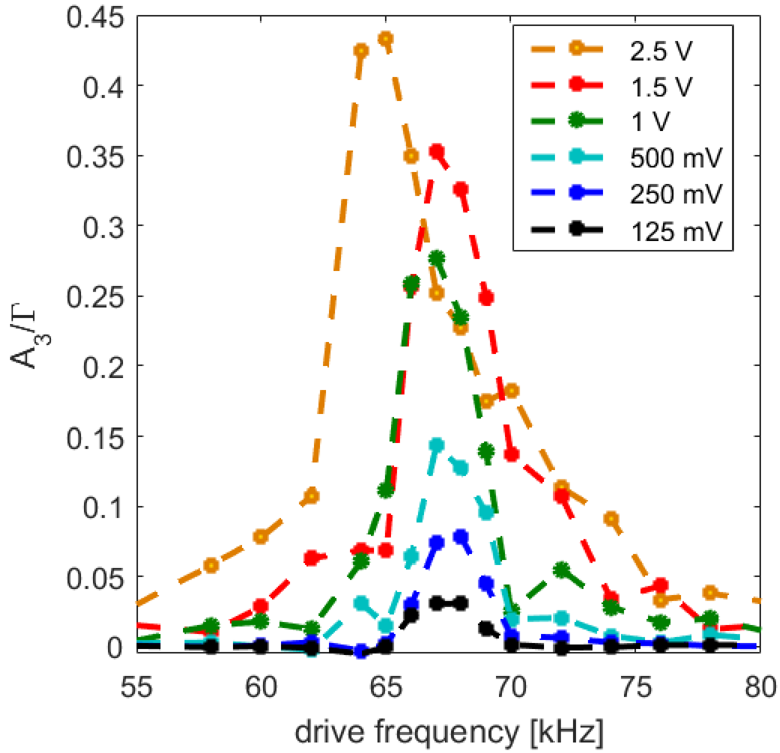


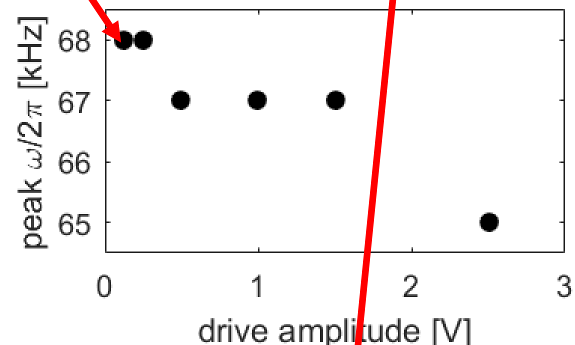
Early stage: resonance features

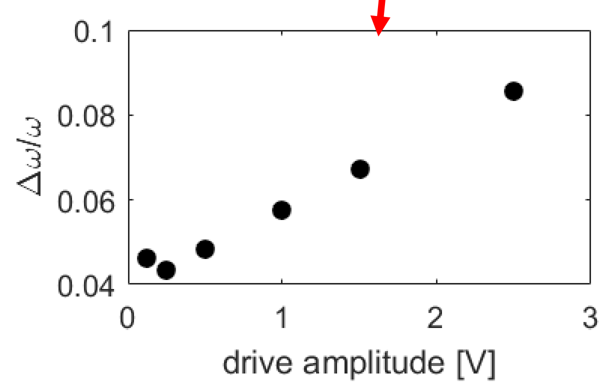
- Caveat: $\boldsymbol{\omega_l} = \omega_{rot}\left[l - 1 + \left(\frac{R_p}{R_W}\right)^{2l}\right]$ $\omega_{rot} = \frac{en}{\varepsilon_0 B}$ $n_0(r) = n_0 H(R_P - r)$ **only for uniform density, within linear regime**

V-states: nonlinear oscillators, $\boldsymbol{\omega_l} = \mathrm{f}(\boldsymbol{A_l})$ with $\boldsymbol{A_l}$ mode amplitude → resonance downshifted and broadened

l=3 excitation





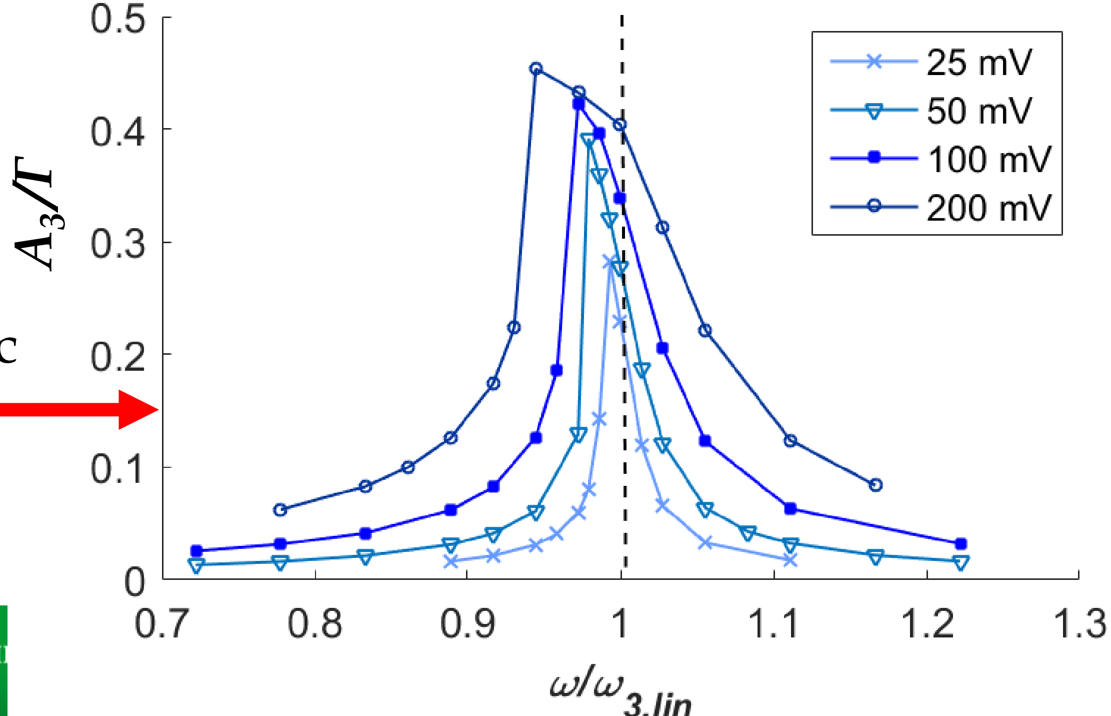


Early stage: resonance features

- Caveat: $\boldsymbol{\omega}_l = \omega_{rot}\left[l - 1 + \left(\frac{R_p}{R_W}\right)^{2l}\right]$ $\qquad \omega_{rot} = \frac{en}{\varepsilon_0 B}$ $\qquad n_0(r) = n_0 H(R_P - r)$
  only for uniform density, within linear regime

V-states: nonlinear oscillators, $\boldsymbol{\omega}_l = f(A_l)$ with A_l mode amplitude
→ resonance <u>downshifted</u> and <u>broadened</u>
out of resonance, beatings limit growth

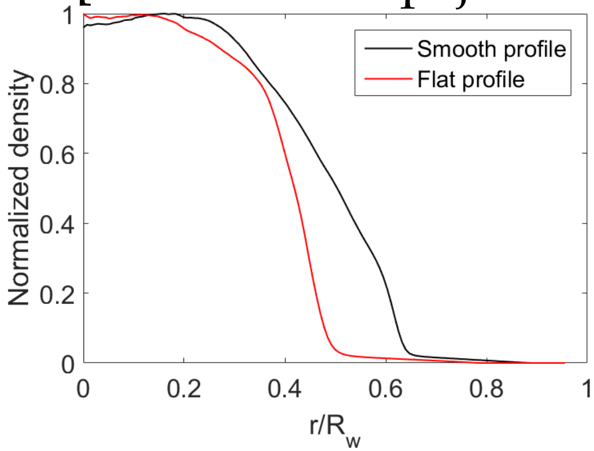
Also backed by systematic 2D PIC (particle-in-cell) electrostatic simulations

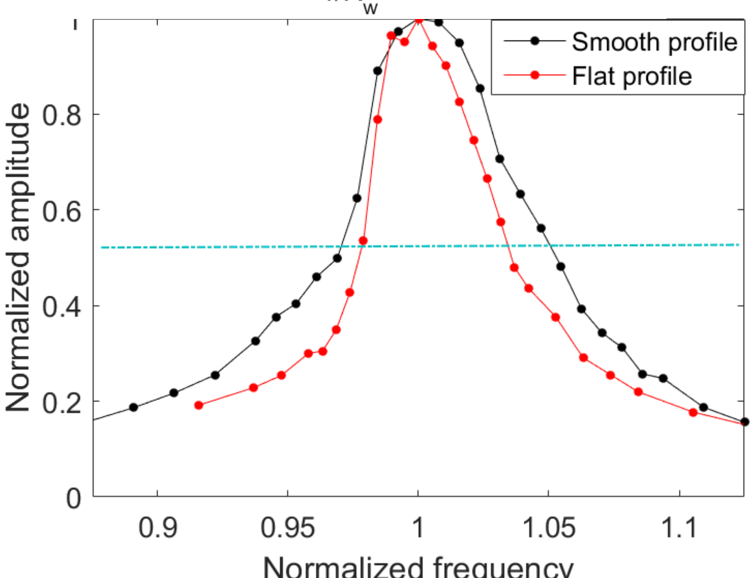


Resonance: Influence of vorticity profile

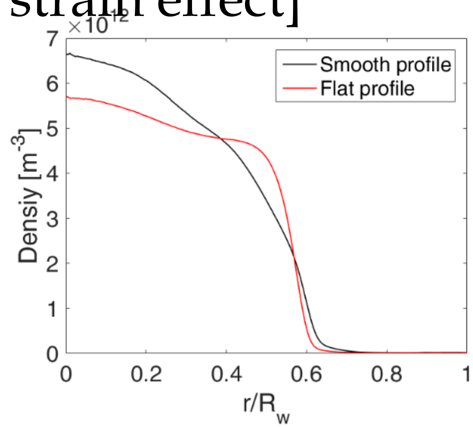
Resonance is **broadened by smooth profile**: **Fall-off region** is the key (KH) wider vortex-strain interaction region across $\boldsymbol{\omega_l}$ spread
[Central clump: just a ‘total circulation’, vorticity/strain effect]

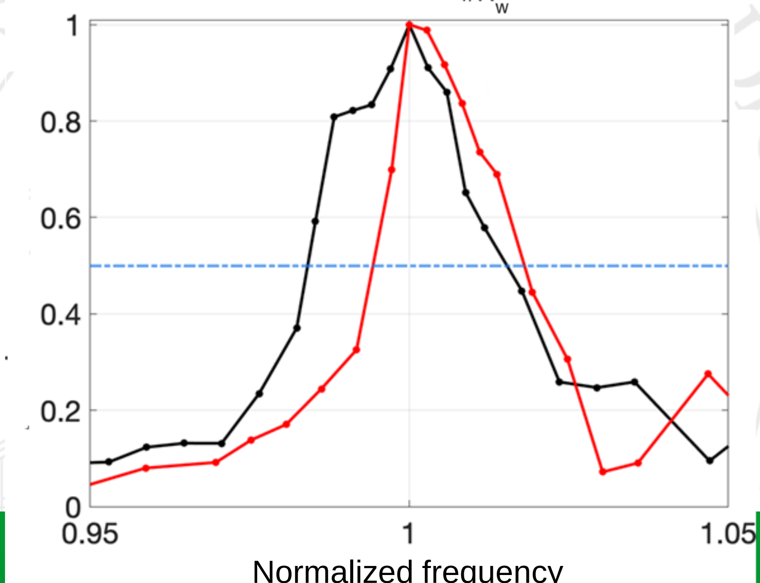
l = 2 excitation





l = 3 excitation

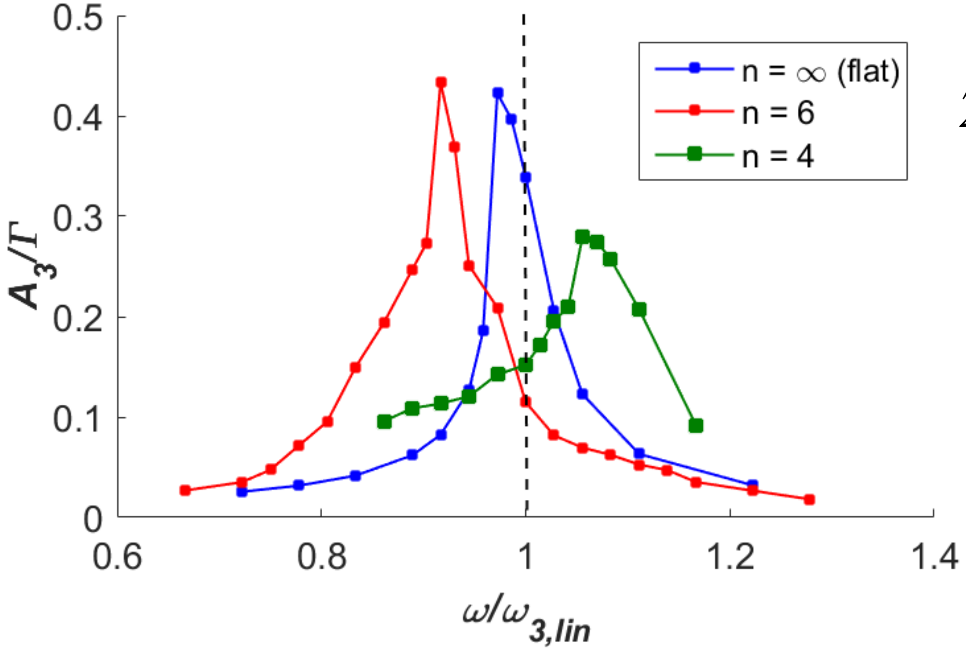




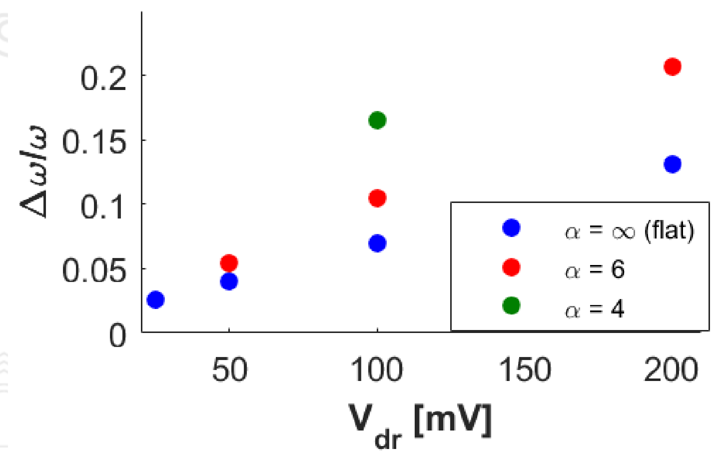
Resonance: Influence of vorticity profile

Resonance is **broadened by smooth profile**: **Fall-off region** is the key wider vortex-strain interaction region across $\boldsymbol{\omega_l}$ spread
[Central clump: just a 'total circulation', vorticity/strain effect]

Fall-off region: $n(r) = n_0\, exp[-(r/R_p)^a]$ α → flat (Rankine), α = 2 Gaussian



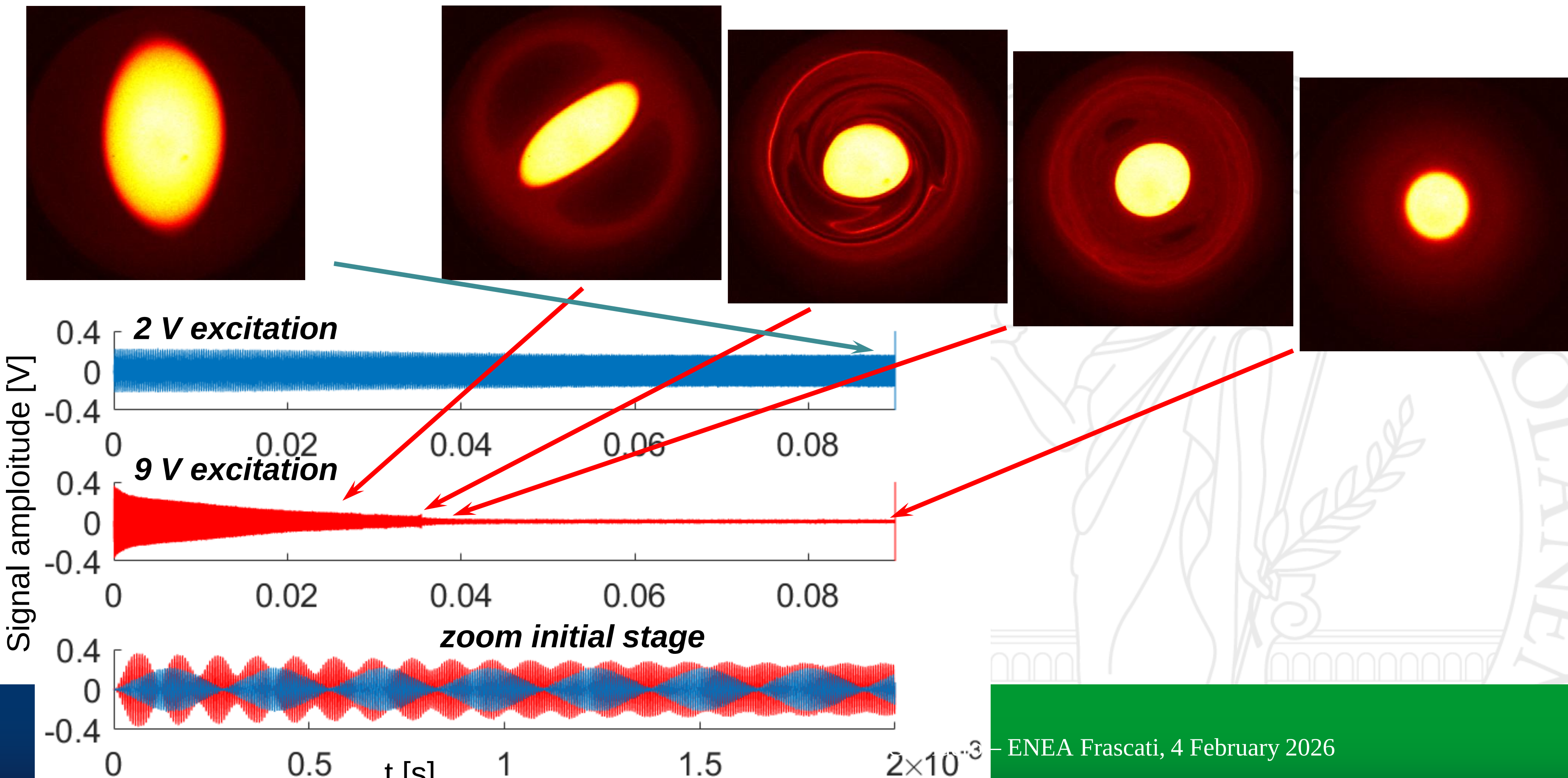
2D PIC simulations



Forced evolution (continuous excitation)

After first saturation, V-state can persist over long times (>$\underline{10^3 \cdot \tau_{rot}}$), with slowly-damped amplitude beatings, as long as enough core vorticity is conserved.

Strong vorticity stripping (also depending on strain/vorticity ratio)
→ collapse and mode cascade to an axisymmetric core in a low-vorticity halo.

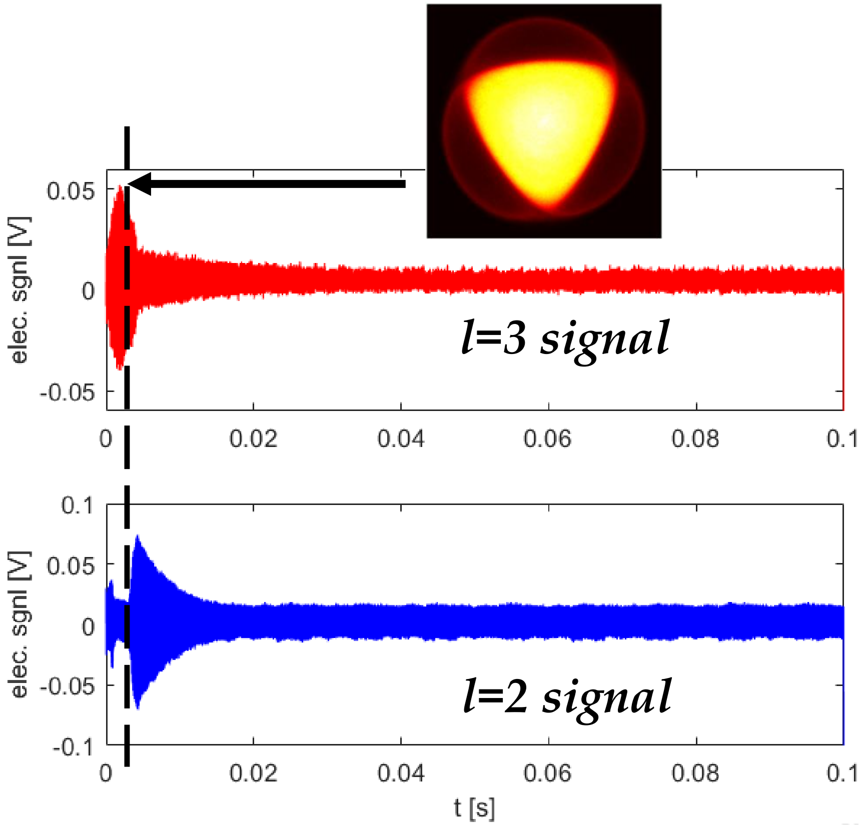


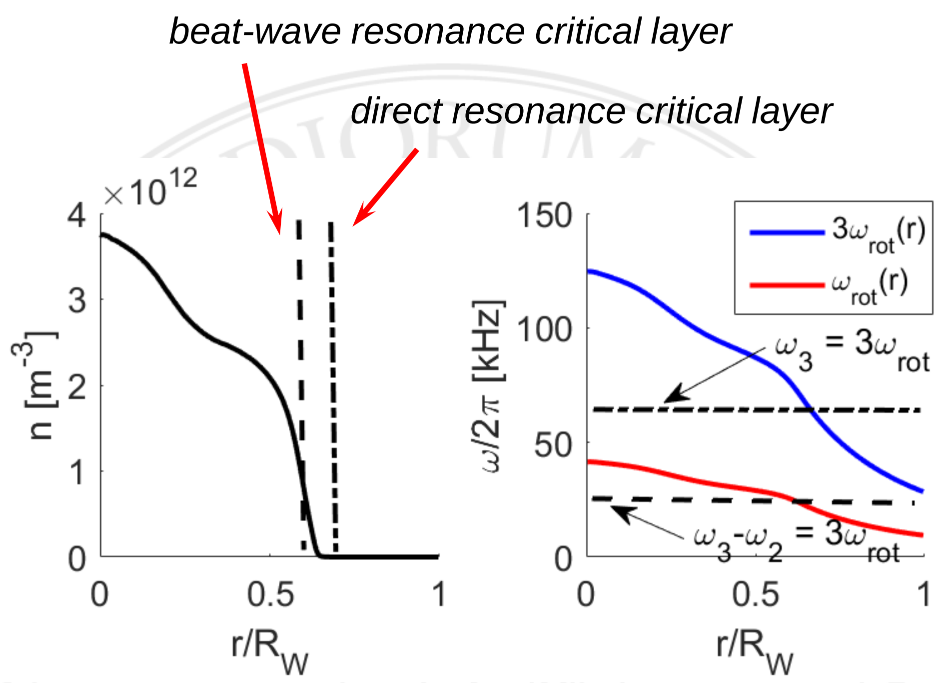
Forced evolution: resonant instability

Collapse and mode cascade: again, **a resonant process**

Spatial, direct or beat-wave resonance rotation - mode frequencies

[Dubin et al., Phys. Fluids 2024]





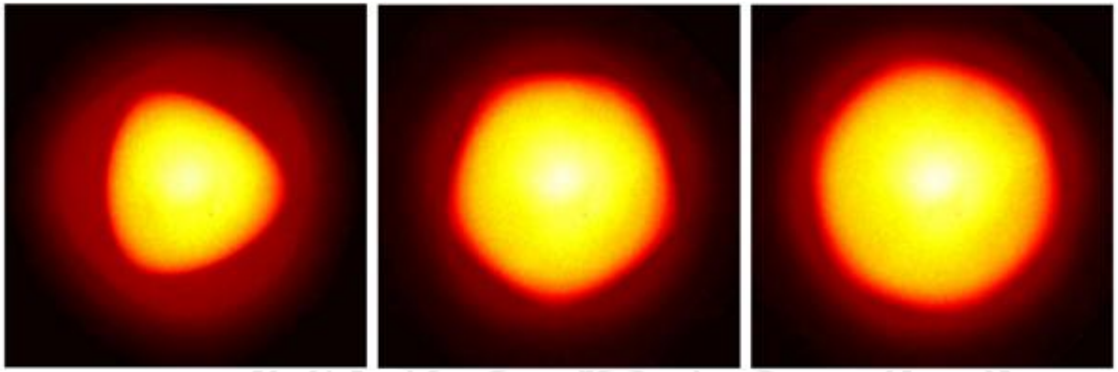
Again, vorticity profile influence (smooth extends to larger critical radius)

→ expect smooth vortex to be less stable [especially *l*>2]

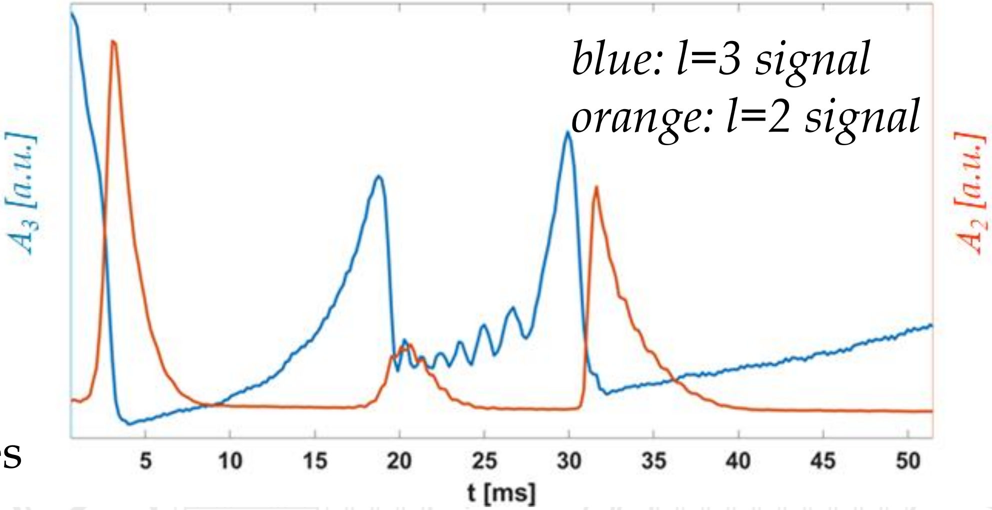
Continuous drive – multiple couplings

The smearing of the vorticity profile in the diffuse background helps attaining:

Persistent deformation ($>10^3 \cdot \tau_{rot}$)

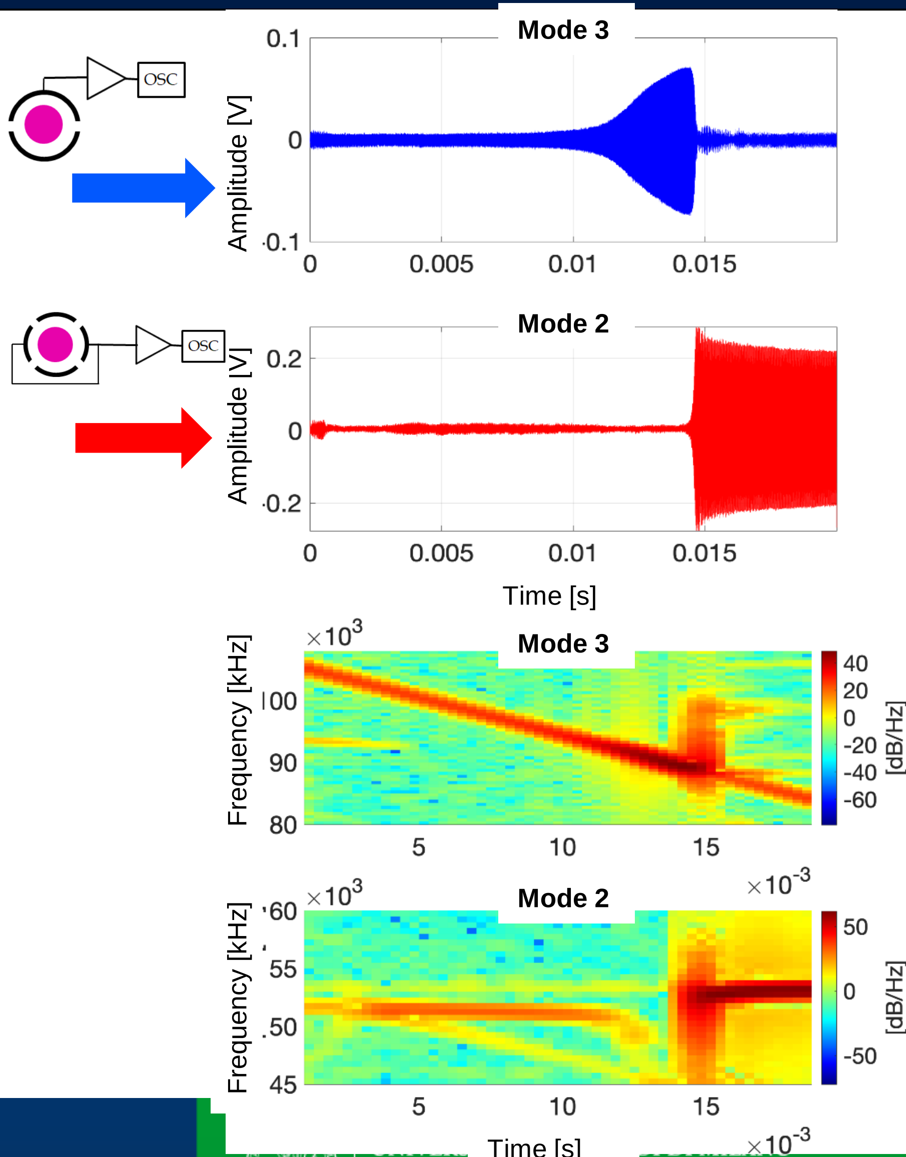


Repeated mode coupling
→ For high-*l* modes, a sufficiently intact core can undergo repeated mode growth/decay stages, even in the presence of (low) dissipation to the background vorticity bath



Did I say? These are **resonant** processes

Slowly-varying strain: Autoresonance



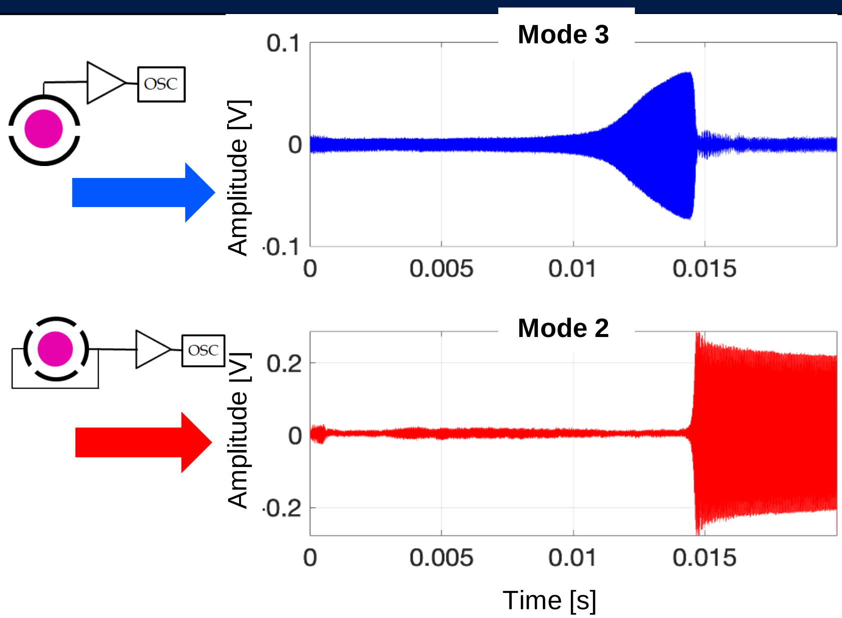
Autoresonance: A nonlinear oscillator locks to a frequency time-varying drive and is bound to grow as $\omega_l = \omega_l(\mathbf{A}_l)$

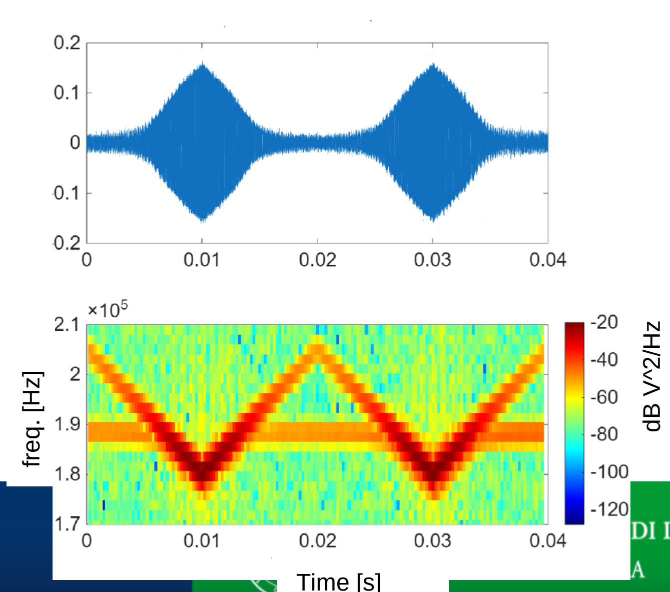
Threshold phenomenon

Above the threshold, drive frequency is in charge (drive amplitude is not)

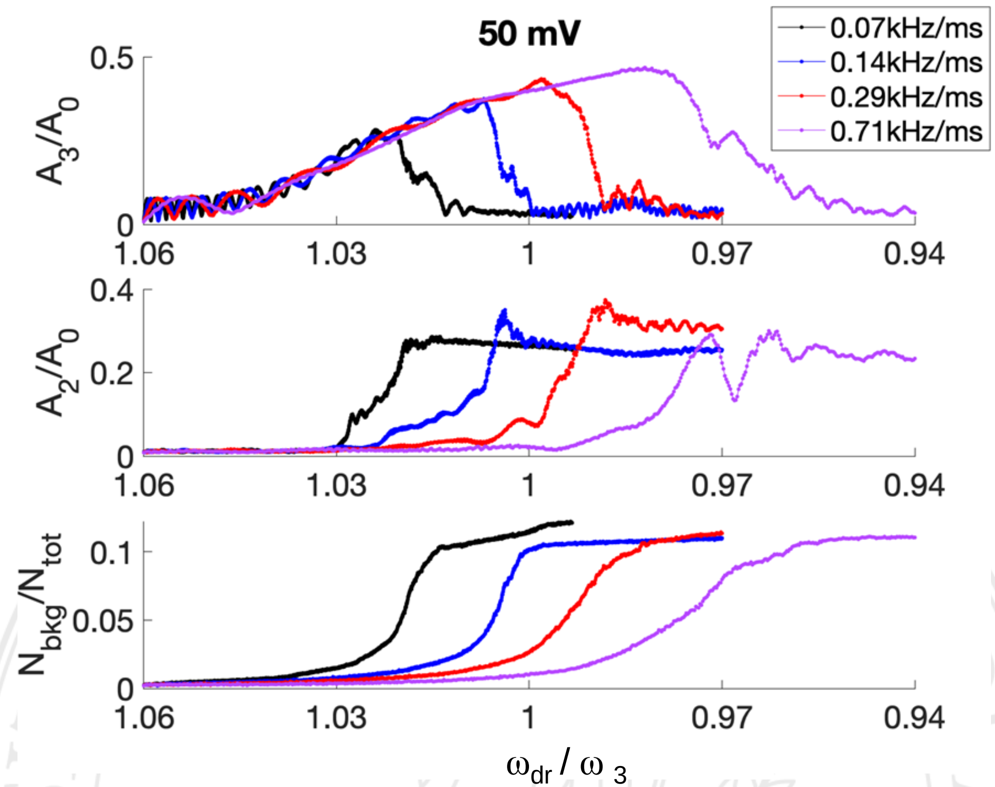
→ Downward frequency sweep to go along mode growth (for $l>1$)

Slowly-varying strain: Autoresonance





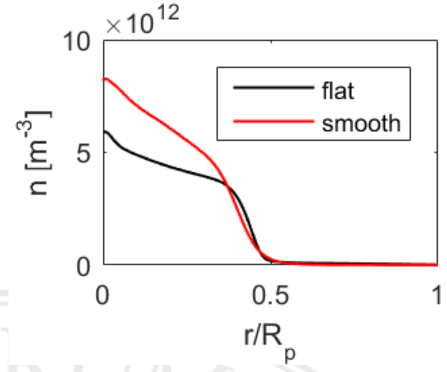
2D PIC simulations

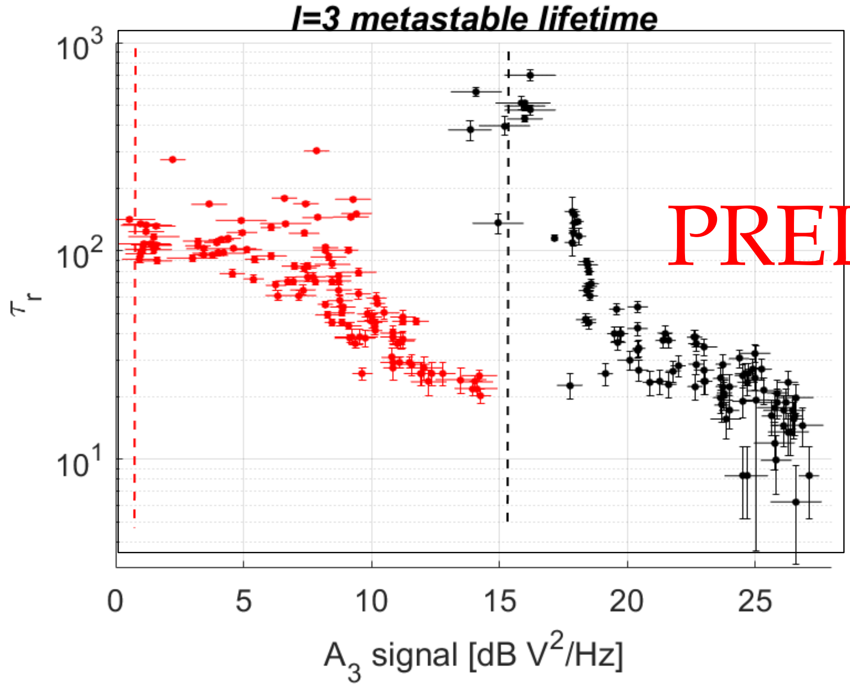


Faster sweep (above threshold)
→ precise control on V-state amplitude
→ higher deformation (competes with dissipation)

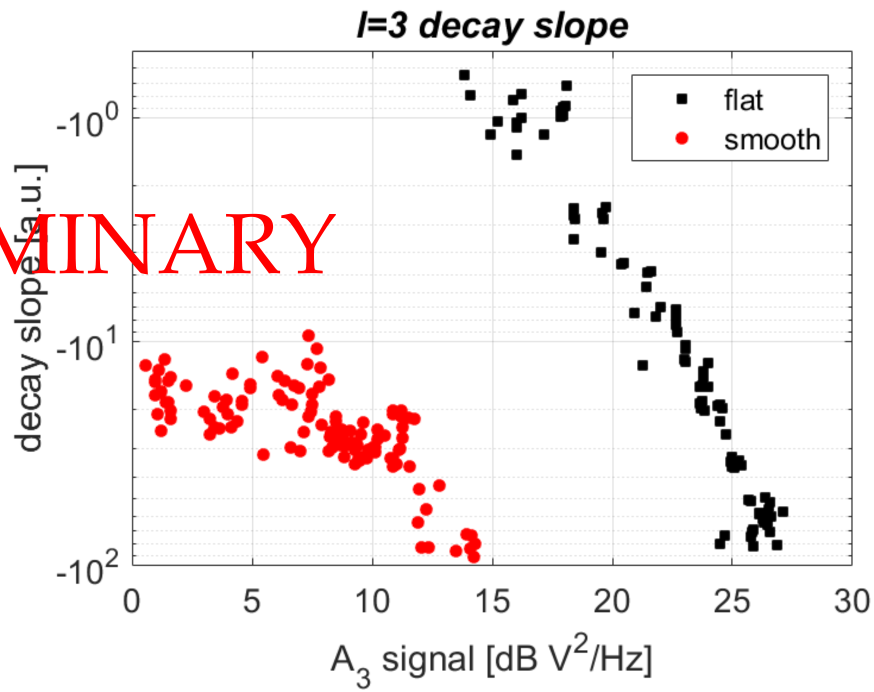
Free evolution: V-state stability

Quasi-stable stage sets in after releasing strain field, followed by decay (see aforementioned instabilities). Both related to the initial amplitude of the V-state → higher amplitude corresponds to shorter stability and faster decay (to be investigated further; what is the role of the density profile?).





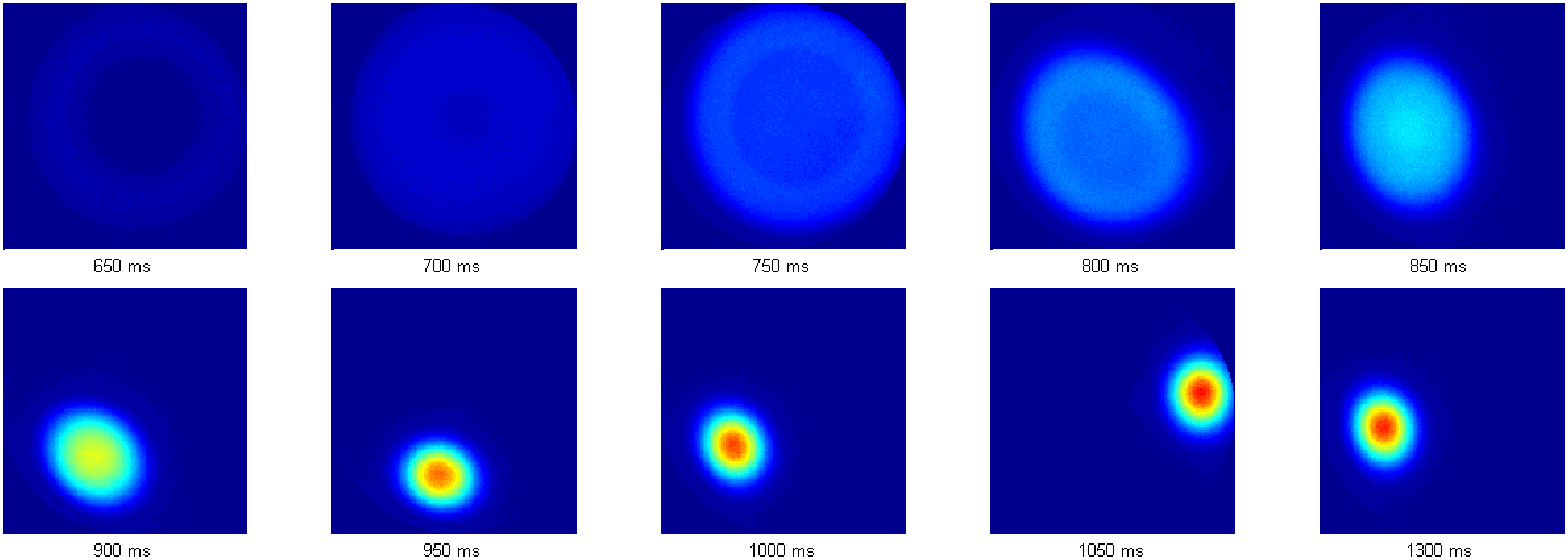
PRELIMINARY



A collection of oddities

Self-organizing structures in non-conserving systems

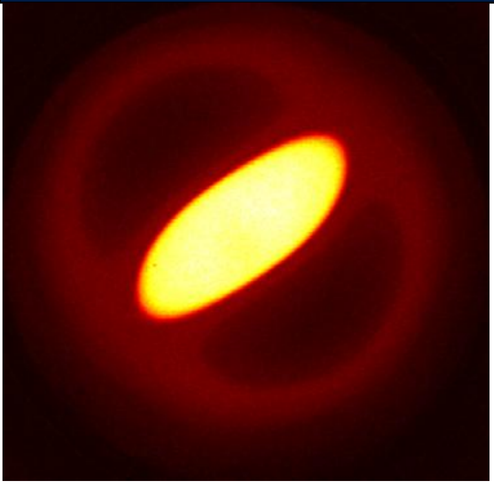
Dynamics and steady state: single vortex



Diffuse, low-density plasma typically formed at large radii (higher electric field, more heating and ionization)

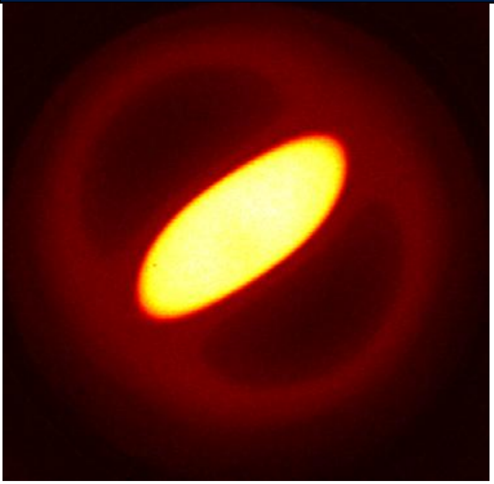
Density profile may evolve to **high-density structures**, eventually showing **enhanced stability** with respect to common sources of destructive instability (e.g., impulsive perturbations, resistive-wall dissipation) **as long as RF is on**

Weirdos



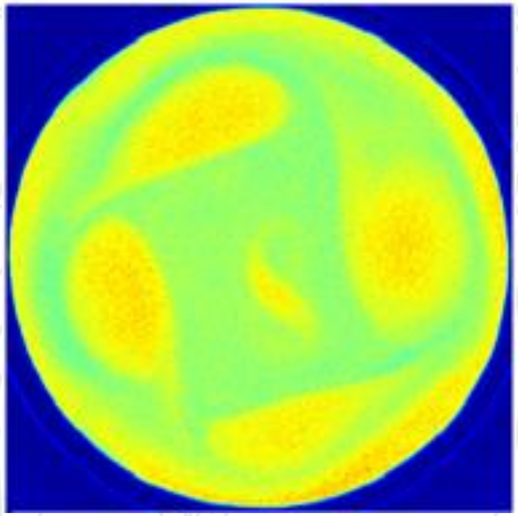
Cat's eye

Weirdos

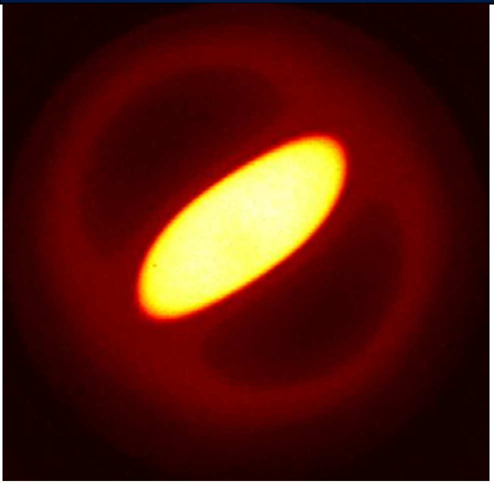


Cat's eye

Goldfish aquarium

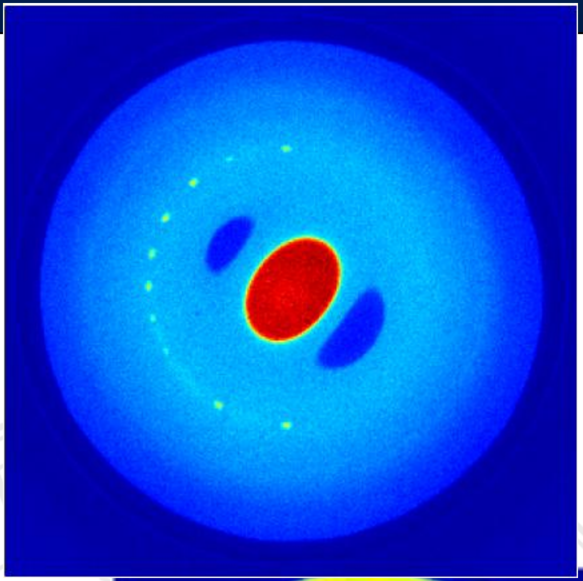


Weirdos

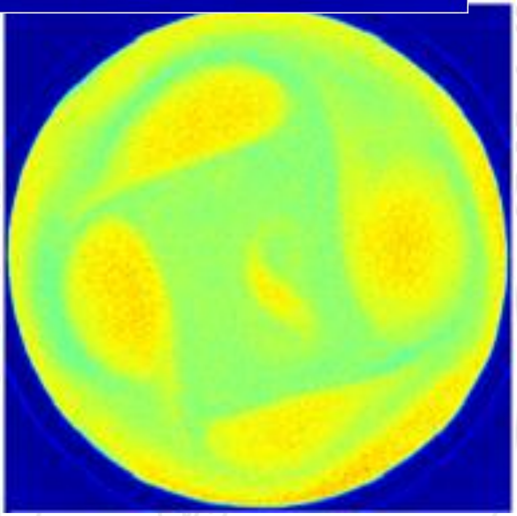


Cat's eye

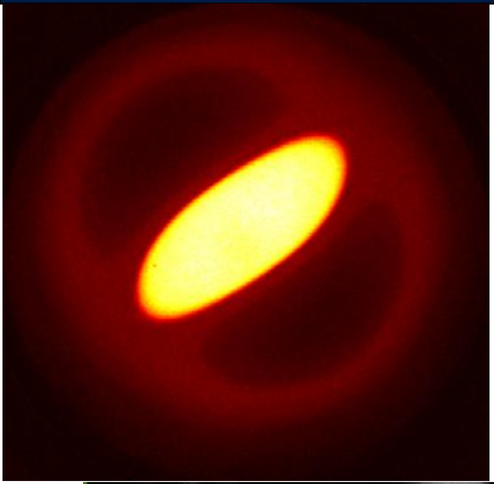
Vortex stream



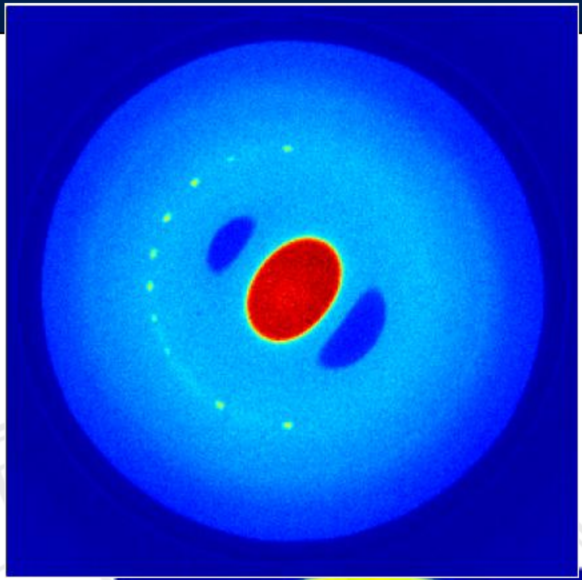
Goldfish aquarium



Weirdos

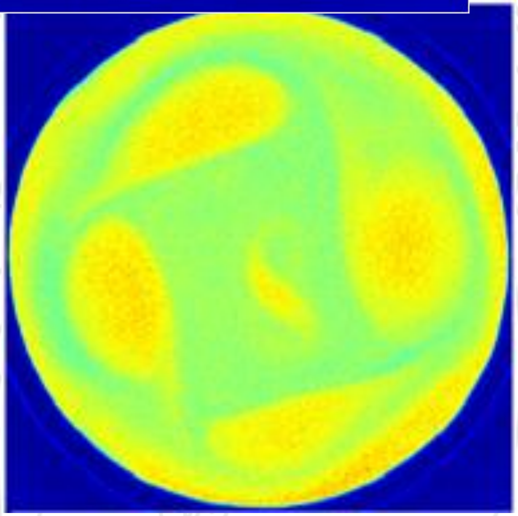


Cat’s eye



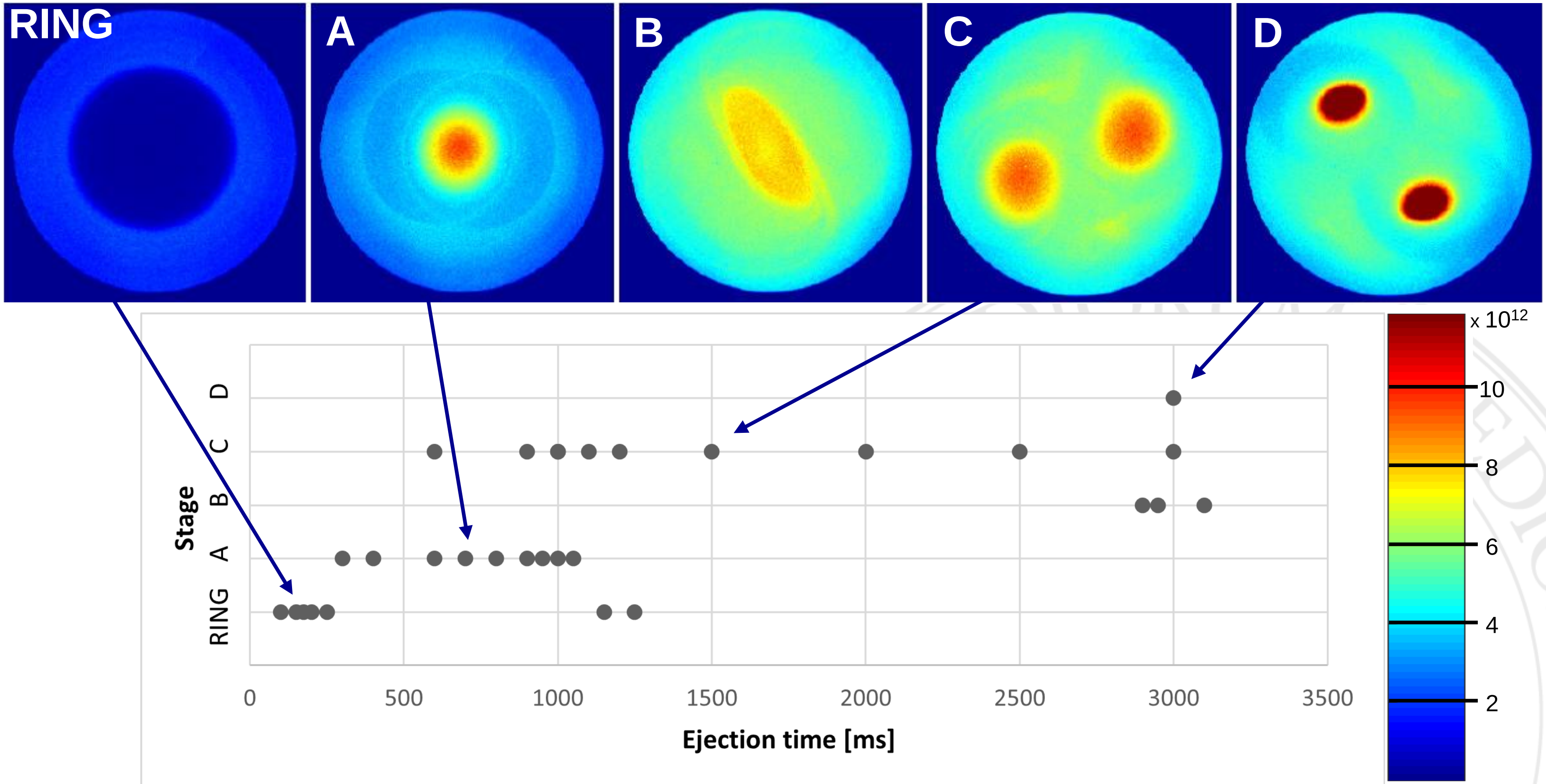
Vortex stream

Goldfish aquarium

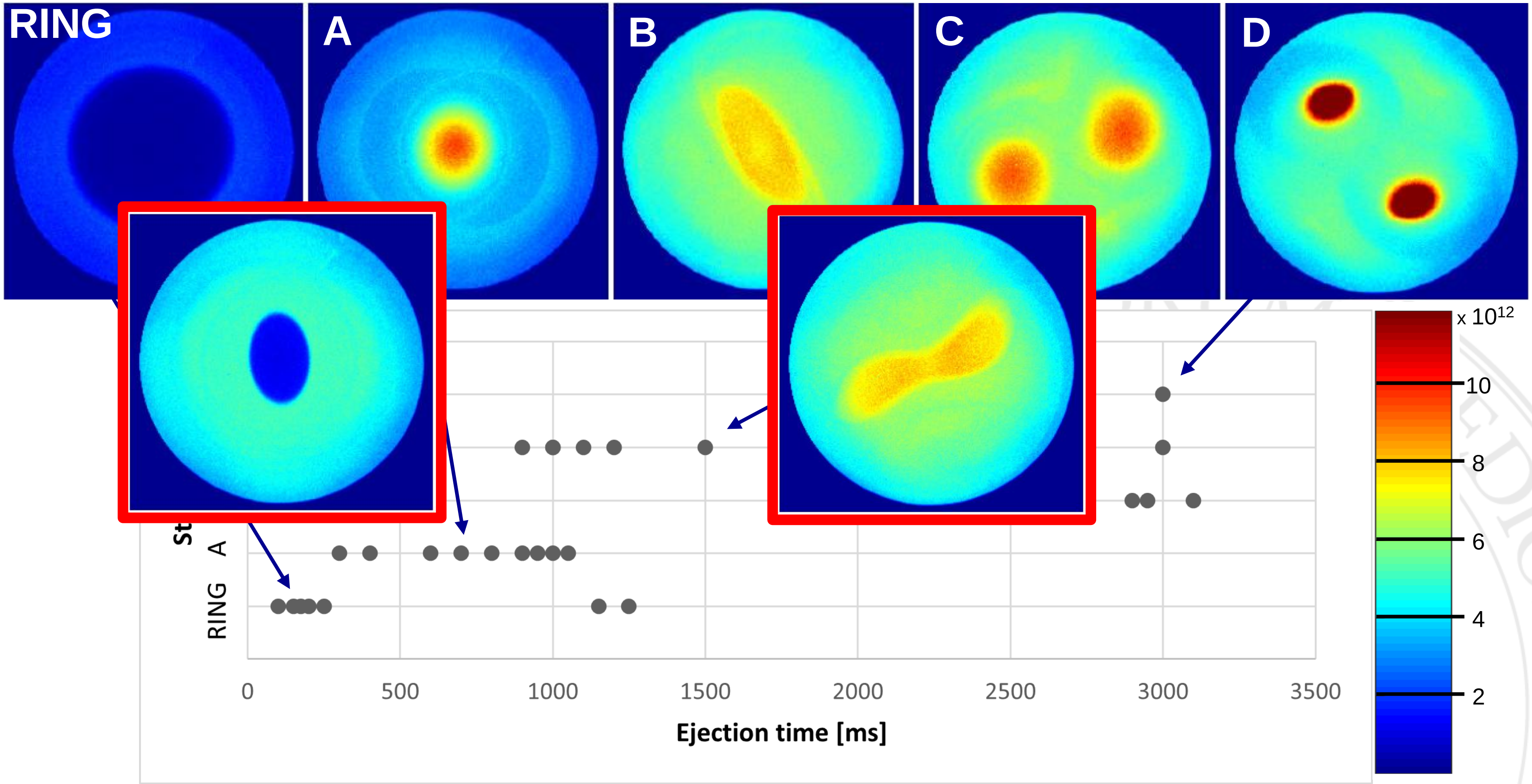


Plasma thong (adults only)

Double vortex



Double vortex



Formation on the edge; accretion with quadrupolar structure and tearing into 2 vortices

Conclusions and outlook

- Contrary to intuition, still much to understand about 2D vortices
- Trapped NNPs represent the best experimental environment
- ‘Plasma approach’ sheds light on the resonance character of phenomena, offers quantitative assessment
- Long-lived coherent structures survive due/despite dissipation and modification of vorticity gradients
- More to do to improve on systematic measurement, quantitative analysis, elimination of 3D effects
- Small-scale experiments with interdisciplinary and educational value for students

Thanks for your attention

G. Maero et al., ‘Fundamental physics and applications using nonneutral plasma’, Adv. Phys.: X, 2024

More stuff

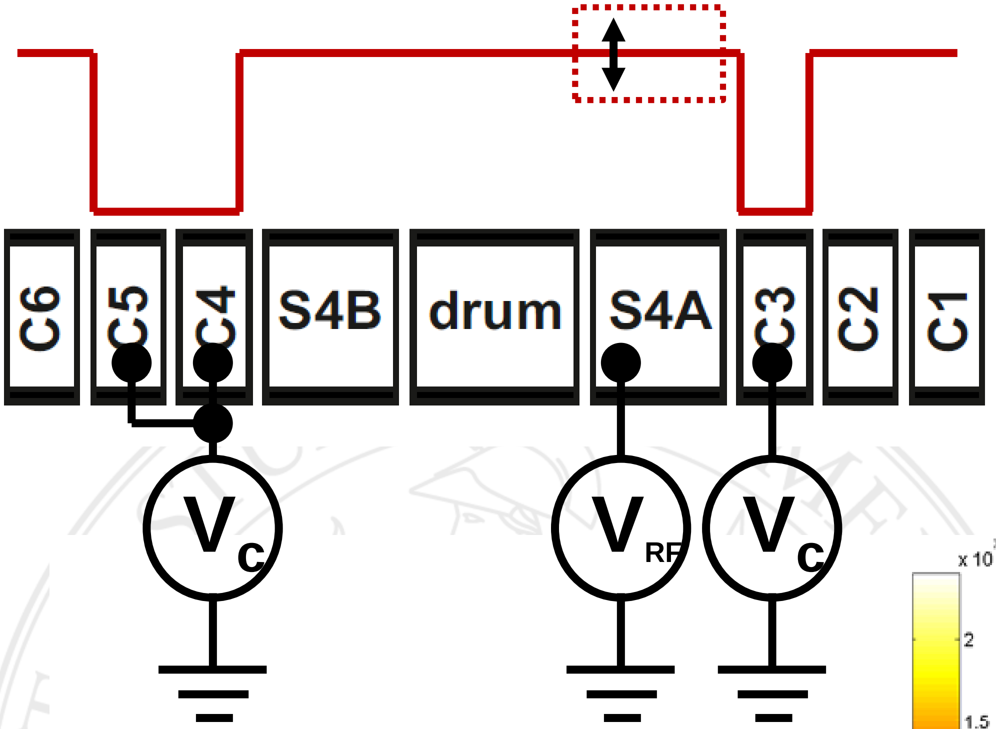
A step back: Plasma generation

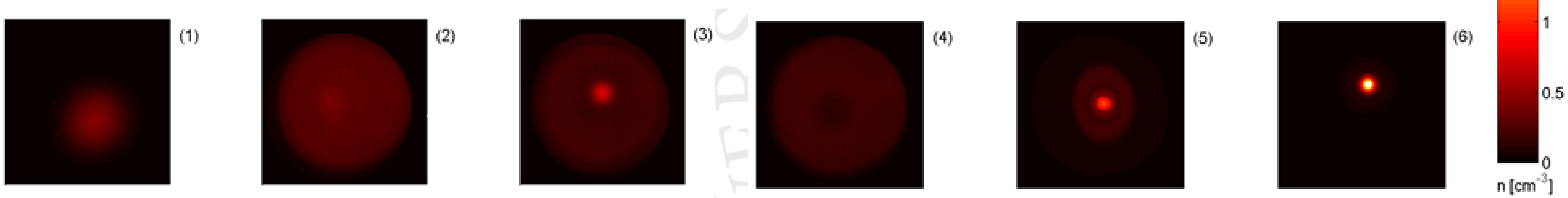
RF excitation on one of the inner electrodes
→ heating of free electrons in the background gas
→ ionization and e- accumulation

Typical RF parameters:
$f(t)=V_{RF}\sin(2\pi\nu_{RF}t)$, V_{RF}~1-5 V, ν_{RF}~**1-20 MHz**

Balance established ~ **seconds**

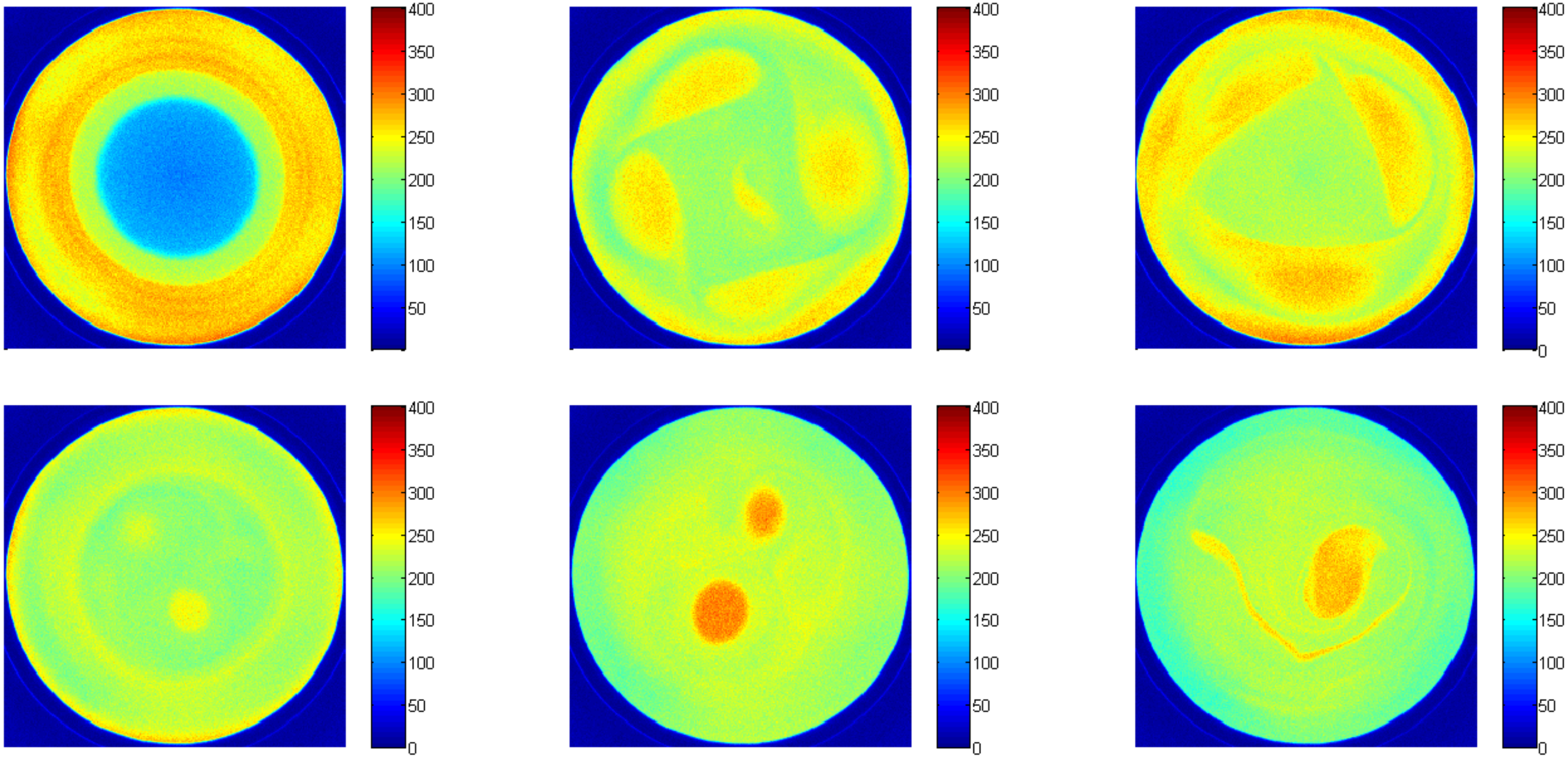
Many different configurations, stable as long as RF is on





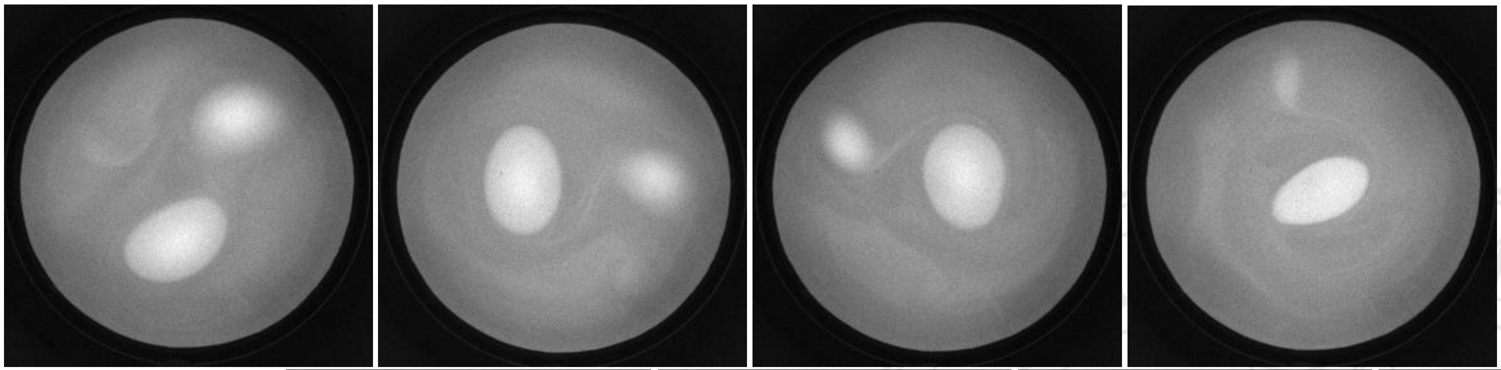
Formation and evolution: Early structures

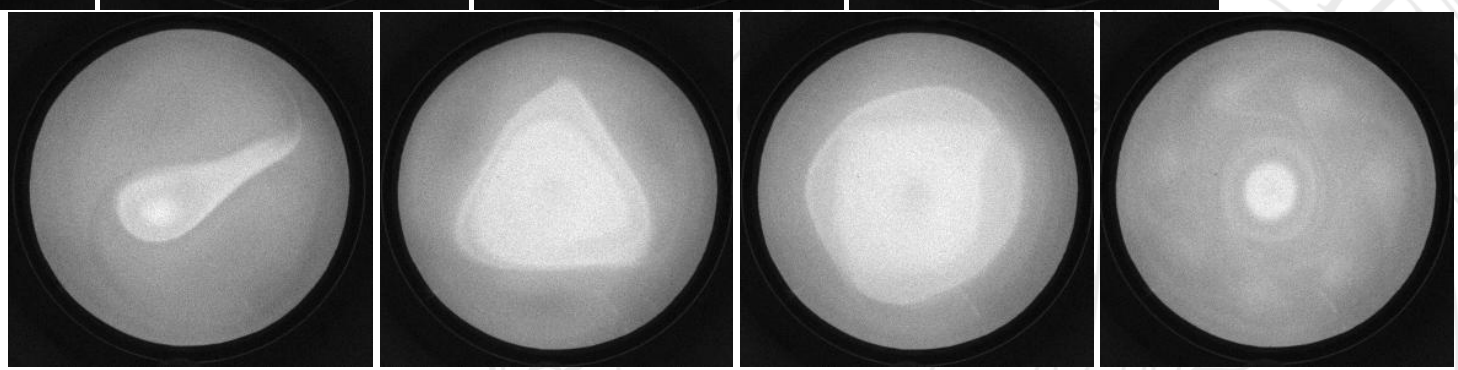
Diffuse, low-density plasma typically formed at large radii (higher electric field, more heating and ionization). But then non-axisymmetric structures may emerge…



Formation and evolution: More structures

Asymmetric coalescence





‘Exotic’ vortices and structures

Rotating-field mode excitation

- Example: Rotating dipole field on a 4-sector electrode (m = 0,…,3)
- Excitation amplitude V_d, frequency ω_d, σ =±1 co-/counter-rotation

$$V_m = V_d \cos(\omega_d t + \sigma m \pi / 2)$$

potential on m-th sector; Δφ = π/2 phase between adjacent sectors

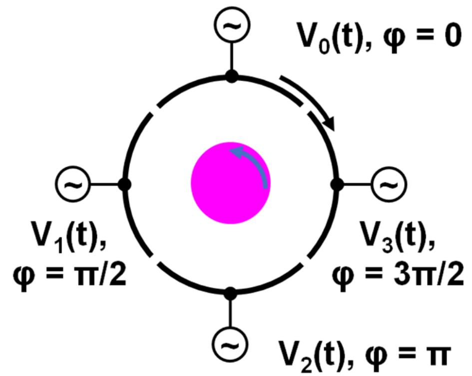
$$\delta\phi(r = R_W, \theta, t) = \sum_{m=0}^{3} V_m(t)[H(\theta - m\pi/2) - H(\theta - (m+1)\pi/2)]$$

overall perturbation at trap wall

Fourier expansion coefficients for perturbation

$$c_l^+(t) = \begin{cases} \epsilon e^{iw_d t} & l = 4k+1 \\ \epsilon' e^{-iw_d t} & l = 4k+3 \end{cases}$$

$$c_l^-(t) = \begin{cases} \epsilon e^{-iw_d t} & l = 4k+1 \\ \epsilon' e^{iw_d t} & l = 4k+3 \end{cases}$$



to be plugged into linearized perturbation theory starting from step-wise profile $n_0(r) = n_0 H(R_P - r)$

Rotating-field mode excitation

- Resonance appears for l = 4k+1, σ =-1 at $\omega_d = \Omega_l$ (i.e. $\Omega_1, \Omega_5, \Omega_9, \ldots$) etc.

$$\delta\phi(r = R_P, \theta, t) = \epsilon\left(\frac{R_P}{R_W}\right)^l\left[\frac{l\omega_D - \Omega_l}{\omega_d - \Omega_l}\exp(il\theta - i\Omega_l t) + \frac{l\omega_D - \omega_d}{\Omega_l - \omega_d}\exp(il\theta - i\omega_d t)\right]$$

resonant at $\omega_d = \Omega_l$ (i.e. $\Omega_1, \Omega_5, \Omega_9, \ldots$)

- The result is generalized for any number N of sectors, e.g.

***N* = 4**, $\Delta\varphi = \pi/2$

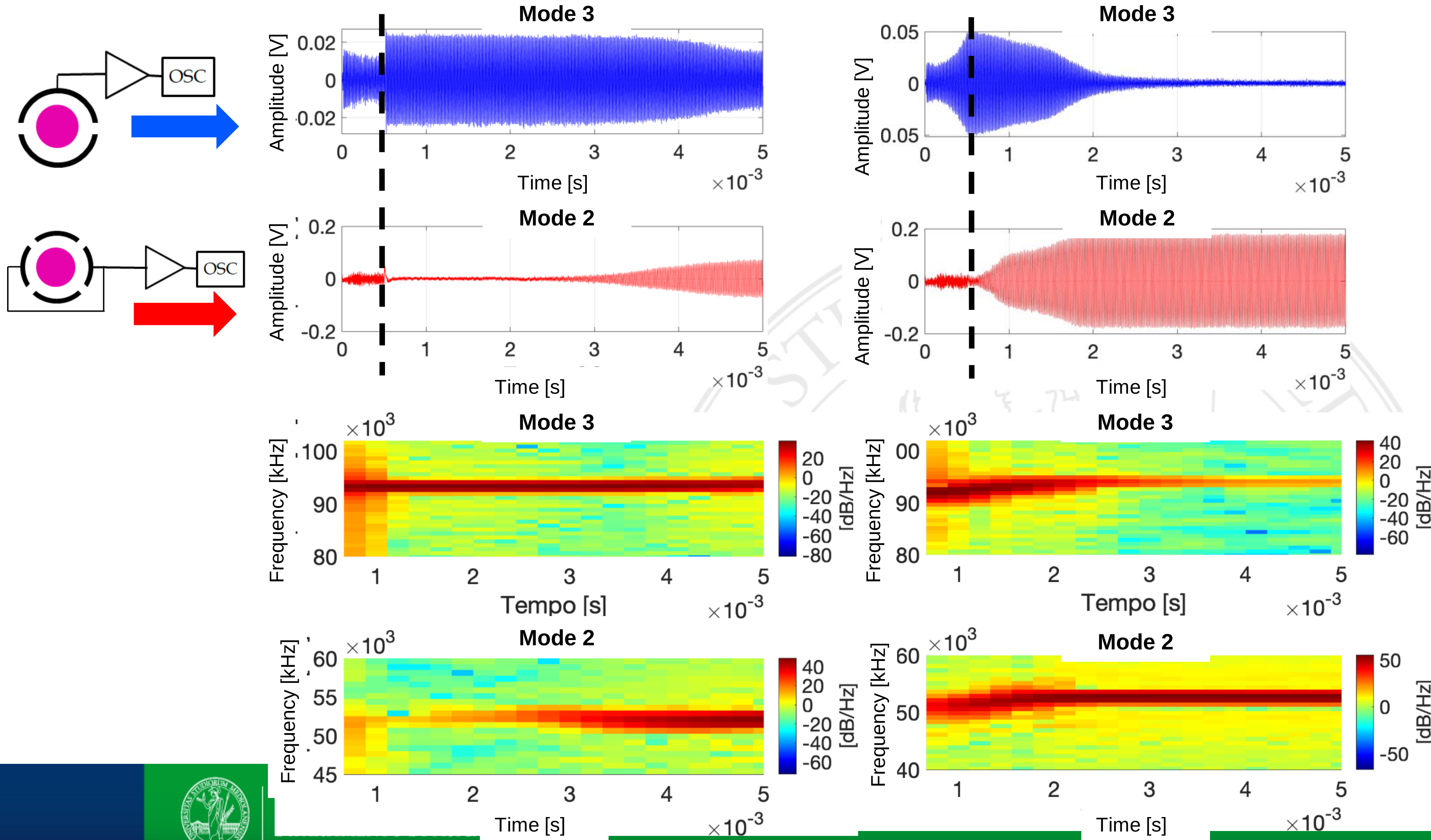
- l = 4k+**1**, σ =-1
- l = 4k+**3**, σ =+1
- l = 4k+**2** (σ = ± 1)

***N* = 8**

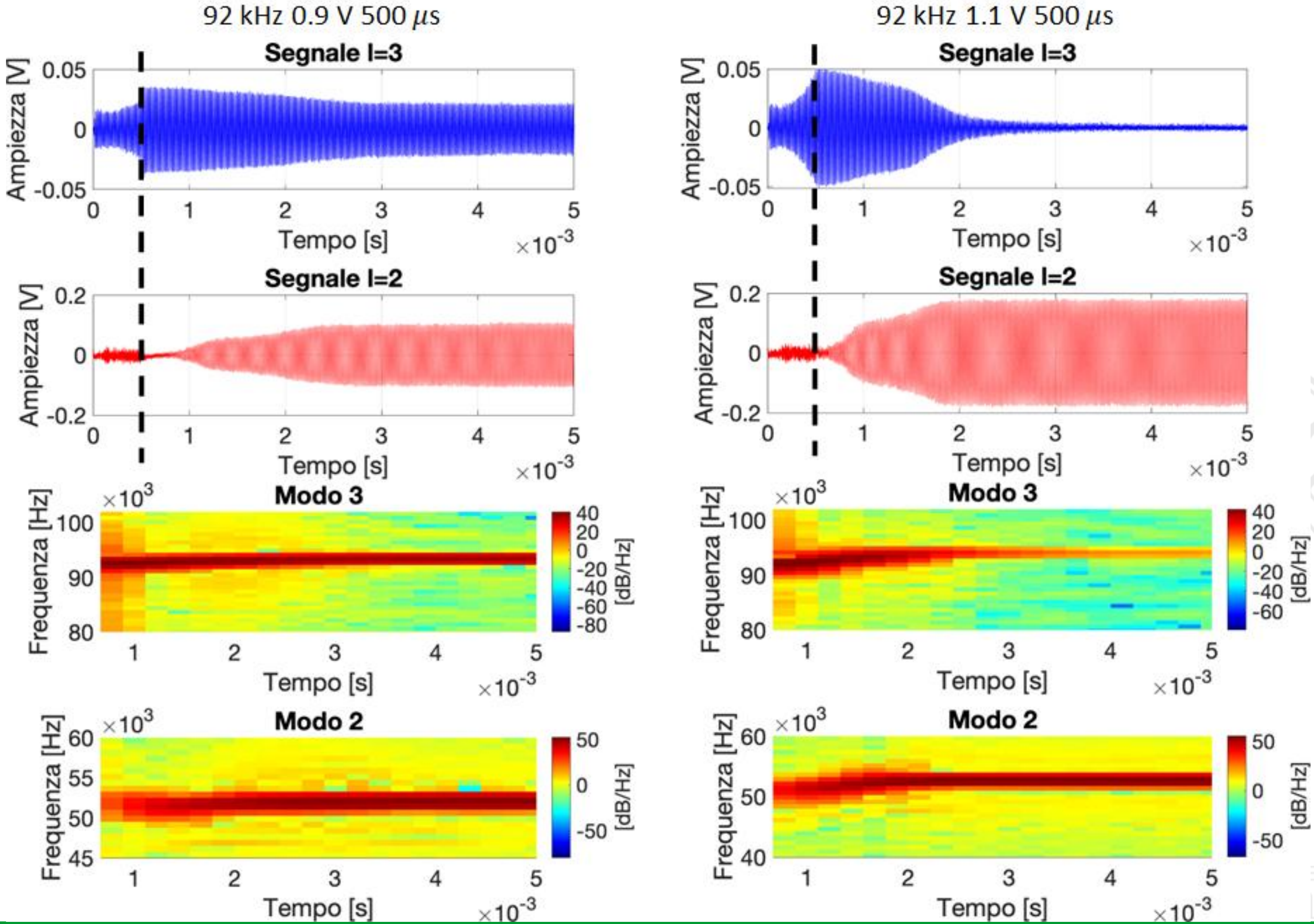
	σ =-1 (co-rot.)		σ =+1 (counter-rot.)	
[D]	$\Delta\varphi = -\pi/4$,	l = 8k+**1**	$\Delta\varphi = \pi/4$,	l = 8k+**7**
[Q]	$\Delta\varphi = -\pi/2$,	l = 8k+**2**	$\Delta\varphi = \pi/2$,	l = 8k+**6**
[S]	$\Delta\varphi = -3\pi/4$,	l = 8k+**3**	$\Delta\varphi = 3\pi/4$,	l = 8k+**5**
[O]	$\Delta\varphi = -\pi$,	l = 8k+**4**	$\Delta\varphi = \pi$,	l = 8k+**4**

Free evolution: Amplitude-dependent stability

92 kHz 0.9 V 500 μs | 92 kHz 1.1 V 500 μs



V-state excitation and free evolution



UNIVERSITÀ DEGLI STUDI DI MILANO
DIPARTIMENTO DI FISICA