Energization and Transport of Cosmic Rays

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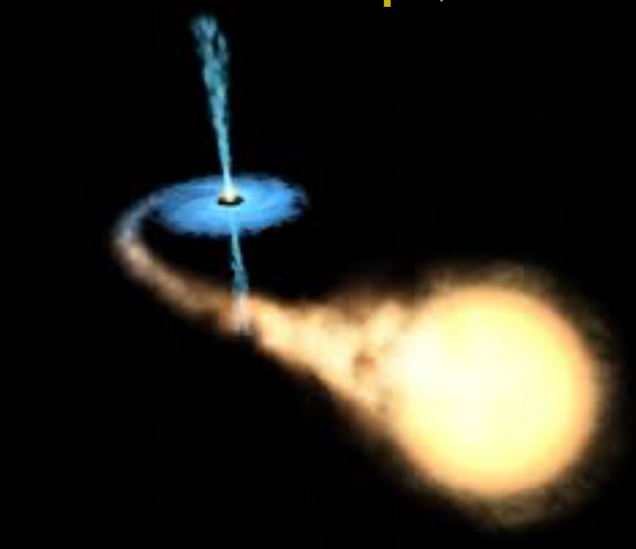
NON THERMAL PHENOMENA ARE UBIQUITOUS

Universal need for acceleration mechanisms

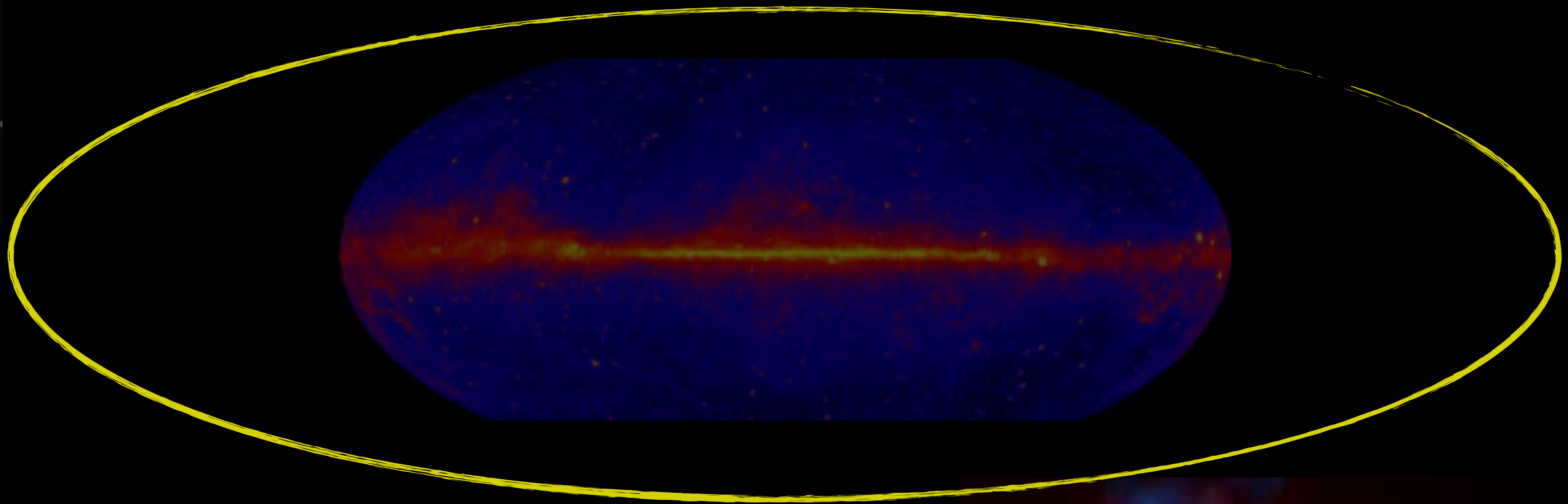
Sun

µQSO



SNRs

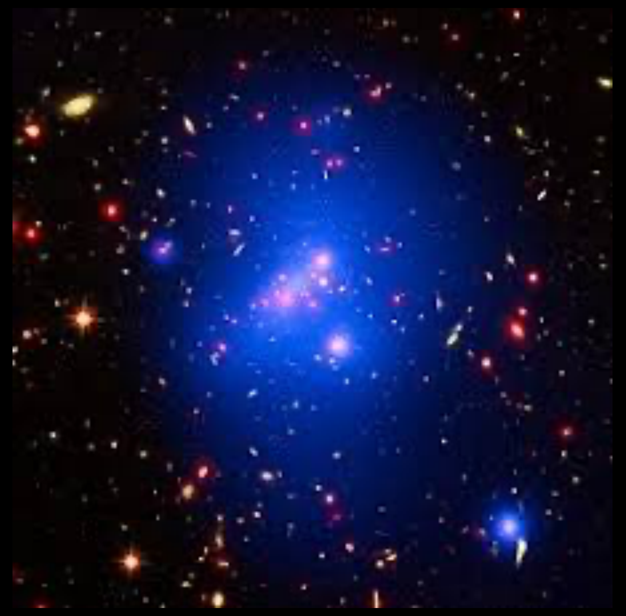
AGN



PWNe

Star Clusters

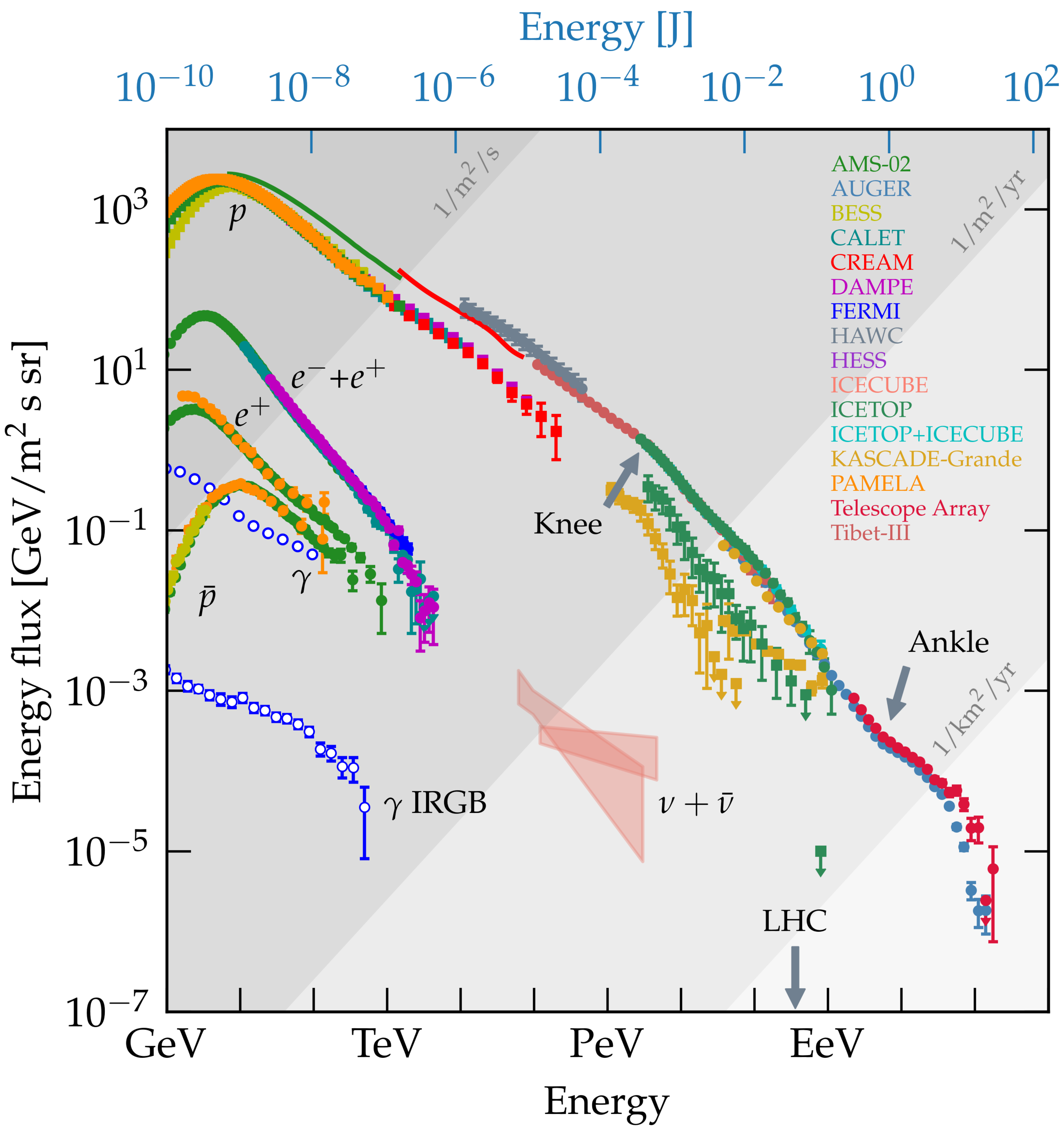
Starburst galaxies



Clusters of galaxies


SS 433
discontinuity
particle acceleration site
microquasar
SS433
discontinuity
particle acceleration site
manatee nebula
supernova remnant

Cosmic Rays - a short introduction



- We now measure with good accuracy the spectra of individual primary components (p, He, C, …) as well as secondary (Be, B, antiprotons, positrons, …)
- The all-particle spectrum shows a marked knee at about 2 PeV and a clear suppression at 10^{20} eV
- At the knee there is also evidence for a transition from lighter to heavier nuclei
- The generally accepted view is that CR remain Galactic in origin up to the so-called second knee (10^{17} eV) where extragalactic CR kick in
- *It is not yet clear whether the suppression at the end of the spectrum reflects an intrinsic maximum energy or rather photo disintegration of nuclei*

A Plasma View of Cosmic Rays

Cosmic Rays are "Mostly" positively charged particles propagating around in a plasma "mostly" made of protons and electrons in approximately thermal equilibrium.

This definition applies to both the acceleration regions and the interstellar space between sources

In general, cosmic ray diffusion leads to establishing **spatial gradients** —> **electric currents**

The background plasma reacts to the presence of cosmic rays by creating **return currents**, in order to retain charge neutrality and these are responsible for **exciting instabilities** under certain conditions

This phenomenon is of crucial importance for both acceleration and propagation of cosmic rays

A Plasma View of Cosmic Rays

- In the Galaxy a global gradient can be established between the disc (where the sources are) and a free escape boundary (halo), of order n_{CR}/H

- By the same token, a gradient can be established near sources on scales L such that the contribution of the source dominates upon the sea of CRs (gradient n_{CR}/L)

- If acceleration occurs at shocks, the upstream generates a precursor of size $D(E)/u_{sh}$ correspondent to the distance that can be reached by accelerated particles. A gradient is established on such a distance

- Finally, far from the shock in the upstream region there is a current of particles escaping the acceleration region, at the highest energies

Plasma Physics of Cosmic Rays

On typical time scales of transport, collisions are unimportant, hence:

$$\frac{\partial f_\alpha}{\partial t} + \vec{v} \cdot \vec{\nabla} f_\alpha + \frac{q_\alpha}{c}\left(\vec{v} \times \vec{B}\right) \cdot \frac{\partial f_\alpha}{\partial \vec{p}} = 0$$

α = thermal protons, thermal electrons, CR

+ Maxwell Equations connecting the EM fields to the source term (charge density and currents)

$$n_p + n_{CR} = n_e$$
$$n_e v_e \approx J_{CR} = n_{CR} v_D$$

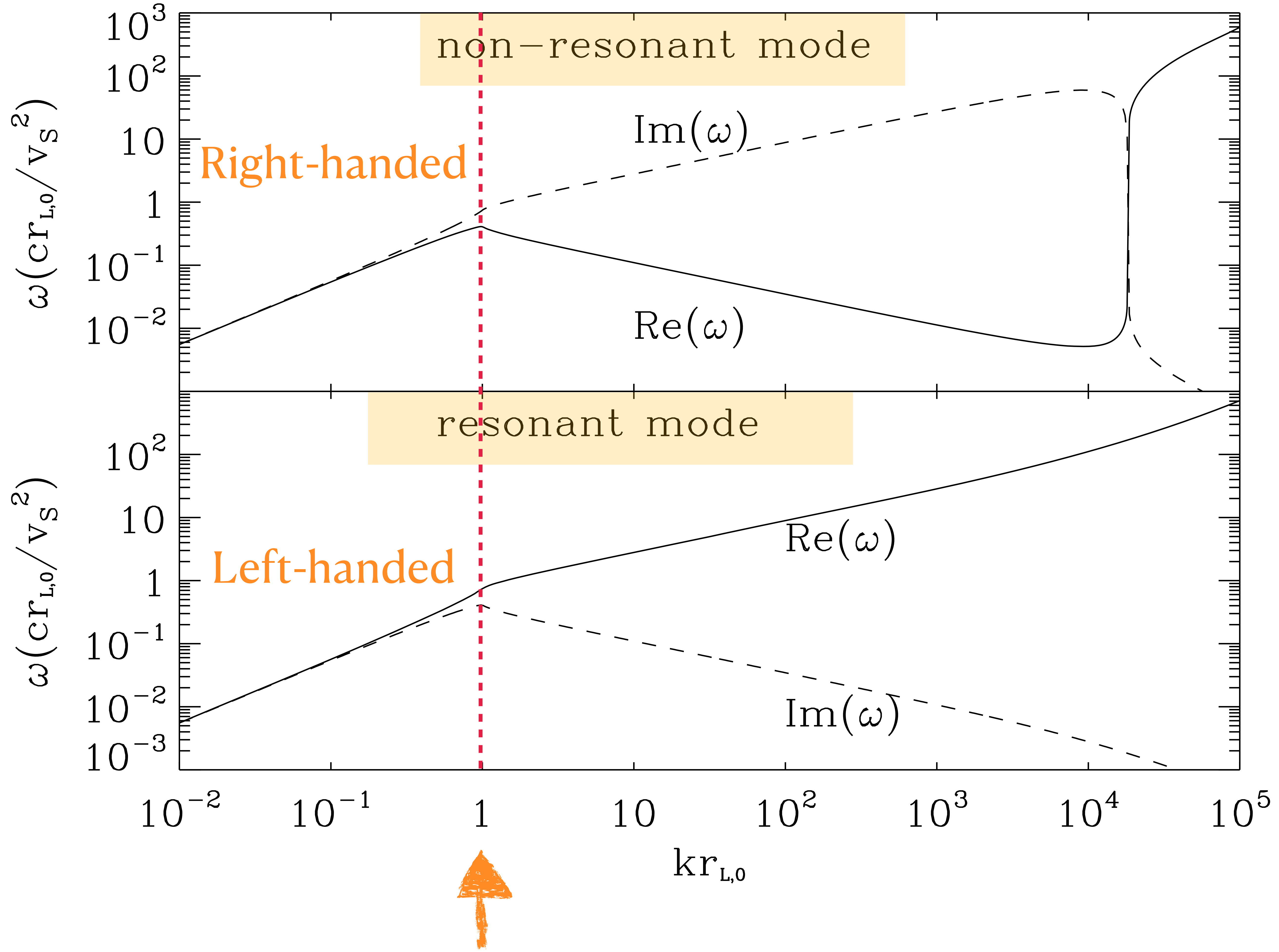
→

$$v_e = \frac{n_{CR}}{n_p + n_e} v_D \approx \frac{n_{CR}}{n_p} v_D << v_D$$

Perturbation of these equations leads to the well known dispersion relation:

$$\frac{c^2k^2}{\omega^2} = 1 + \sum_\alpha \frac{4\pi^2 q_\alpha^2}{\omega} \int_0^\infty \mathrm{d}p \int_{-1}^{+1} \mathrm{d}\mu \frac{p^2 v(p)(1-\mu^2)}{\omega + kv(p)\mu \pm \Omega_\alpha} \left[\frac{\partial f_\alpha}{\partial p} + \left(\frac{kv}{\omega} + \mu\right) \frac{1}{p} \frac{\partial f_\alpha}{\partial \mu}\right]$$

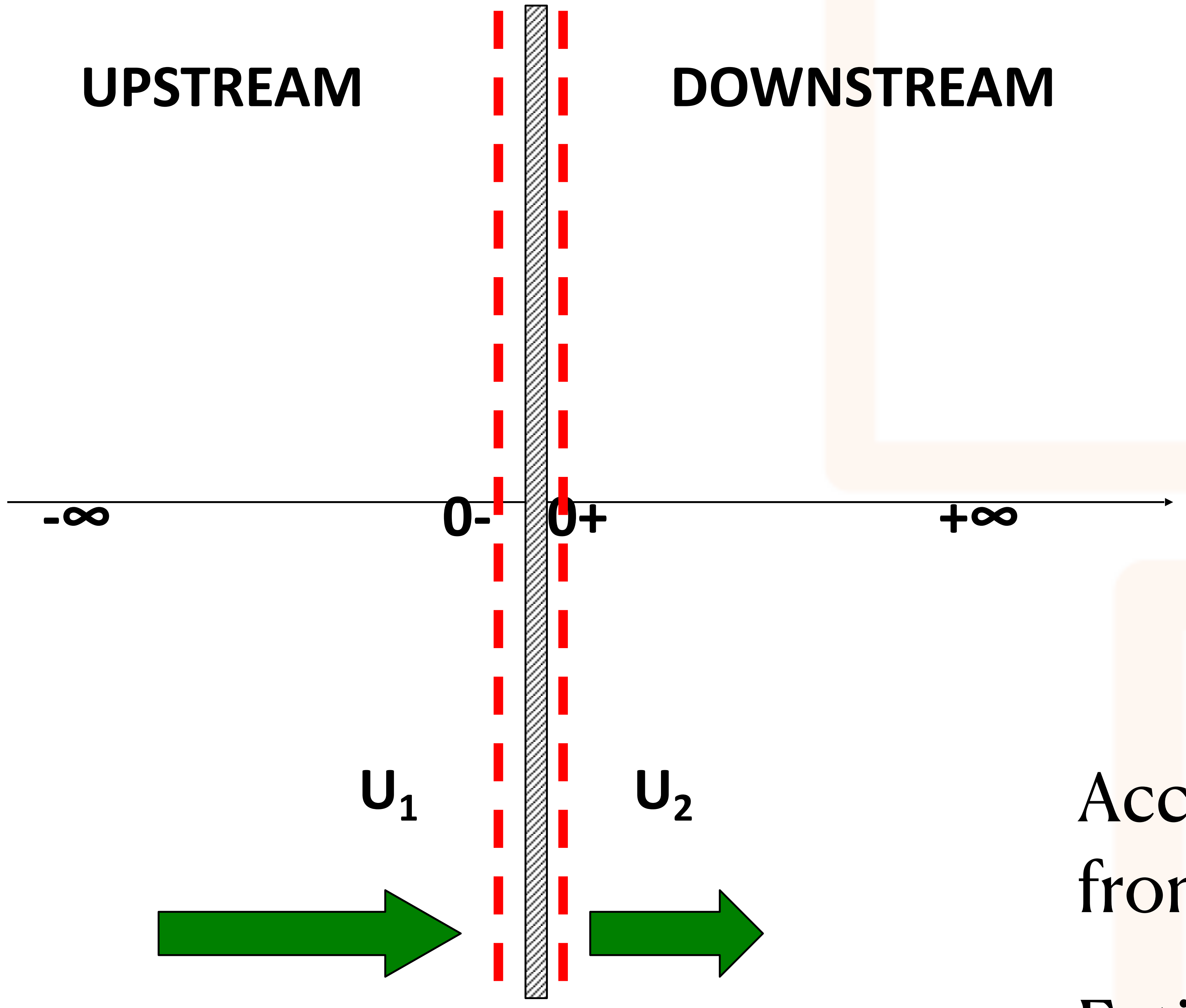
STREAMING INSTABILITIES



wavenumber k resonant with the gyration of particles in the CR current

- The LH modes represent weakly modified Alfven modes with an imaginary part that peaks at wavelength = Larmor radius of the particles
- The RH modes (Bell 2004) grow the fastest at $k_{max} r_L >> 1$ and are quasi-purely growing (Im(ω)>>Re(ω))
- By their very nature these latter modes are non-resonant and can in fact be obtained also in a treatment in which the background plasma is treated as MHD
- These modes are excited only if the current is strong enough

Basics of diffusive shock acceleration



$$\frac{\partial f_0}{\partial t} + u\frac{\partial f_0}{\partial z} - \frac{1}{3}\left(\frac{du}{dz}\right) p\frac{\partial f_0}{\partial p} = \frac{\partial}{\partial z}\left[D\frac{\partial f_0}{\partial z}\right]$$

Advection Compression Diffusion

FOR A PLANE PARALLEL SHOCK:

$$\frac{du}{dz} = (u_2 - u_1)\delta(z)$$

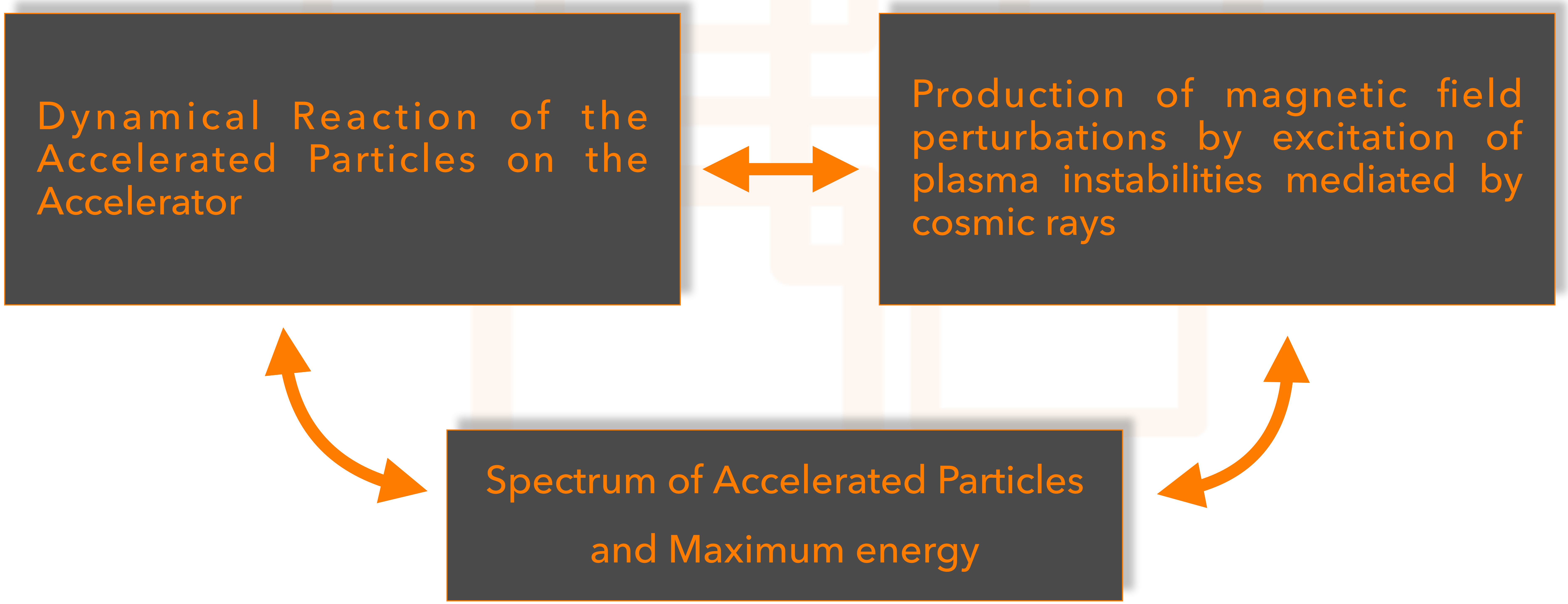
Acceleration due to the irreducible induced electric field across the shock front

Estimate of the acceleration time: $\tau acc(E) \simeq \frac{D(E)}{v_s^2} \propto \frac{E}{Bv_s^2}$

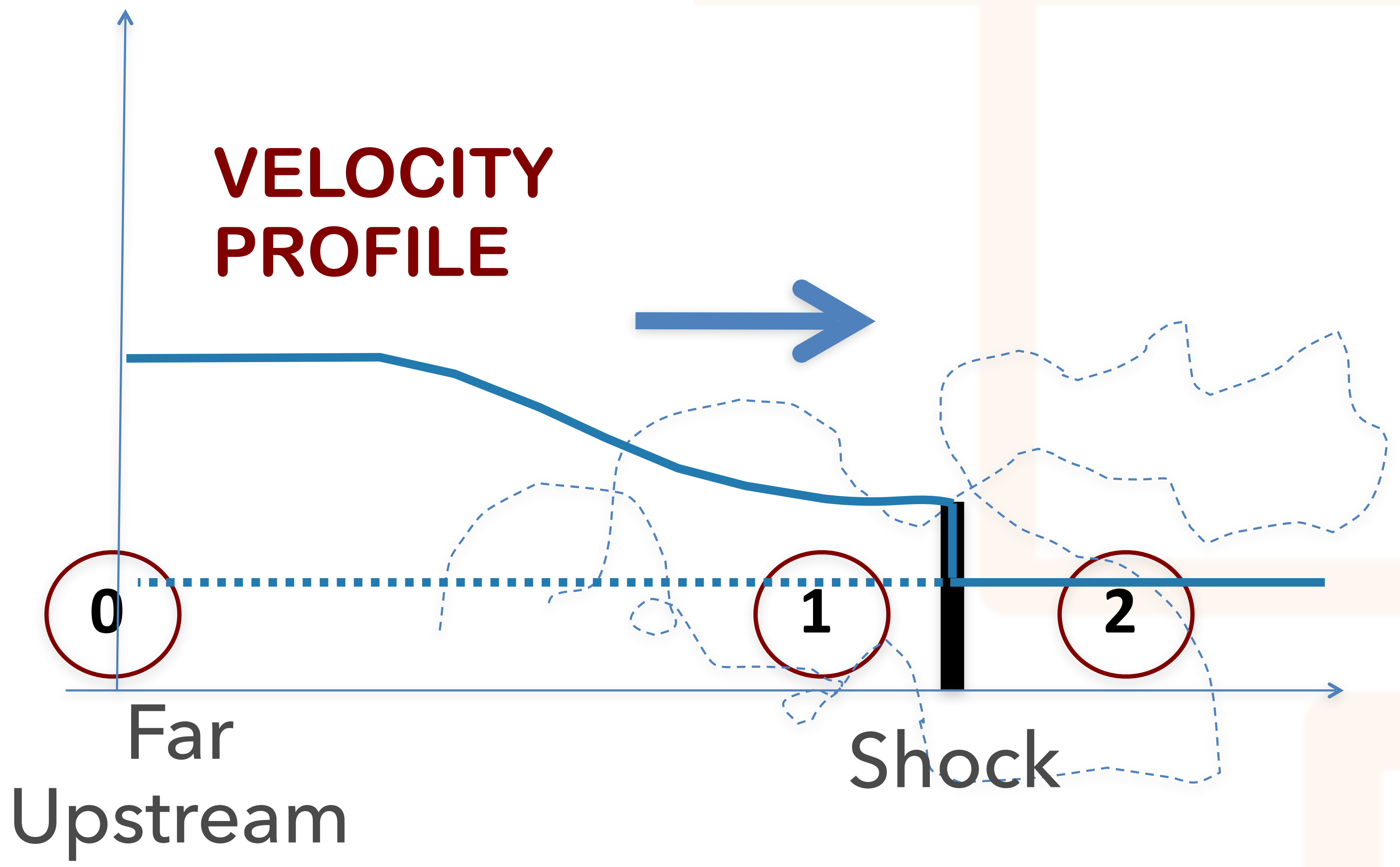
Acceleration to high E requires large v_s and/or large B

Modern Theory of DSA in SNRs

These theories aim at a description of the interplay between accelerated particles and the accelerator itself – the theory becomes non-linear and often untreatable analytically, but Physics is clear



Dynamical Reaction of Cosmic Rays



- Compression factor becomes a function of energy
- Spectra are not perfect power laws (concavity)
- Gas behind the shock is cooler because part of the energy has been used to energise CR

$$P_g(x) + \rho u^2 + P_{CR} = P_{g,0} + \rho_0 u_0^2$$

$$\frac{P_g}{\rho_0 u_0^2} + \frac{u}{u_0} + \frac{P_{CR}}{\rho_0 u_0^2} = \frac{P_{g,0}}{\rho_0 u_0^2} + 1$$

$$\frac{u}{u_0} \approx 1 - \xi_{CR}(x)$$

$$\xi_{CR}(x) = \frac{P_{CR}(x)}{\rho_0 u_0^2}$$

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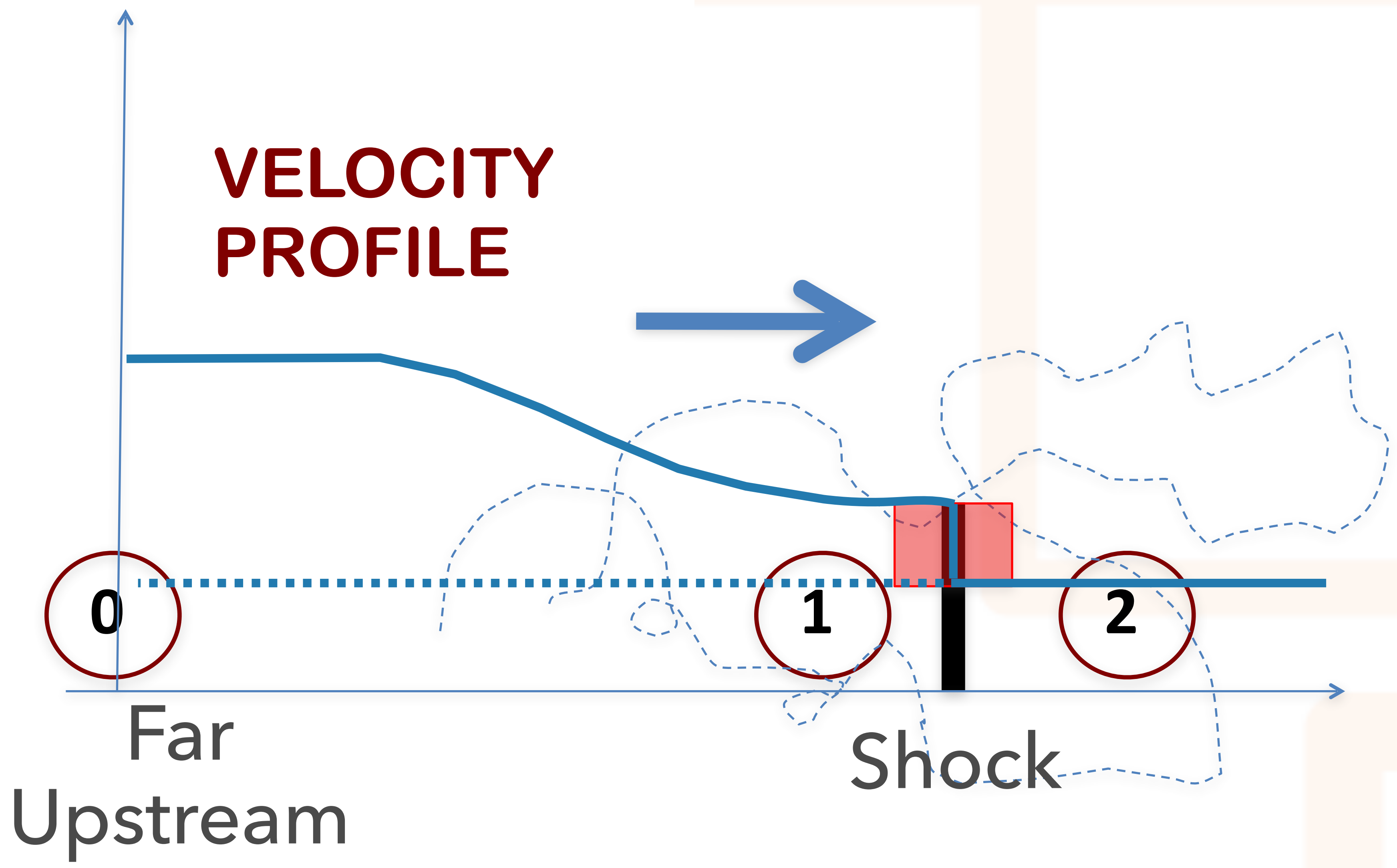

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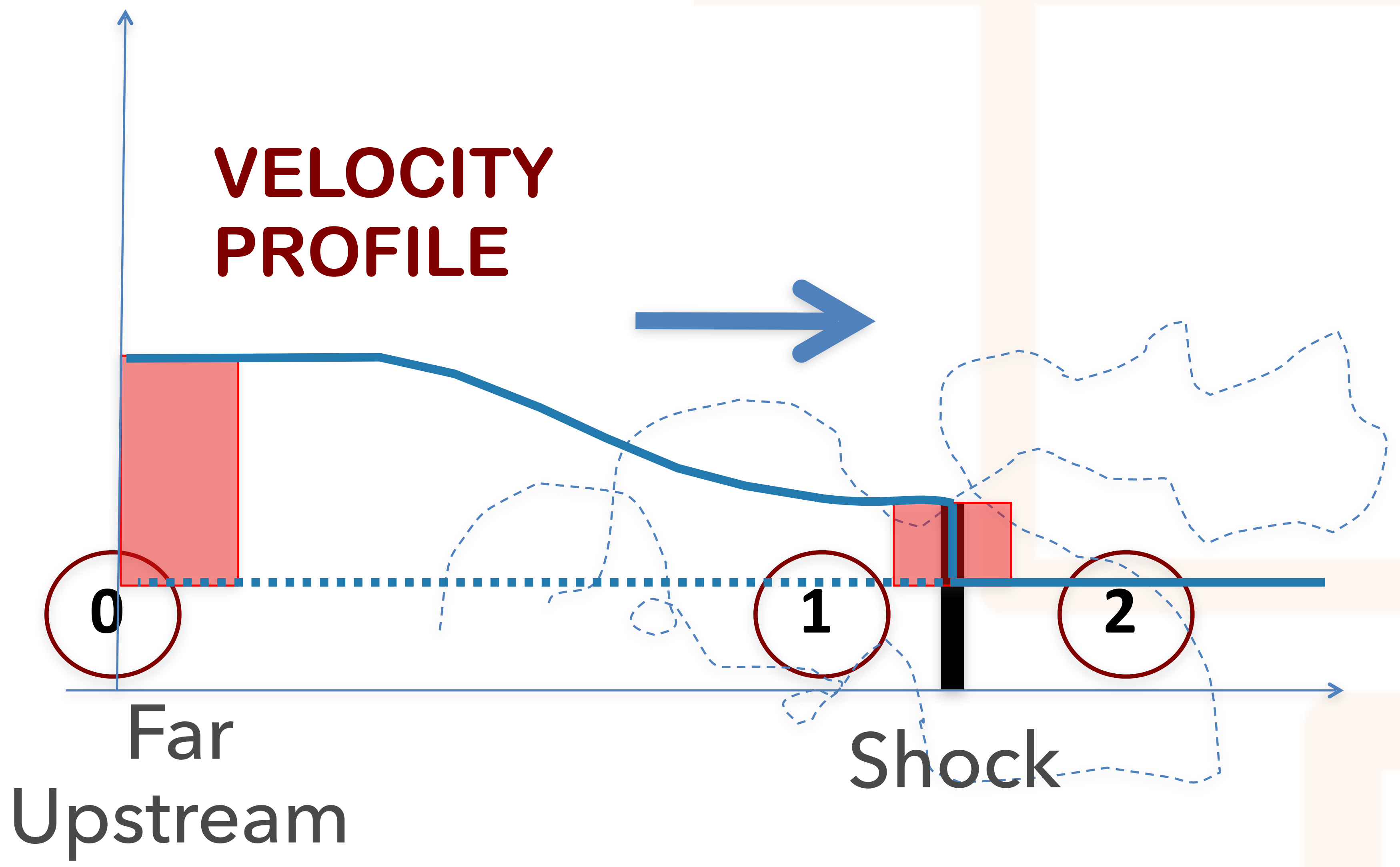

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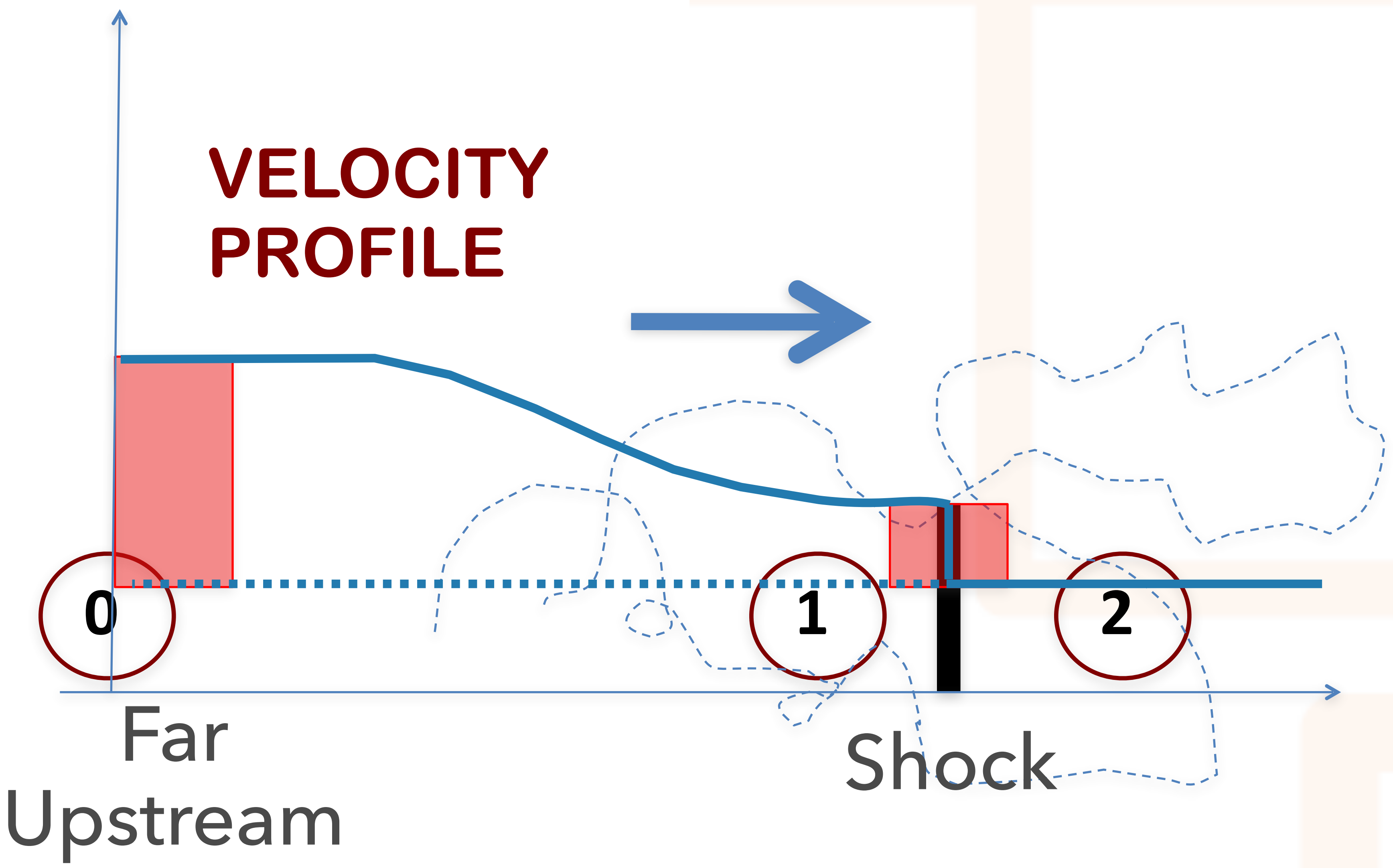

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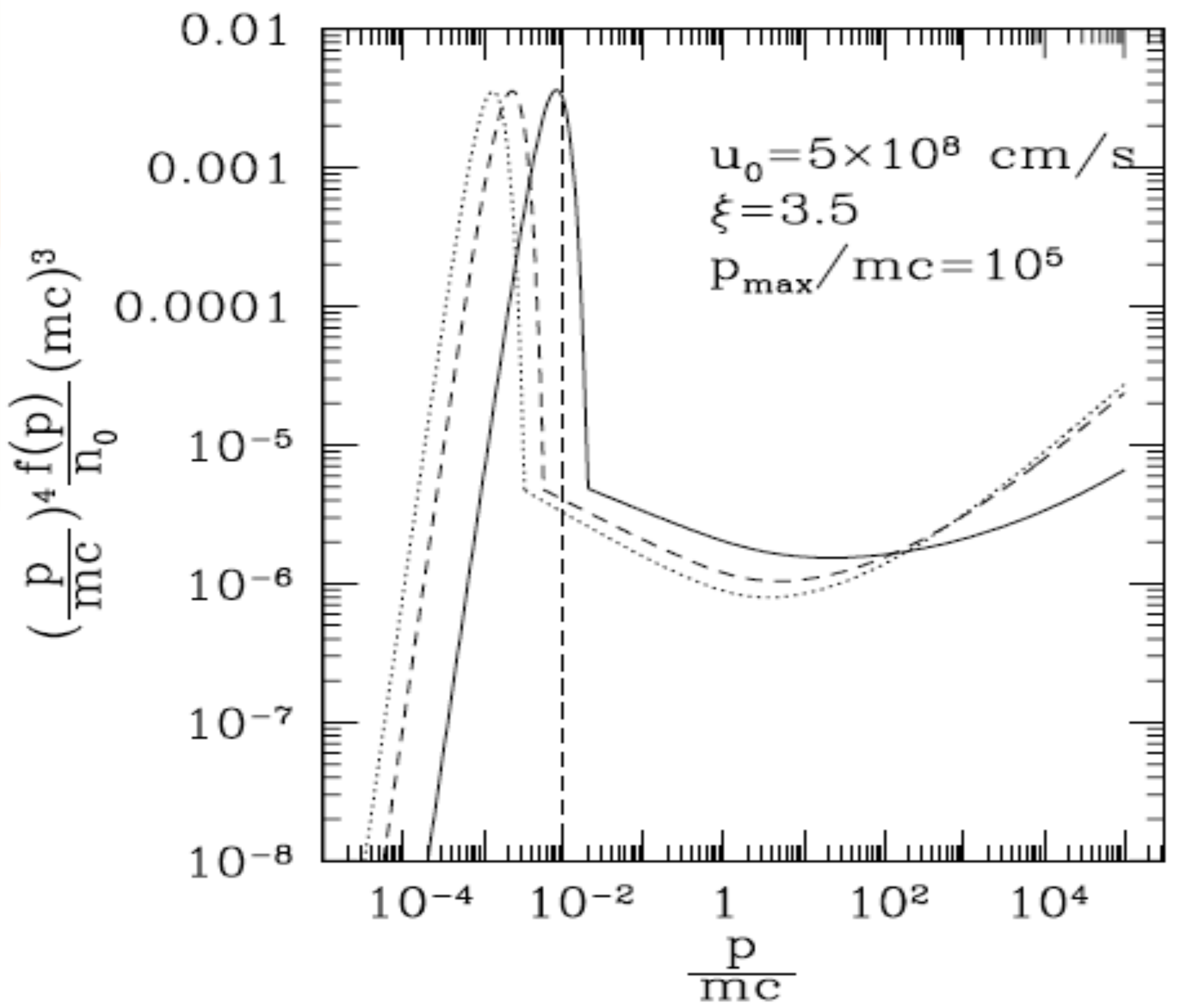
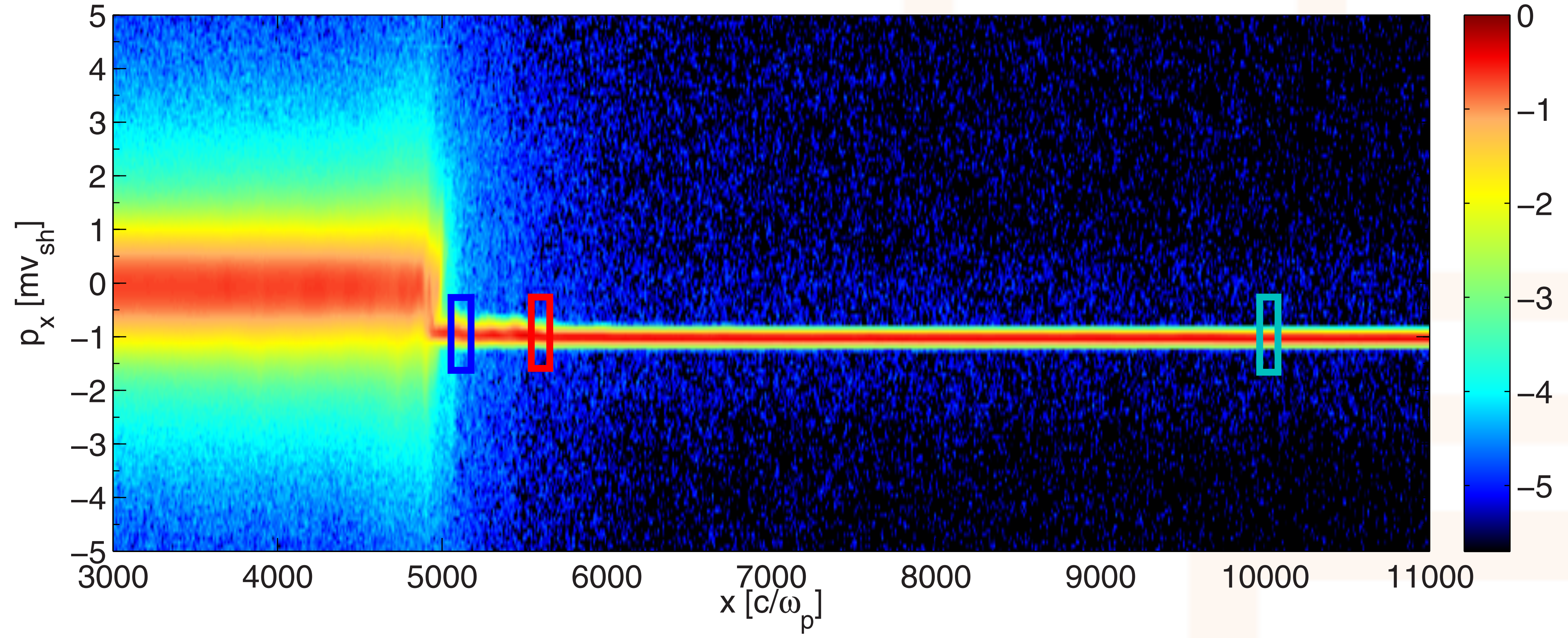


Malkov 2001, PB 2002, PB, Gabici & Vannoni 2005,...

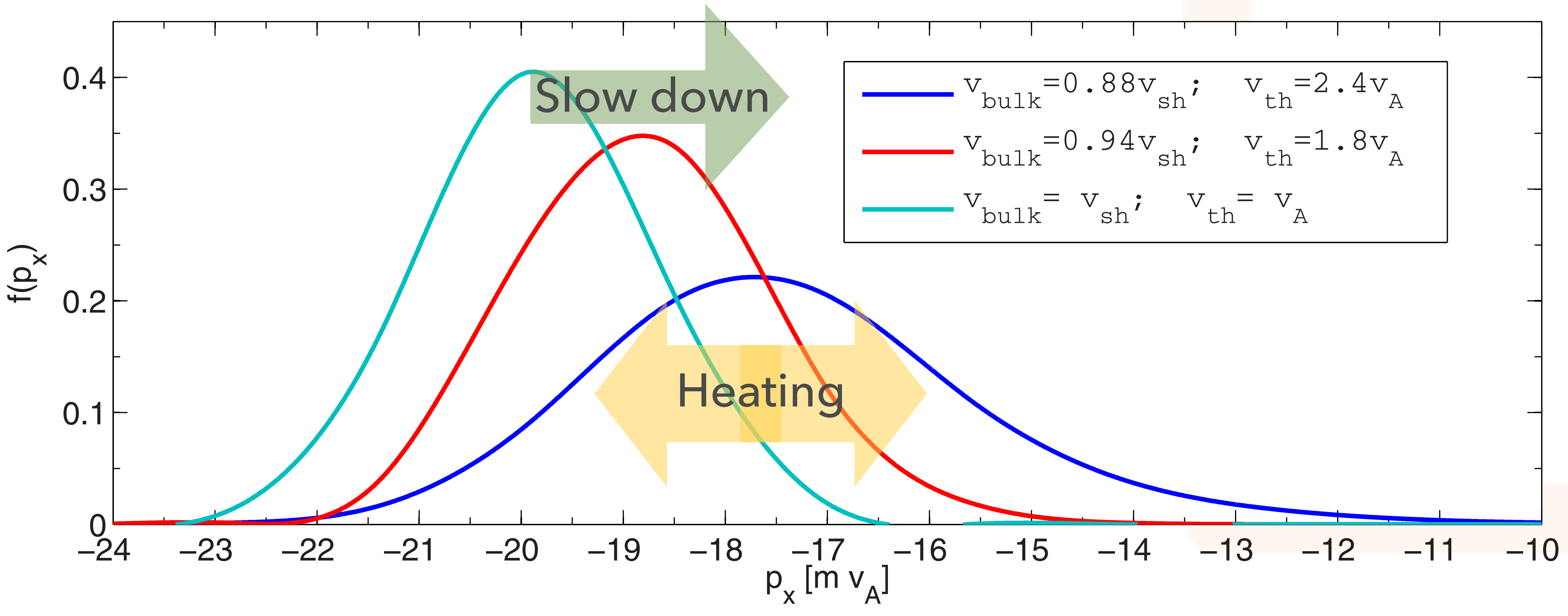
Dynamical Reaction of Cosmic Rays

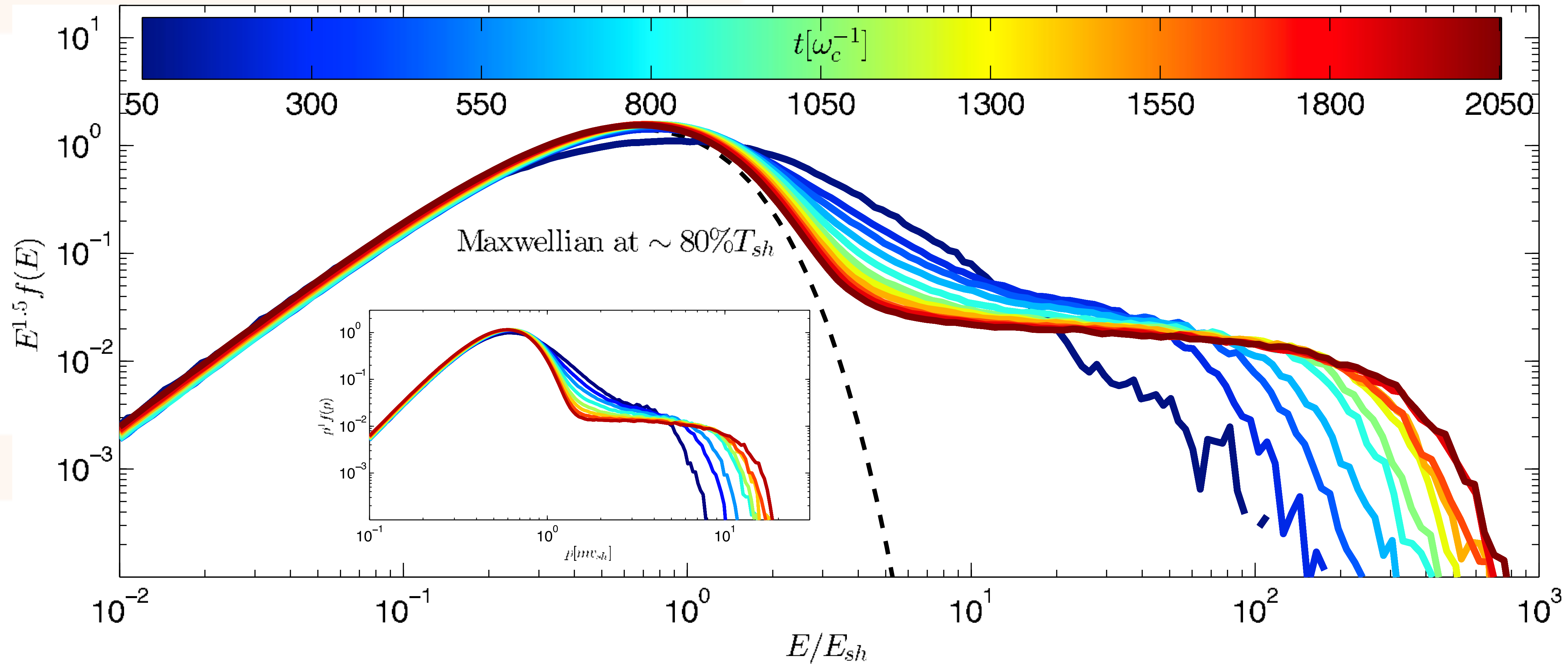


Hybrid simulations now confirm that the shock is modified by the accelerated particles…

They also confirm that some level of heating occurs also upstream, resulting in lower Mach number and a reduced curvature

As a result: spectra close to power laws and efficiency of order 10%





Caprioli & Spitkovsky 2014

Magnetic Field Amplification (MFA)

The single most important non linear effect that makes DSA interesting if the turbulent amplification of magnetic fields induced by the accelerated particles

The necessary condition for the process to be important for acceleration is that enough power is created in magnetic fields on the scale of the gyration radius of the particles you want to accelerate

The fields are needed to explain the narrow X-ray filaments in virtually all young SNR and to account for the maximum energy

The main channels that have been investigated, both analytically and numerically, are:

RESONANT STREAMING INSTABILITY
Kulsrud & Pearce 1969, Bell 1978, Lagage & Cesarsky 1982

NON RESONANT HYBRID STREAMING INSTABILITY
Bell 2004, Amato & PB 2009

ACOUSTIC INSTABILITY AND TURBULENT AMPLIFICATION
Drury & Falle 1986, Berezniak, Jones & Lazarian 2012

MFA through Resonant Streaming Instability

This is a phenomenon of the utmost importance for both Galactic CR transport and particle acceleration at shocks…

It requires that you have particles drifting at $v_D > v_A$ – **in the case of DSA $v_D \sim v_{shock} >> v_A$**

A small perturbation δB grows exponentially with a growth rate that can be easily estimated as:

$$\Gamma_{res}(k) = \frac{n_{CR}(>E)}{n_i}\frac{v_{shock}}{v_A}\Omega_{cyc} = \frac{\xi_{CR}}{\Lambda} M_A \left(\frac{v_{shock}}{c}\right)^2 \frac{c}{r_L(E)}$$

Assuming a spectrum E^{-2}

$$M_A = \frac{v_{shock}}{v_A}$$

Alfvenic Mach number of the Shock

$$\Lambda = \ln(E_{max}/m_p c^2) \sim 10$$

…and the instability grows on scales $k \sim 1/r_L(E)$ –– ***Once δB becomes of order B_0 (pre-existing field) the process stops!***

Imposing that the acceleration time equals the beginning of Sedov-Taylor:

$$E_{max} \approx 20\ TeV \left(\frac{v_{shock}}{10^9\ \mathrm{cm/s}}\right)^2 \left(\frac{B_0}{3\mu G}\right)\left(\frac{T_{Sedov}}{150\ \mathrm{yrs}}\right)$$

Lagage & Cesarsky 1982

In general the instability is quenched by ion-neutral damping or non linear Landau damping depending on environment, at earlier times…

Some useful considerations

Adopting Quasi-Linear Theory as a benchmark, one can write the diffusion coefficient as:

$$D(E) = \frac{1}{3} v \frac{r_L(E)}{F(k)} |_{k=1/r_L} \qquad F(k) = \left(\frac{\delta B(k)}{B_0}\right)^2$$

In the presence of resonant streaming instability, F(k)= constant if the spectrum of accelerated particles is $\sim E^{-2}$, so that diffusion is linear in E (Bohm diffusion)

If one requires that E_{max}=1 PeV it is easy to infer that:

$$F(k)|_{k=1/r_L(1PeV)} \approx 140 \left(\frac{v_{shock}}{10^9 \text{ cm/s}}\right)^{-2} \left(\frac{B_0}{3\mu G}\right)^{-1} \left(\frac{T_{Sedov}}{150 \text{ yrs}}\right)^{-1} >> 1$$

…namely for a SNR to be a PeVatron, **one cannot be in the regime of resonant streaming instability**!

THE BELL INSTABILITY
[a.k.a. Non resonant Hybrid Instability]

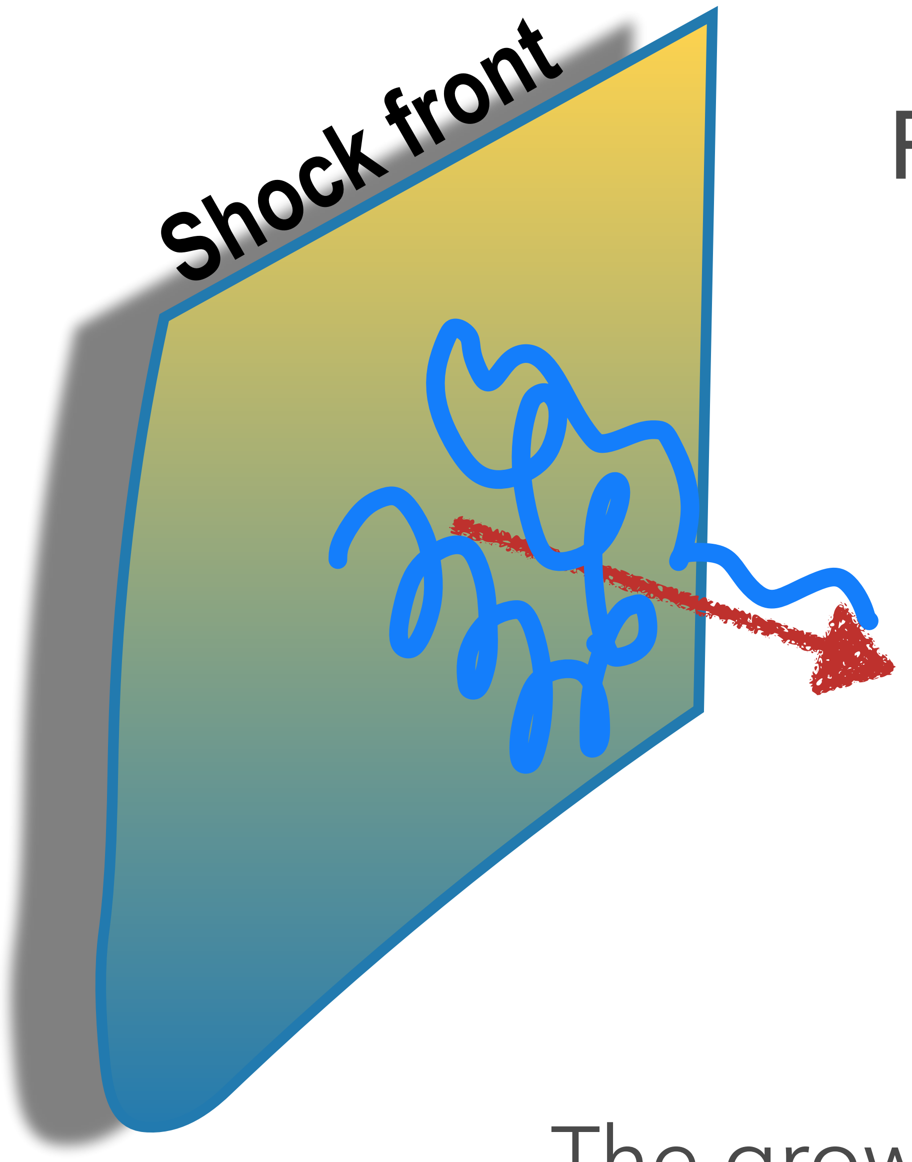
This instability was discovered in 2004 by T. Bell and attracted immediately much attention, for several reasons:

1) *Under certain conditions (see below) it grows much faster than the RSI discussed earlier*
2) *The level of δB reached seems to compare well with those inferred from thickness of X-ray filaments*
3) *It is potentially capable to allow acceleration to larger energies*

Reasons of concern…

a) *it develops on very small scales compared with the Larmor radius – in the beginning no scattering*
b) *it grows the "wrong" polarisation in the linear regime… again, in the beginning no scattering*

The Bell Instability
[a.k.a. Non Resonant Hybrid Instability]



Protons of given energy upstream of the shock represent a current $J_{CR}=n_{CR}(>E)\, e\, v_{shock}$

The background plasma cancels the CR positive current with a return current created by a slight relative motion between thermal electrons and protons, thereby creating a two stream instability that grows the fastest on scales $l \sim 1/k_{max}$ where

$$k_{max} B_0 = \frac{4\pi}{c} J_{CR}$$

The growth occurs at a rate that can be approximated as: $\Gamma_{max} = k_{max} v_A$

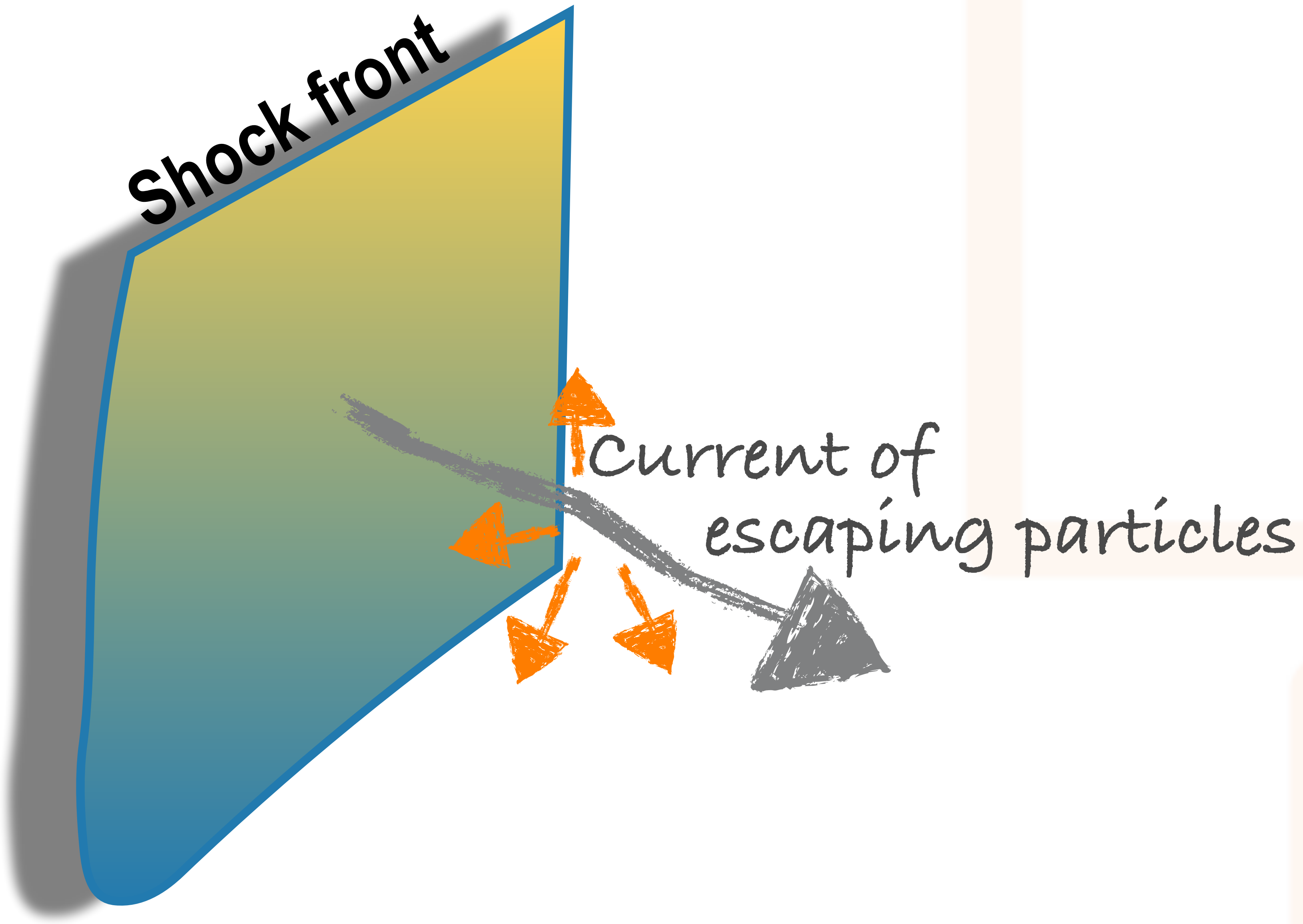
The condition for this instability to develop is that $k_{max}>1/r_L(E)$, which is equivalent to requiring that:

$$n_{CR}(>E) E \frac{v_{shock}}{c} > \frac{B_0^2}{4\pi} \quad \Rightarrow \quad M_A > \left(\frac{\Lambda}{\xi_{CR}} \frac{c}{v_{shock}}\right)^{1/2} \approx 500 \left(\frac{\xi_{CR}}{0.1}\right)^{-1/2} \left(\frac{v_{shock}}{10^9 \text{ cm/s}}\right)^{-1/2}$$

for E^{-2} spectrum

Only works in very young SNR with large Alfvenic Mach number

Saturation and Maximum Energy in SNR



The current of escaping particles acts as a force on the background plasma in the direction perpendicular to both the current and the amplified field:

$$\rho \frac{dv}{dt} \sim \frac{1}{c} J_{CR} \delta B \longrightarrow \Delta x \sim \frac{J_{CR}}{c\rho} \frac{\delta B(0)}{\gamma_{max}^2} exp(\gamma_{max} t)$$

The current is weakly disturbed until the transverse displacement becomes of order the Larmor radius in the amplified field…this condition leads to the following saturation condition:

$$\frac{\delta B^2}{4\pi} = n_{CR}(> E) E \frac{v_{shock}}{c} \approx \frac{\xi_{CR}}{\Lambda} \rho v_{shock}^2 \frac{v_{shock}}{c}$$ independent on scale (Bohm Diff)

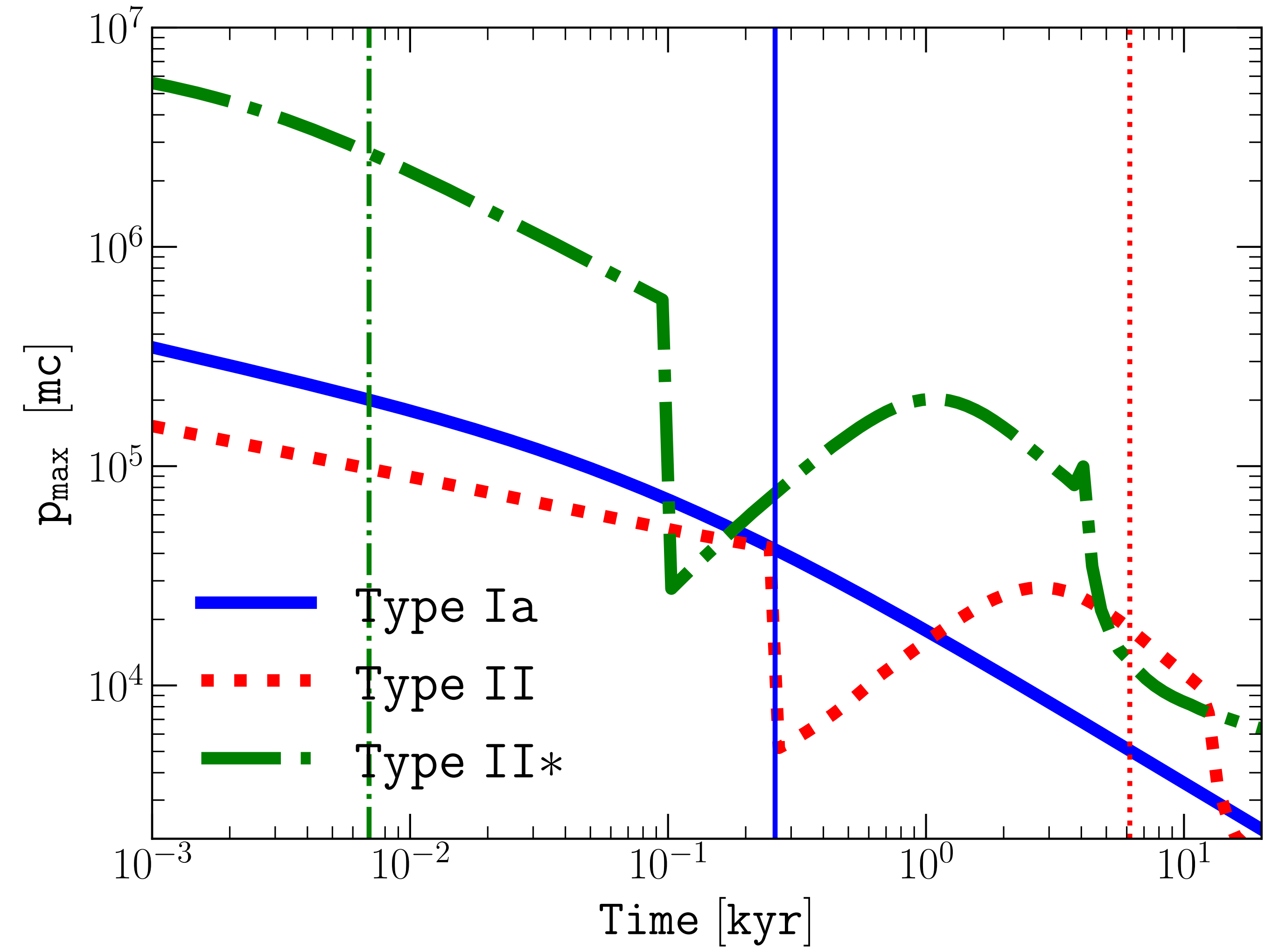
The maximum energy at time T is estimated by requiring that the growth time of the instability equals ~T/5 (5 e-folds) - this condition dominates upon the condition on acceleration time:

$$E_{max} \approx \frac{\xi_{CR}}{10\Lambda} \frac{\sqrt{4\pi\rho}}{c} e R_{shock}(T) v_{shock}^2(T)$$

The time dependence of R_{shock} and v_{shock} is different depending on the type of SNR

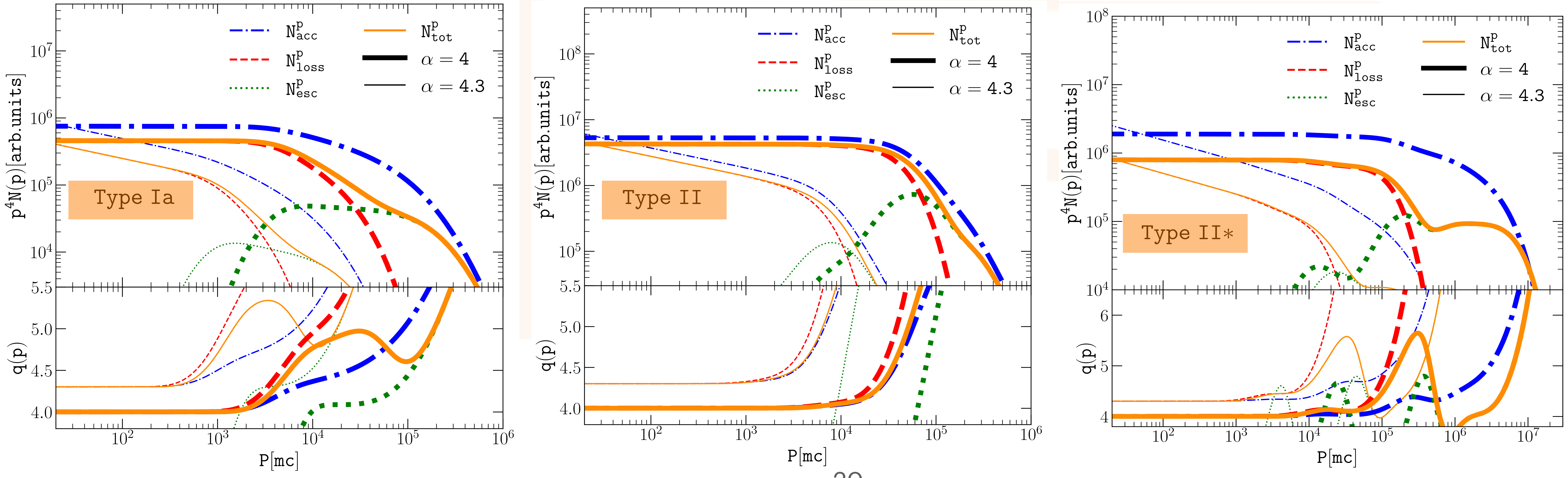
Maximum Energy of CR in SNR

Cristofari, PB & Amato 2020



CR accelerated at SNR are liberated in two stages:

1) particles leave the remnant at each time t with energy $E_{max}(t)$
2) the particles trapped downstream (lower E) lose energy adiabatically and escape at the end of the SNR life
3) The time integrated spectrum drops at an **effective E_{max} that is the maximum energy reached at the beginning of Sedov**
4) The cut is NOT exponential, it's a power law reflecting time dependence in the ejecta dominated phase



Cristofari, PB & Caprioli 2022

Role of CR induced Instabilities during Transport

Excitation of instabilities during Galactic transport	Excitation of Resonant and non-Resonant Instability around sources
The global gradient (current) of Galactic CR can be estimated from observations The resonant instability grows on time scales of order 10^3 years for GeV particles (No non-resonant SI) The growth is mainly limited by NLLD The effect stops at <TeV where other processes must kick in Recent insights from hybrid simulations into NLLD put this into a new framework	The gradients can be expected to be much larger near sources For middle age supernova remnants, the escaping cosmic rays with E<TeV can be confined near sources for long times For young supernovae the current can be strong enough to excite the non-resonant instability The confined cosmic rays accumulate substantial overpressure and can excavate cavities in the surrounding medium by exerting $-\nabla P$ force

THE ROLE OF NLLD IN REGULATING STREAMING INSTABILITIES

- Classical picture -

- In the absence of any damping process, SI leads to a reduction of a super-Alfvenic drift velocity because particles start scattering

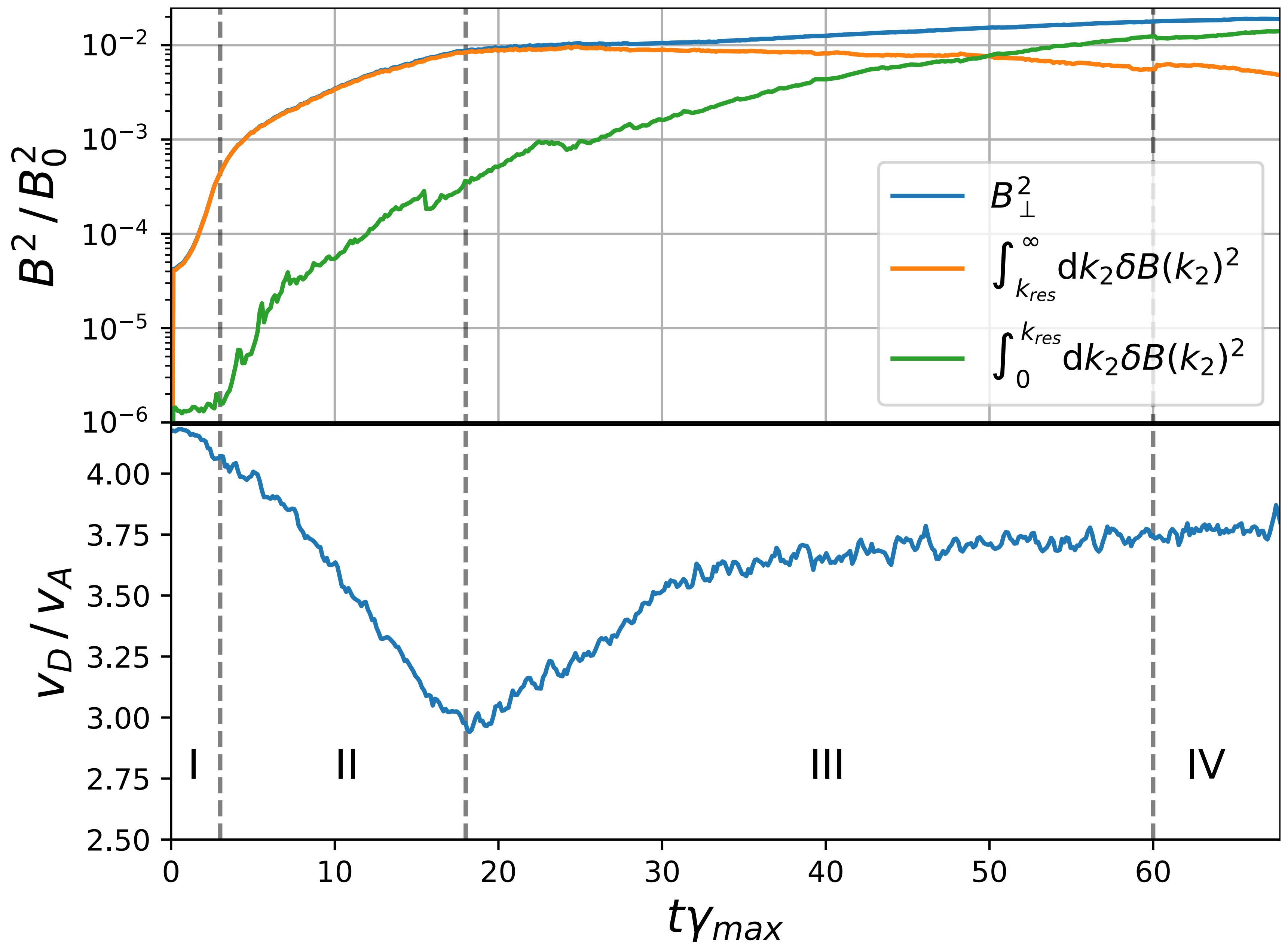
- As a result the process leads to a situation in which the drift becomes Alfvenic (namely the particles are now isotropic in the reference frame of the Alfven waves)

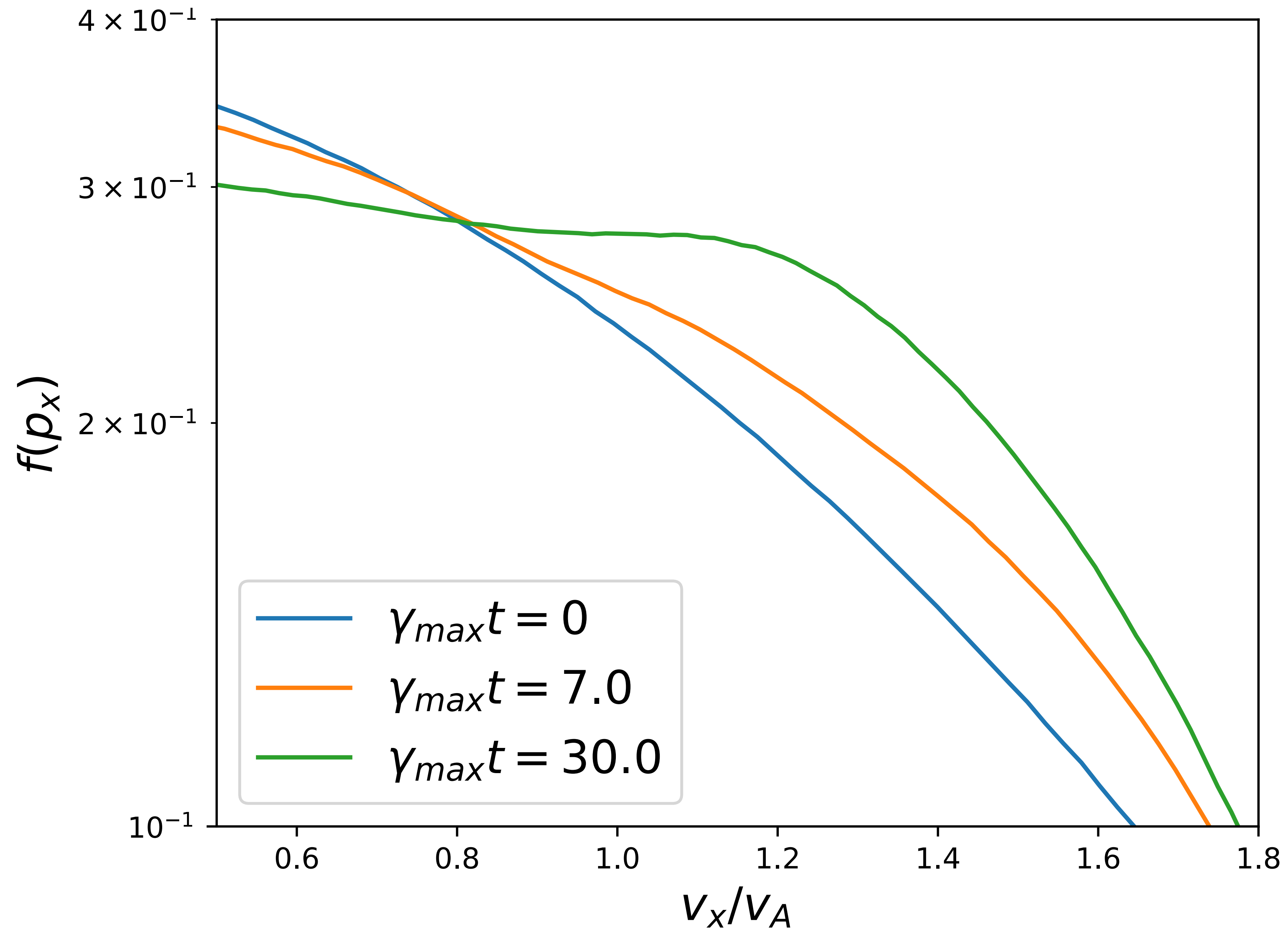
- In the presence of damping, wave growth continues until damping kicks in and the growth stops, hence the drift remain super-Alfvenic at saturation of the instability:

$$\Gamma_{CR}^{SI}(k) = \Gamma_{NLLD}(k) \rightarrow$$ Equilibrium Spectrum of perturbations as a function of k and hence diffusion coefficient

THE ROLE OF NLLD IN REGULATING STREAMING INSTABILITIES

- Hybrid PIC Simulations - Schroer+2025,2026





The excitation of the resonant SI slows down the super-Alfvenic drift

Low k modes lead more damping and slight increase of the drift speed

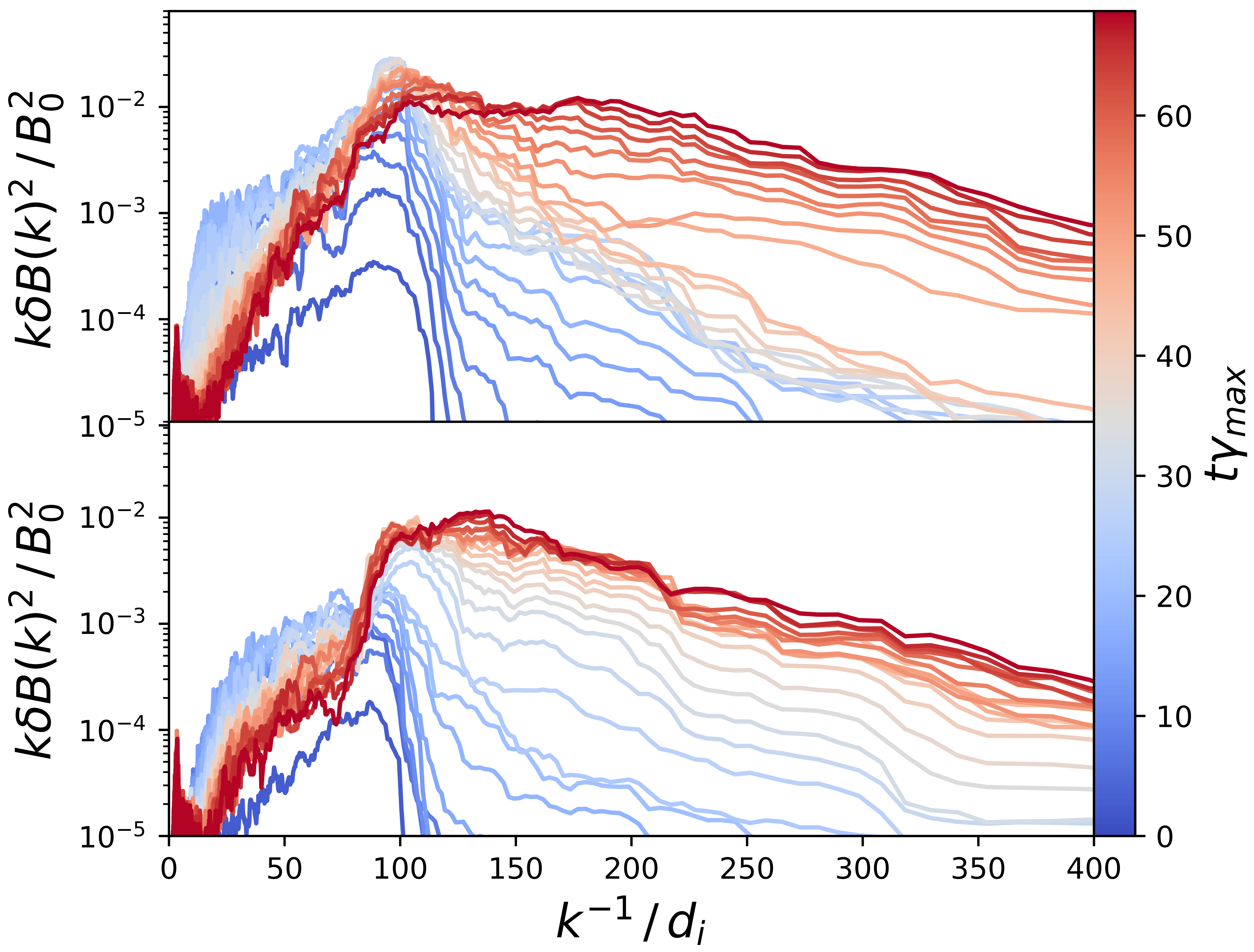
Eventually the drift speed saturates

We see heating of the thermal plasma in coincidence with damping

The damping saturates when a plateau is reached in the thermal distribution

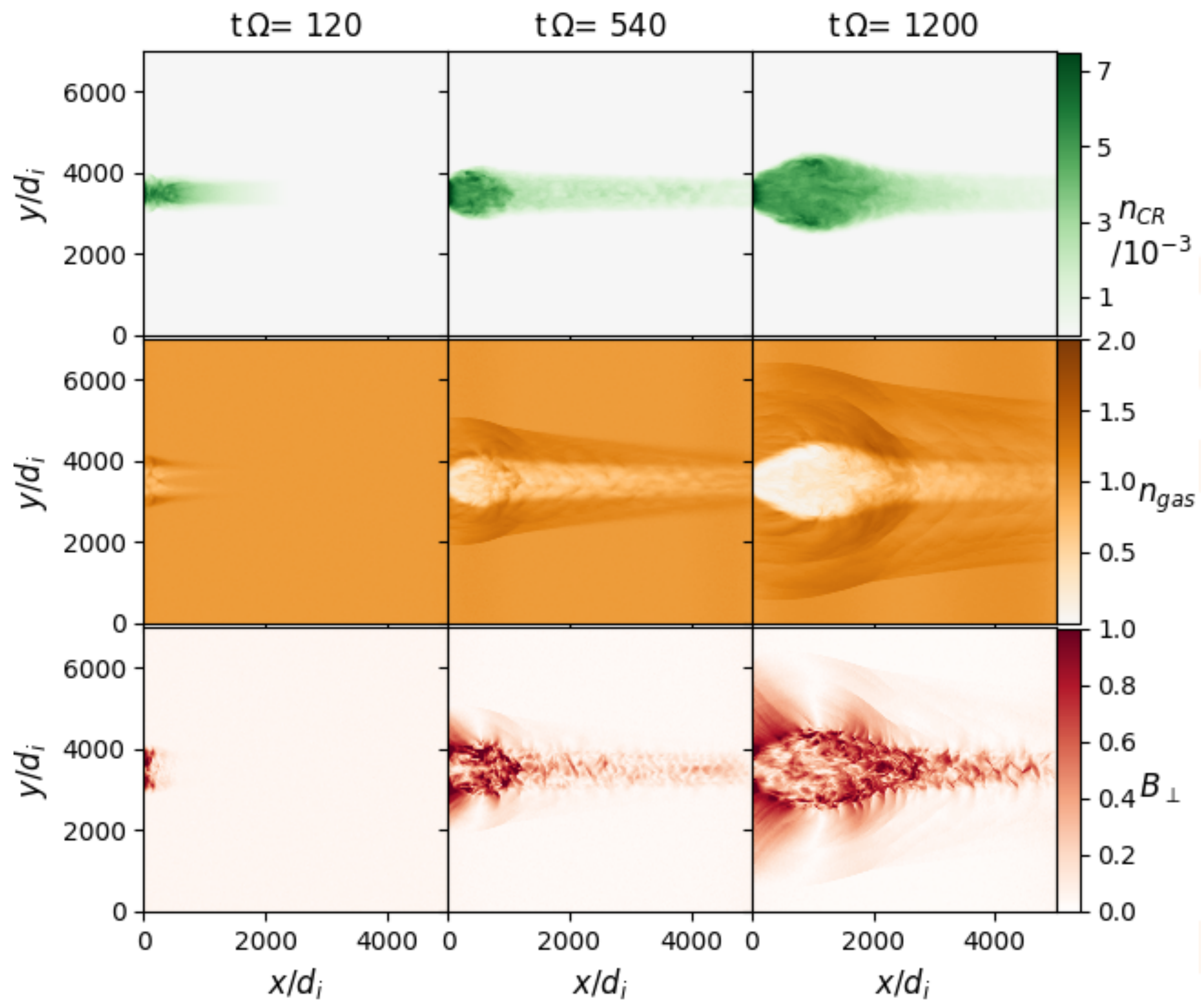
THE ROLE OF NLLD IN REGULATING STREAMING INSTABILITIES

- Hybrid PIC Simulations - Schroer+2025,2026



- Initially the resonant SI leads to the growth of perturbations on the scale of the larmor radius of the particles
- When NLLD sets in, we see that the perturbations at the resonant scale saturate but power appears to larger scales (inverse cascade)
- These perturbations contribute to the damping but also can scatter with higher energy particles
- Even more important, if Alfvenic perturbations were pre-existing they would dominate the damping with catastrophic implications for CR scattering
- The waves are either completely self-generated or none of them are...

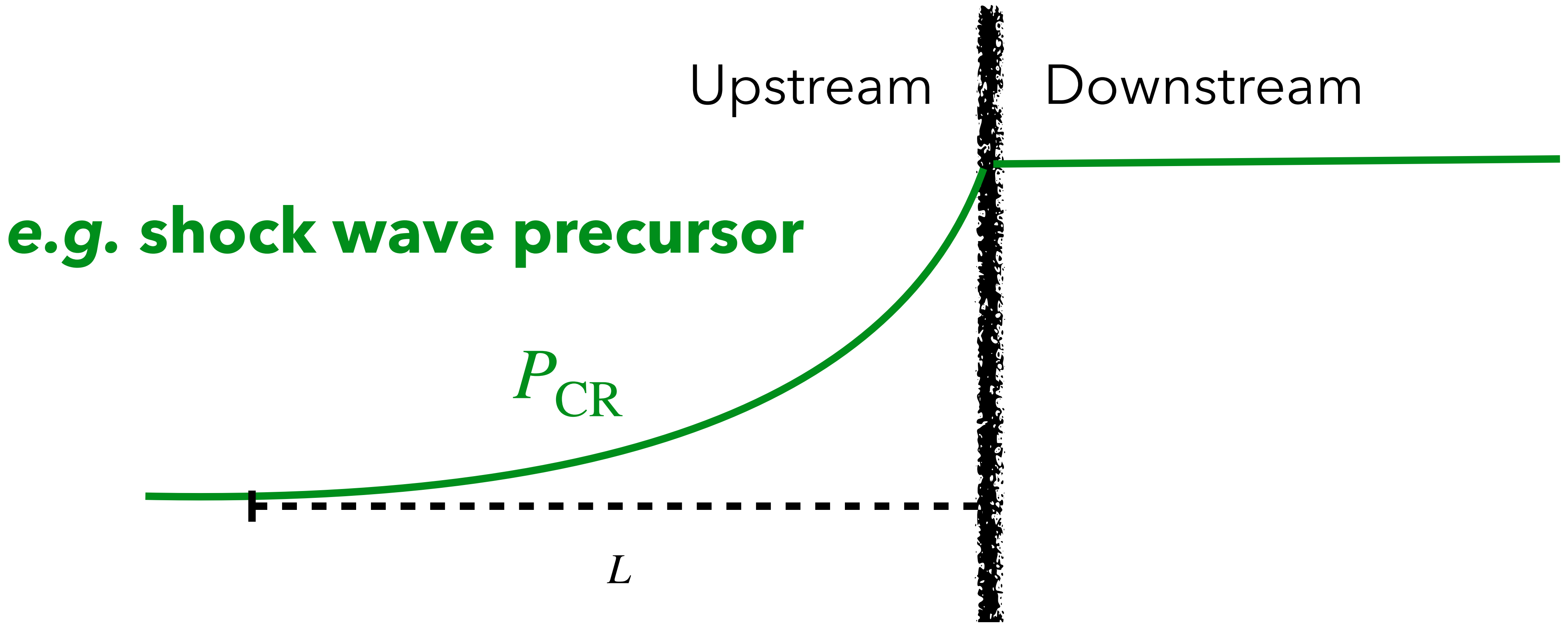
Hybrid PIC simulations



Schroer+, 2021 and 2022

- The excitation of the instability leads to strong particle scattering, which in turn increases CR density near the source
- The pressure gradient that develops creates a force that leads to the inflation of a bubble around the source
- The same force evacuates the bubble of most plasma
- There is no field in the perp direction to start with, but CR create it at later times (suppressed diffusion, about 10 times Bohm)

Acoustic Instability + Kinematic dynamo

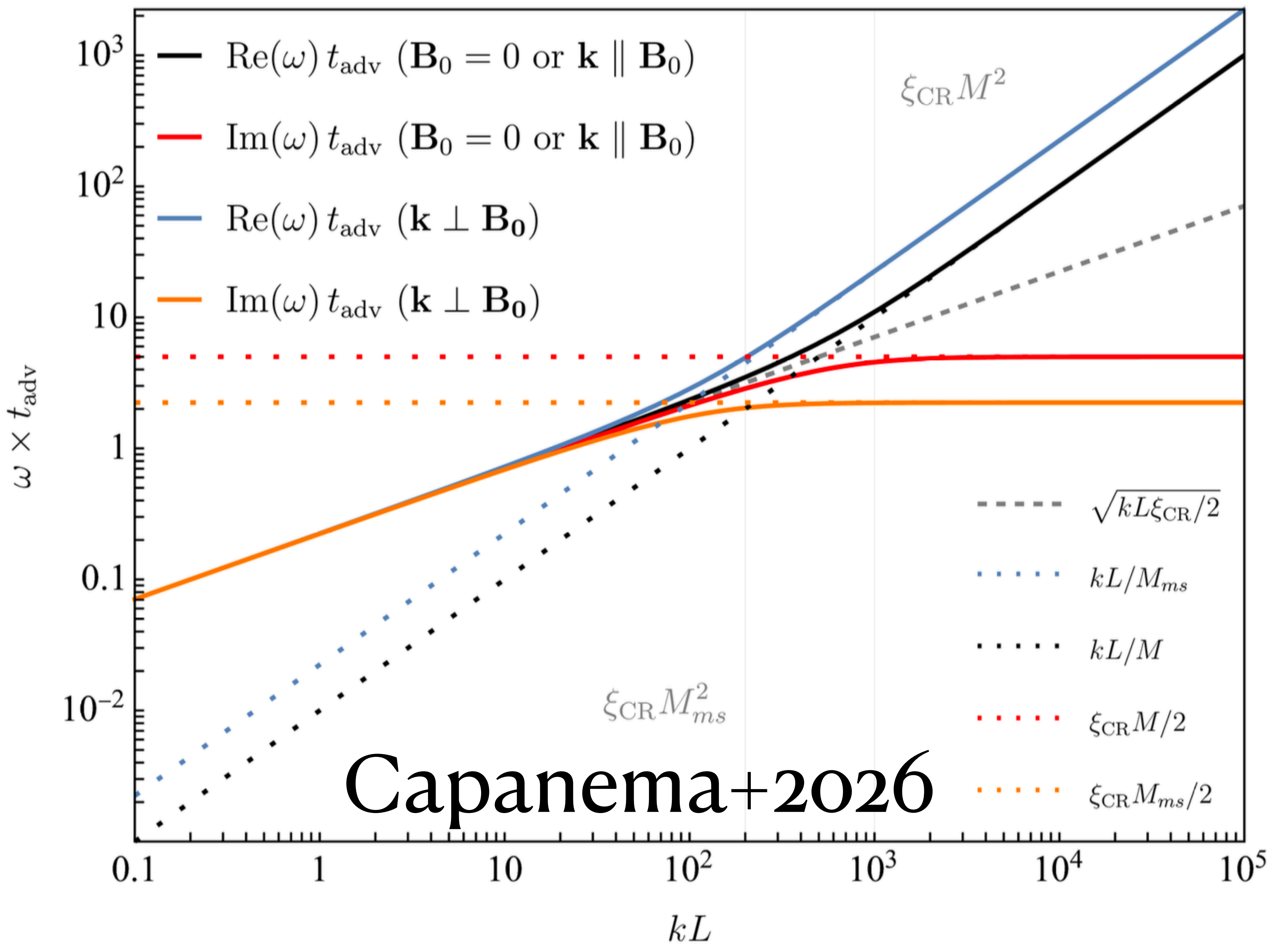


A sound wave or a magneto-sonic wave moving in the pressure gradient of upstream CR is unstable (Drury & Falle 1986, Drury & Downes 2012)

$$P_{CR}(x) = \xi_{CR}\, \rho_0 u_0^2\, \frac{x}{L}$$

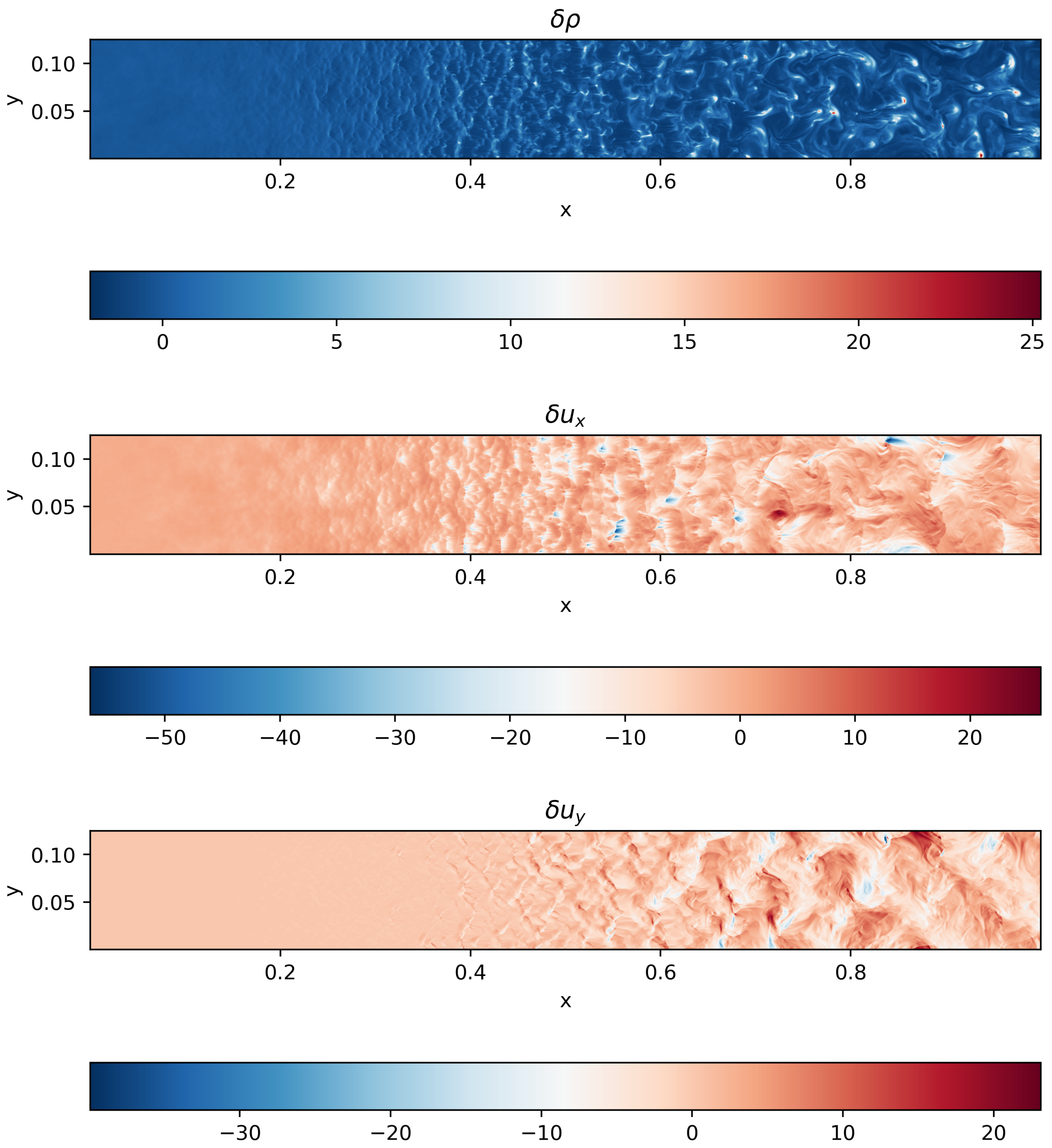
$$\nabla P_{CR}(x) = \xi_{CR}\, \frac{\rho_0 u_0^2}{L}\, \hat{\mathbf{x}}$$

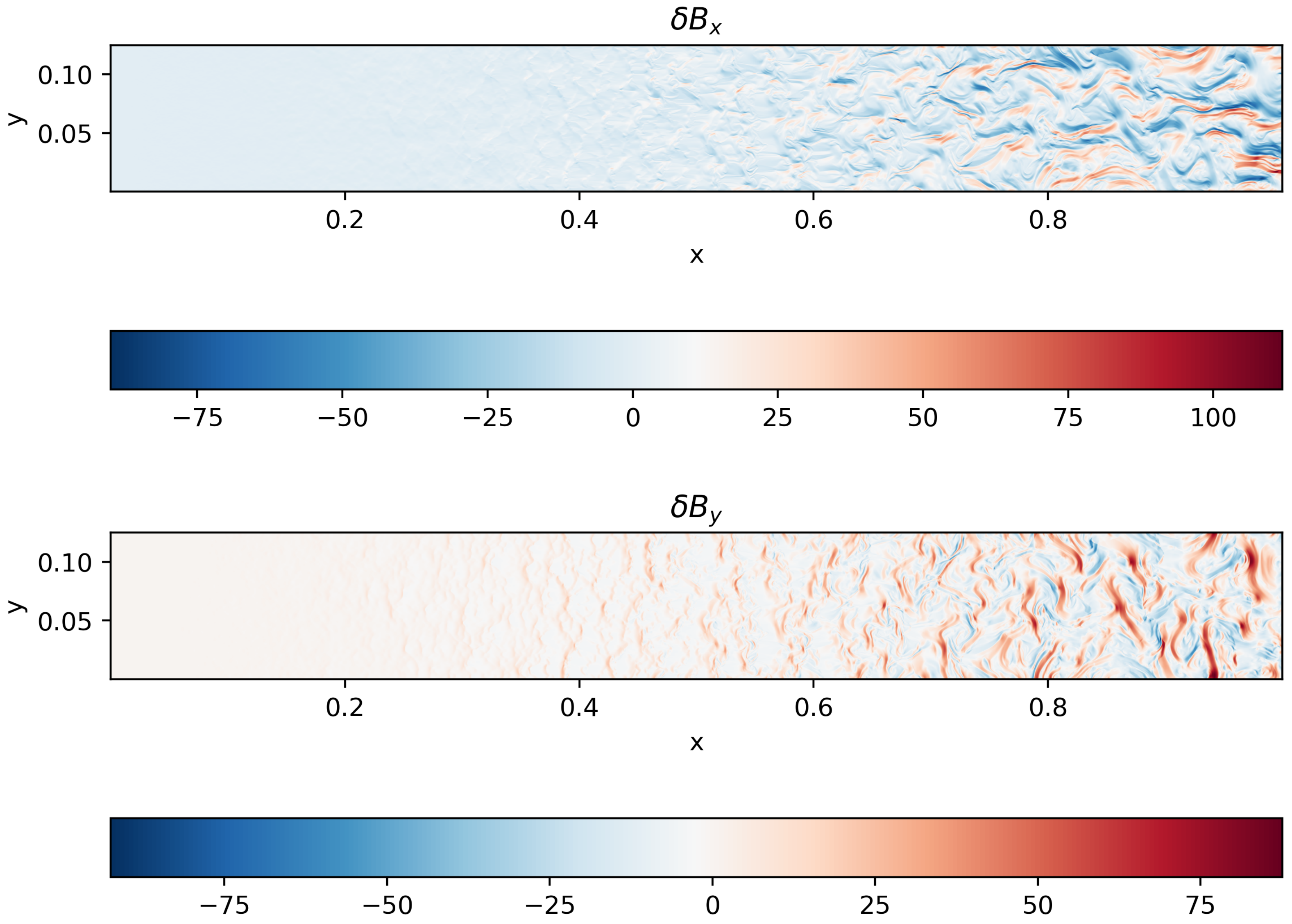
$$\Gamma \rightarrow \begin{cases} \sqrt{\dfrac{k\,\xi_{CR} u_0^2}{2L}}\,, & kL \ll \xi_{CR} M^2 \\ \dfrac{\xi_{CR} M u_0}{2L}\,, & kL \gg \xi_{CR} M^2 \end{cases}$$

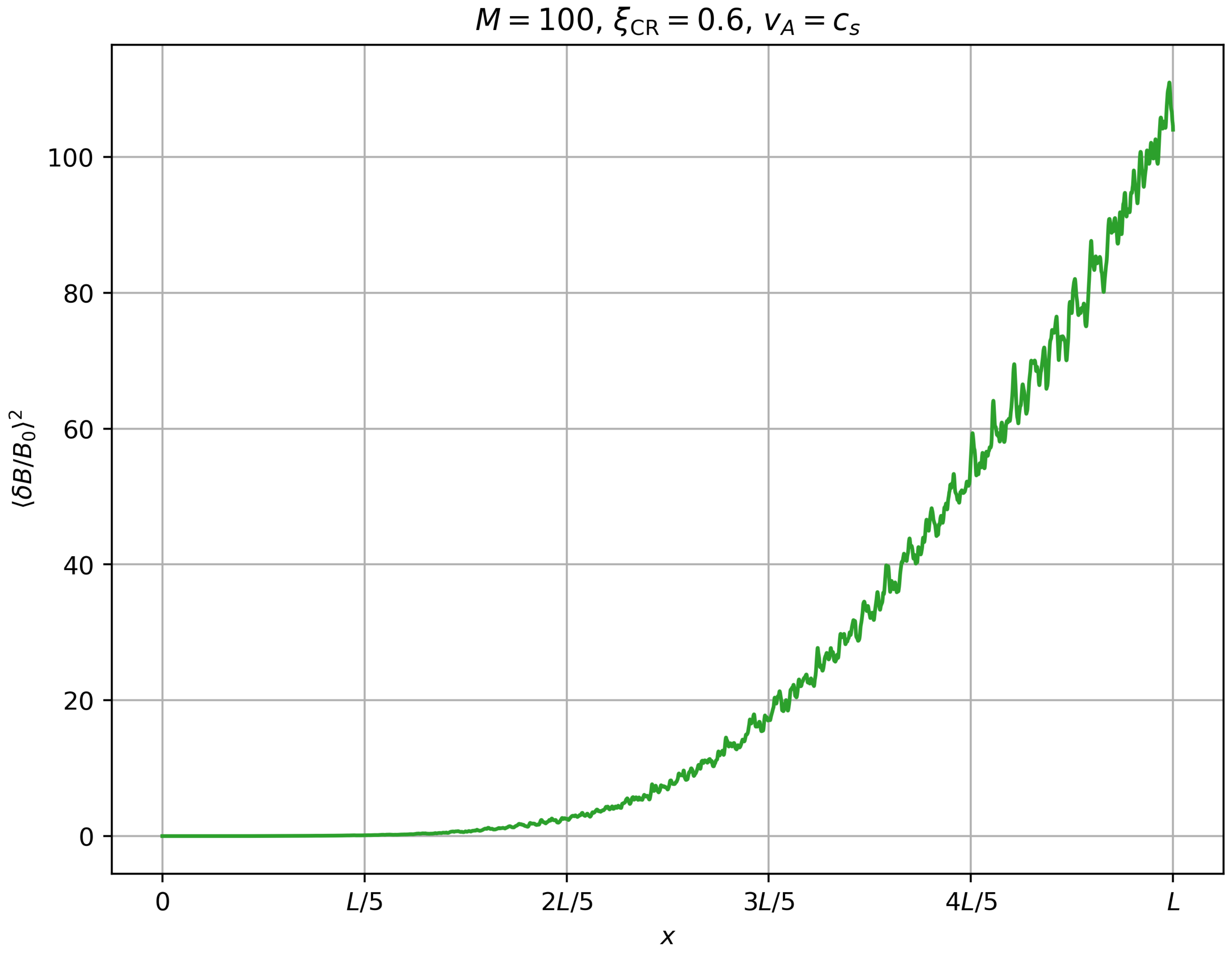


Maximum number of e-folds of growth in one advection time $t_{adv} = L/u_0$ $= \dfrac{\xi_{CR} M}{2}$

ACOUSTIC INSTABILITY + KINEMATIC DYNAMO







MHD (PLUTO) simulations with fixed gradient

Capanema+2026
Beresnyak, Jones, & Lazarian 2009

Summary (1)

- Instabilities induced by non-thermal particles play a central role in both acceleration and transport

- For fast shocks (faster than about 2000 km/s) the non resonant streaming instabilities dominant and leads to $\frac{\delta B^2}{4\pi} = n_{CR}(> E)E\frac{v_{shock}}{c} \approx \frac{\xi_{CR}}{\Lambda}\rho v_{shock}^2\frac{v_{shock}}{c}$

- For slower shocks the resonant instability comes into play and it is limited mainly by NLLD

- In both cases, in the absence of these instabilities, the maximum energy is exceedingly small

- Even in the presence of these instabilities it is unclear whether standard SNR can accelerate to PeV energies

- Other processes (acoustic instability + kinematic dynamo) can contribute to increasing the upstream magnetic field

SUMMARY (2)

- The transport of Galactic cosmic rays with E<1000 GeV is ruled by the excitation of the resonant streaming instability limited by NLLD

- Transport in regions very close to the sources may be affected by both resonant and non-resonant streaming instability

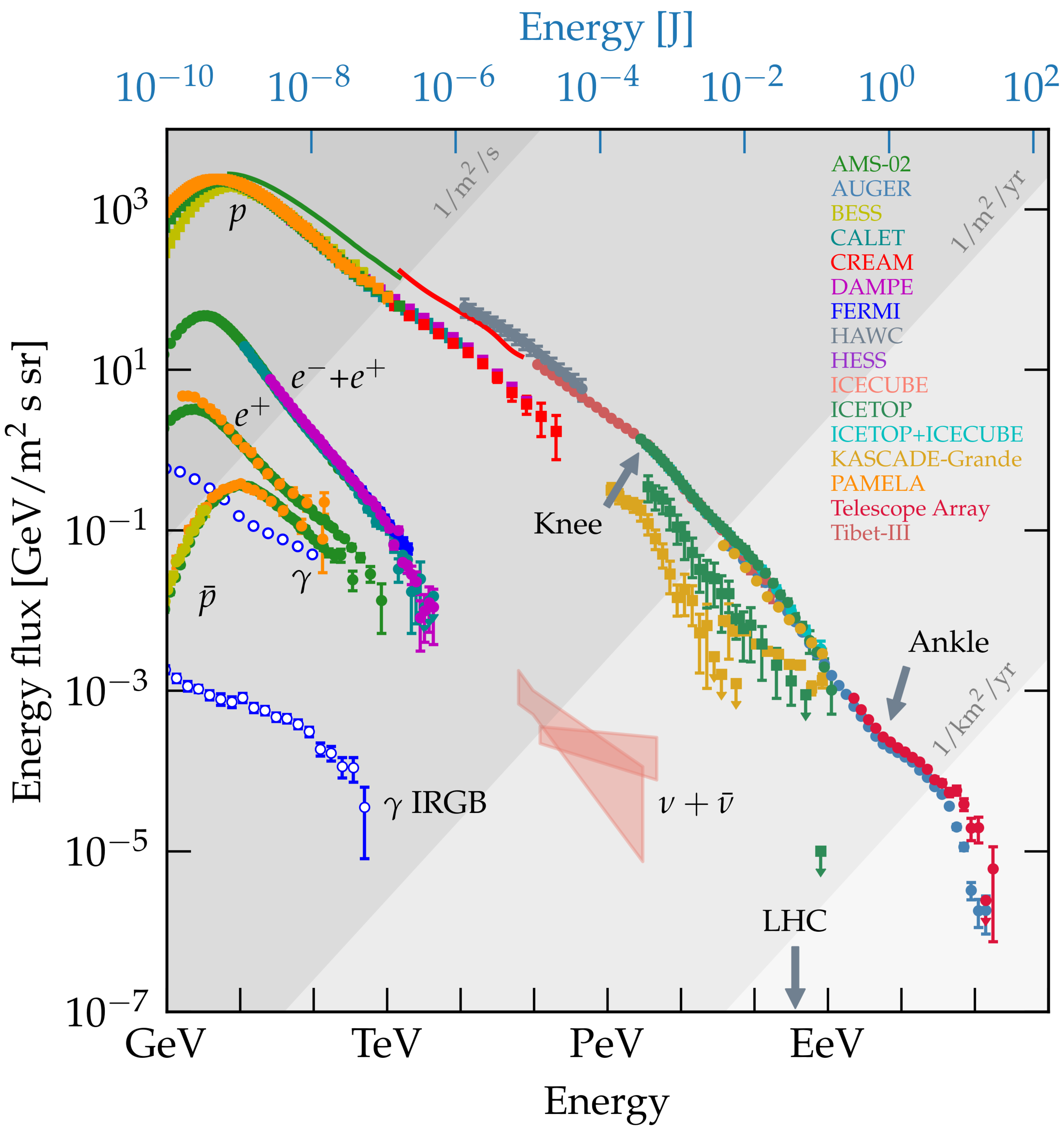
- The local confinement leads to overpressurized regions that excavate cavities in the ISM

- At energies >1000 GeV, CR scattering remains subject of much debate. The feature at 300 GV may be sign of transition from self-generated to external turbulence (PB+2012)

- On global Galactic scales, the self-confinement of CR leads to pressure gradients that shape winds and outflows

Additional Slides

Cosmic Rays - Spectrum



- We now measure with good accuracy the spectra of individual primary components (p, He, C, …) as well as secondary (Be, B, antiprotons, positrons, …)
- The all-particle spectrum shows a marked knee at about 2 PeV and a clear suppression at 10^{20} eV
- At the knee there is also evidence for a transition from lighter to heavier nuclei
- The generally accepted view is that CR remain Galactic in origin up to the so-called second knee (10^{17} eV) where extragalactic CR kick in
- *It is not yet clear whether the suppression at the end of the spectrum reflects an intrinsic maximum energy or rather photo disintegration of nuclei*

Cosmic Rays - Mass Composition

DAMPE 2025

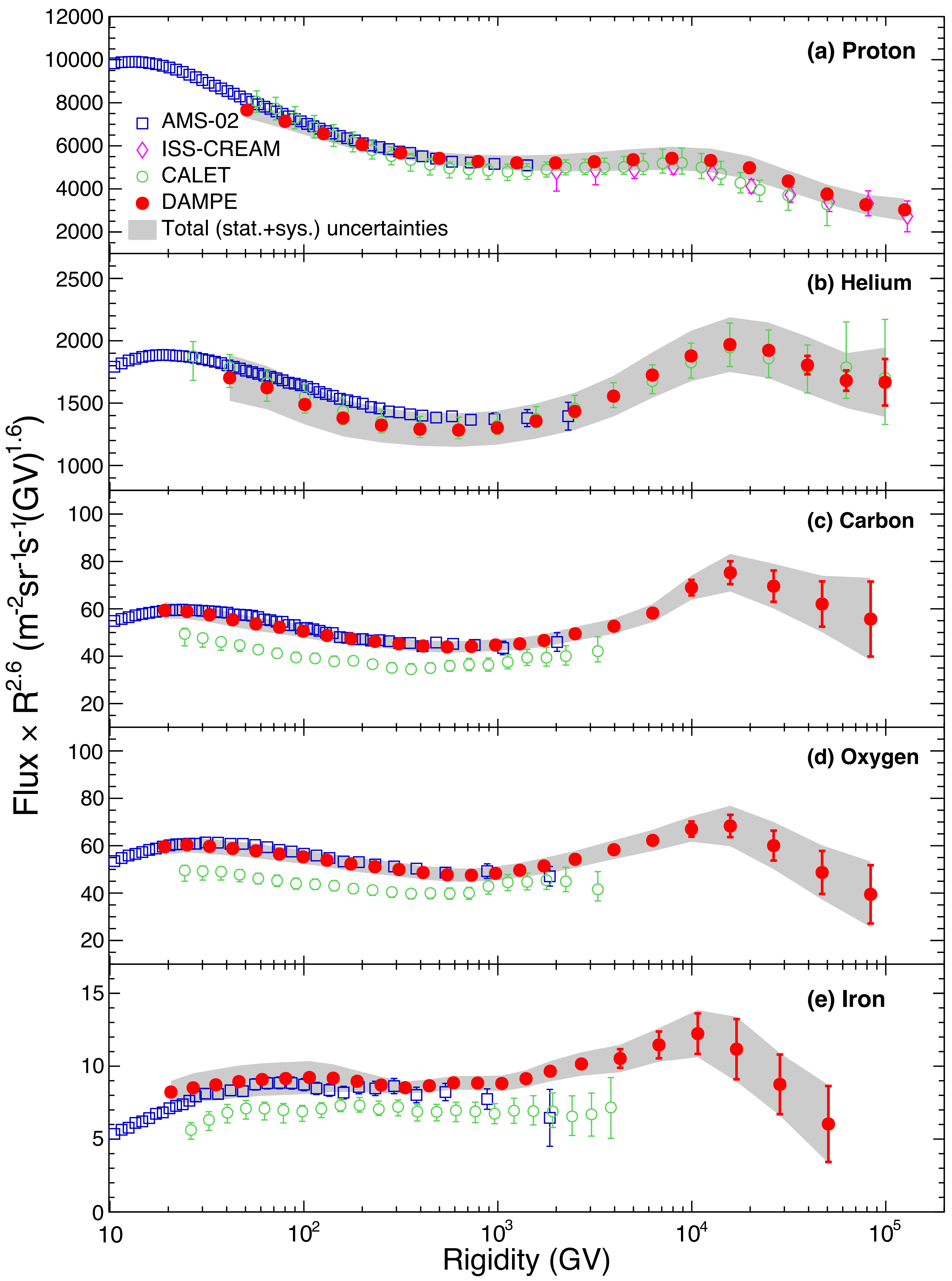
- Virtually all spectra of predominantly primary nuclei show a hardening at rigidity of about 200 GV

- All nuclei show a flux reduction at a rigidity of about 20 TV

- After accounting for transport effects it appears that protons (H nuclei) need to be injected at the sources with spectra steeper than He

- The source spectra of nuclei are all the same but somewhat steeper than He

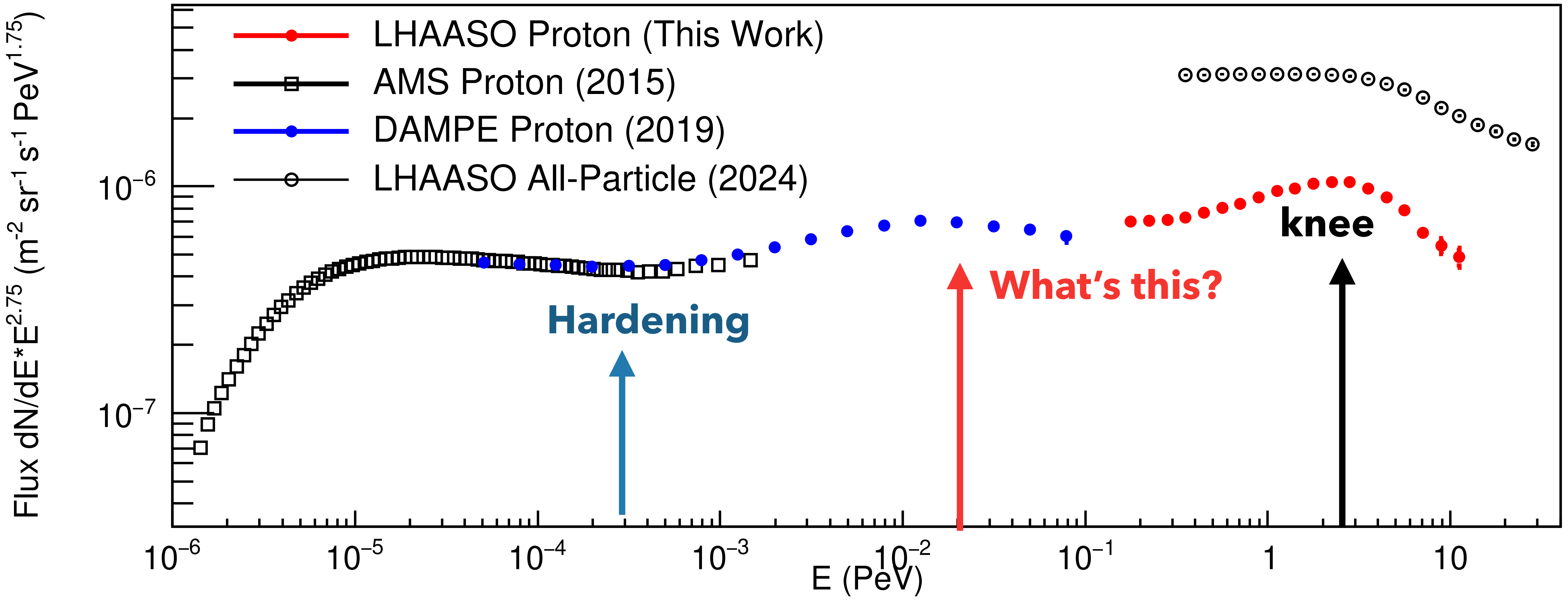
- These findings are difficult to reconcile with the rigidity dependence of both the acceleration and transport processes which is in fact observed

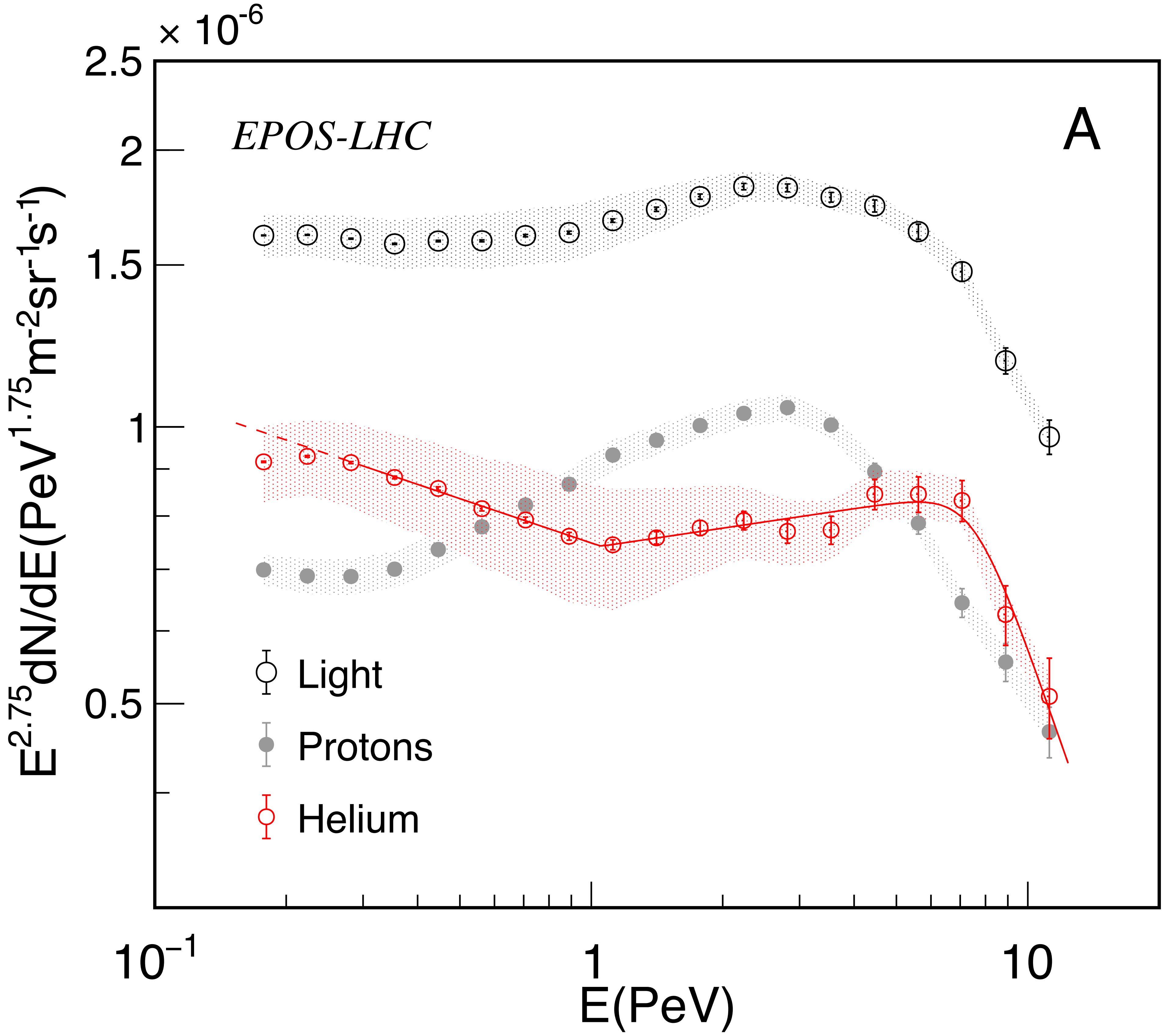


THE KNEE

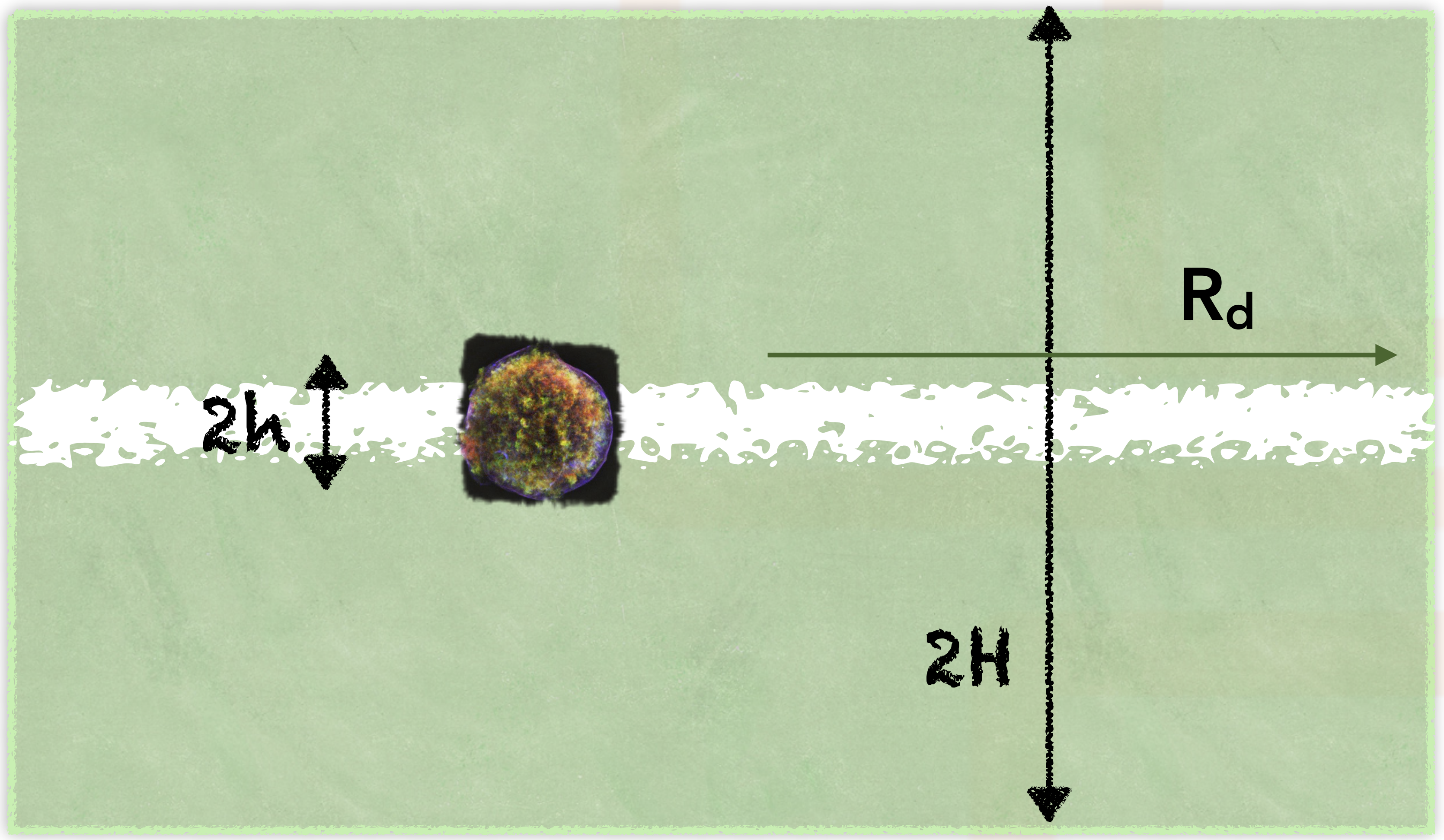
LHAASO 2025

- According to the LHAASO recent measurement, the region of the knee is dominated by the light CR component (p+He)
- The proton component seems to cut at rigidity about 1/2 as for that of He nuclei, consistent with rigidity effects (either acceleration or transport)





A TOY MODEL FOR PROTONS IN OUR GALAXY



HALO ~ several kpc

DISC ~ 300 pc

Assumptions of the model:

1. *CR are injected in an infinitely thin disc*
2. *CR diffuse in the whole volume*
3. *CR freely escape from a boundary*

1 $$Q(p,z) = \frac{Q_0(p)}{\pi R_d^2}\delta(z)$$

2 $$-\frac{\partial}{\partial z}\left[D\frac{\partial f}{\partial z}\right] = Q(p,z)$$

3 $$f(z=H,p) = 0$$

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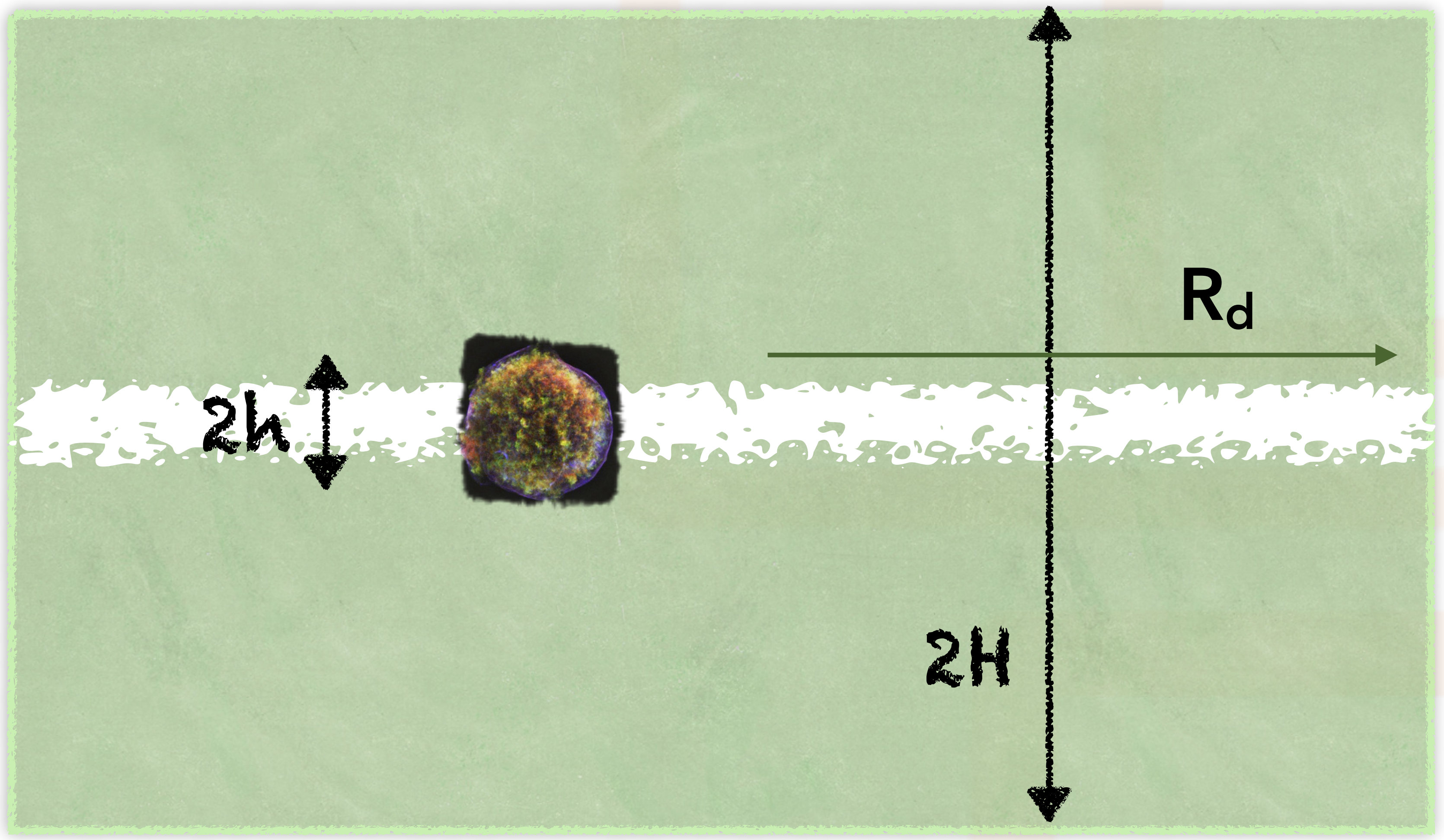

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$$D\frac{\partial f}{\partial z} = Constant \rightarrow f(z) = f_0\left(1-\frac{z}{H}\right)$$

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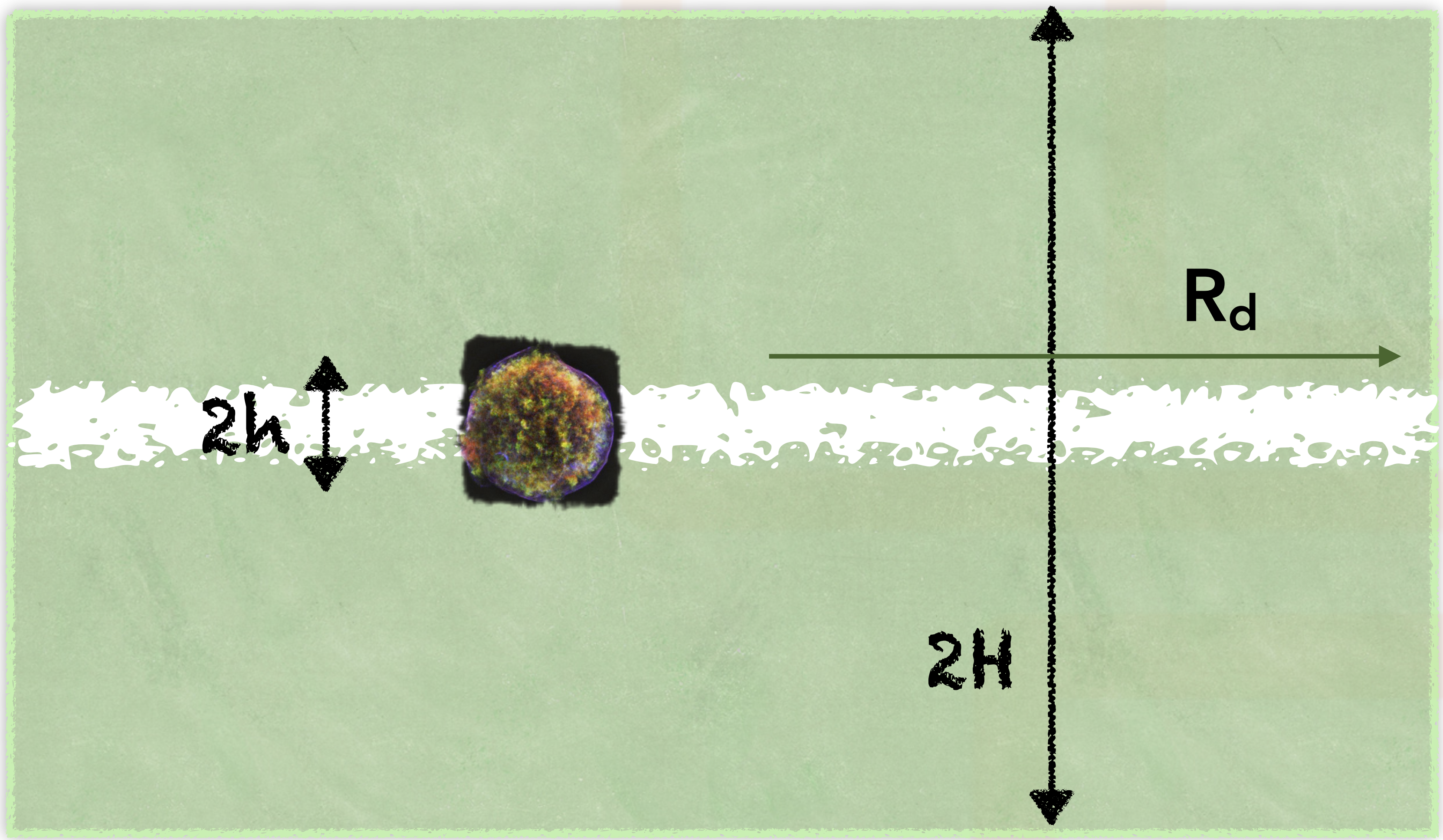

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$$D\frac{\partial f}{\partial z}|_{z=0^+} = -D\frac{f_0}{H}$$

A TOY MODEL FOR OUR GALAXY

Let us now integrate the diffusion equation around z=0

$$-\frac{\partial}{\partial z}\left[D\frac{\partial f}{\partial z}\right] = \frac{Q_0(p)}{\pi R_d^2}\delta(z) \quad \Rightarrow \quad -2D\frac{\partial f}{\partial z}|_{z=0^+} = \frac{Q_0(p)}{\pi R_d^2}$$

and recalling that

$$D\frac{\partial f}{\partial z}|_{z=0^+} = -D\frac{f_0}{H} \quad \Rightarrow \quad f_0(p) = \frac{Q_0(p)}{2\pi R_d^2}\frac{H}{D} = \frac{Q_0(p)}{2\pi R_d^2 H}\frac{H^2}{D}$$

Diffusion Time

Rate of injection per unit volume

Since $Q_0(p) \sim p^{-\gamma}$ and $D(p) \sim p^{\delta}$
$f_0(p) \sim p^{-\gamma-\delta}$

A TOY MODEL FOR OUR GALAXY: escape flux

WHICH CR FLUX WOULD BE MEASURED BY AN OBSERVER OUTSIDE OUR GALAXY?

WE ALREADY ESTABLISHED THAT

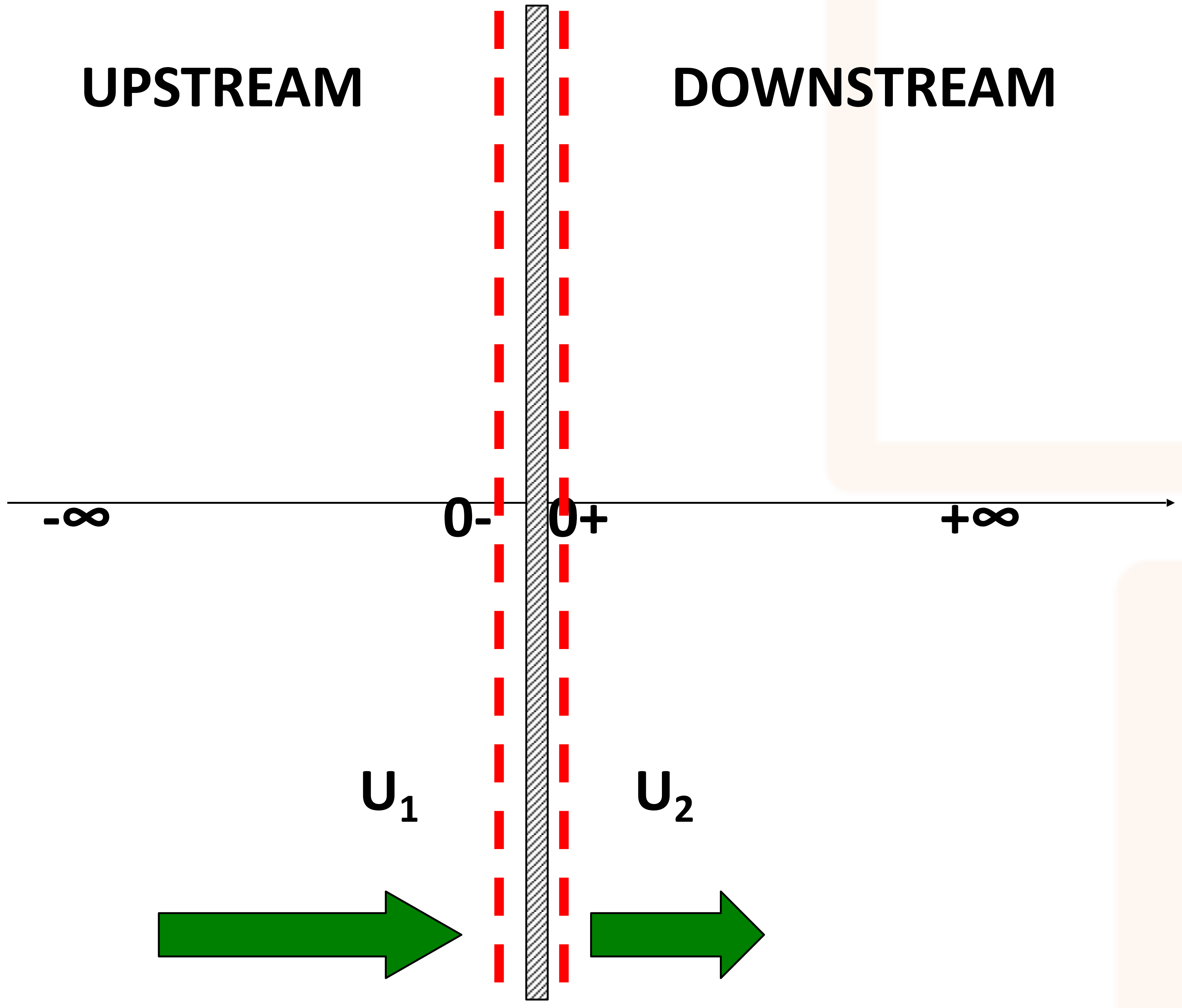
$$D\frac{\partial f}{\partial z} = constant$$

BUT THIS IS EXACTLY THE FLUX ACROSS A SURFACE IN DIFFUSIVE REGIME:

$$\Phi_{esc}(p) = -D\frac{\partial f}{\partial z}|_{z=H} = -D\frac{\partial f}{\partial z}|_{z=0^+} = \frac{Q_0(p)}{2\pi R_d^2}$$

THE SPECTRUM OF COSMIC RAYS OBSERVED BY AN OBSERVER OUTSIDE OUR GALAXY IS THE SAME AS INJECTED BY SOURCES, NOT THE SAME AS WE MEASURE AT THE EARTH!

A SIMPLE APPLICATION:
THE TRANSPORT EQUATION APPROACH TO DSA



$$\frac{\partial f_0}{\partial t} + u\frac{\partial f_0}{\partial z} - \frac{1}{3}\left(\frac{du}{dz}\right)p\frac{\partial f_0}{\partial p} = \frac{\partial}{\partial z}\left[D\frac{\partial f_0}{\partial z}\right]$$

Advection Compression Diffusion

FOR A PLANE PARALLEL SHOCK:

$$\frac{du}{dz} = (u_2 - u_1)\delta(z)$$

LET US ALSO ASSUME STATIONARITY!

It can be argued that downstream of the shock the only solution is the one for which:

$$f(x,p) = \text{constant} = \mathrm{f_0(p)}$$

$$D\frac{\partial f}{\partial z}|_{z\to 0^+} = 0$$

UPSTREAM solution

LET US ASSUME STATIONARITY (LATER WE SHALL DISCUSS IMPLICATIONS)

IN THE UPSTREAM THE EQUATION READS

$$\frac{\partial}{\partial z}\left[D\frac{\partial f}{\partial z}\right] - u\frac{\partial f}{\partial z} = 0 \quad \rightarrow \quad \frac{\partial}{\partial z}\left[D\frac{\partial f}{\partial z} - uf\right] = 0$$

FLUX IS CONSERVED!

THE SOLUTION THAT HAS VANISHING f AND VANISHING DERIVATIVE AT UPSTREAM INFINITY IS

$$f(x,p) = f_0 \exp\left[\frac{u_1 z}{D}\right] \quad \rightarrow \quad D\frac{\partial f}{\partial z}|_{z\rightarrow 0^-} = u_1 f_0(p)$$

AROUND THE SHOCK

INTEGRATING THE TRANSPORT EQUATION IN A NARROW NEIGHBORHOOD OF THE SHOCK WE GET

$$D\frac{\partial f}{\partial z}|_{z\to 0^+} - D\frac{\partial f}{\partial z}|_{z\to 0^-} + \frac{1}{3}(u_2 - u_1)p\frac{df_0}{dp} = 0$$

WHERE WE USED $du/dz=(u_2-u_1)\boldsymbol{\delta}(z)$

REPLACING THE EXPRESSIONS FOR THE DERIVATIVES DERIVED BEFORE:

$$-u_1 f_0(p) + \frac{1}{3}(u_2 - u_1)p\frac{df_0}{dp} = 0 \qquad p > p_{inj}$$

WHICH HAS THE SOLUTION:

$$f_0(p) = Kp^{-\alpha} \qquad \alpha = \frac{3u_1}{u_1 - u_2} = \frac{3r}{r-1}$$